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# Structural Depth Reference Manual for the Civil PE Exam

Third Edition

Alan Williams, PhD, SE, FICE, C Eng



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## STRUCTURAL DEPTH REFERENCE MANUAL FOR THE CIVIL PE EXAM Third Edition

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# Preface and Acknowledgments

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The structural depth section of the civil PE exam requires detailed understanding of engineering codes and their application. My goal in writing this book has been to provide a guide to the relevant codes, and to demonstrate their use in calculations for structures as necessary to help you pass the civil PE exam.

Different from the previous editions, this third edition has been revised and expanded to reflect the most current NCEES design standards. (For a complete list of updated codes, see References and Codes Used to Prepare This Book.) Nomenclature, equations, examples, and practice problems have been checked and updated so that they are congruent with current codes. New text has also been added to sections previously dependent on older editions of exam-referenced codes.

I would like to say thank you to C. Dale Buckner, PhD, PE, for performing the initial technical review of this book, and to the PPI product development and implementation staff who worked on the third edition, including Sarah Hubbard, director of product development and implementation; Cathy Schrott, director of production services; Megan Synnestvedt, Jenny Lindeburg King, and Julia White, editorial project managers; Bill Bergstrom, Lisa Devoto Farrell, Tyler Hayes, Chelsea Logan, Scott Marley, Magnolia Molcan, and Bonnie Thomas, copy editors; Kate Hayes, production associate; Andrew Chan and Todd Fisher, calculation checkers; Tom Bergstrom, technical illustrator; and Amy Schwertman La Russa, cover designer.

Alan Williams, PhD, SE, C Eng, FICE

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# Introduction

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This book addresses the structural depth section of the civil PE examination administered by the National Council of Examiners for Engineering and Surveying (NCEES). The civil PE examination provides the qualifying test for candidates seeking registration as civil engineers. The structural section of the examination is intended to assess the candidate's depth of knowledge of structural design principles and practice. The problems in the examination are intended to be representative of the process of designing portions of real structures.

This book is written with the exam in mind. The national codes are referenced, and the appropriate sections of the codes are explained and analyzed in concise and simple manners. Illustrative examples are also presented. Each example focuses on one specific code issue. This book provides a comprehensive guide and reference for self-study of the subject, and it supplies a rapid and concise solution technique for any particular problem type. In addition to providing clarification and interpretation of the applicable sections of the codes, extensive reference publications are cited to reflect current design procedures.

The text is organized into six chapters, corresponding to the subject areas covered on the NCEES civil PE structural depth exam.

- reinforced concrete design
- foundations and retaining structures
- prestressed concrete design
- structural steel design
- timber design
- masonry design

The American Concrete Institute has published *Building Code Requirements for Structural Concrete*, *ACI 318-08*, and this edition of the code has been adopted by the NCEES for their examinations. The first three chapters of this book conform to the 2008 edition of the ACI code.

For questions involving structural steel design, the candidate may choose to use either the allowable stress design (ASD) method or the load and resistance factor (LRFD) method. In Ch. 4 of this book, both methods have been used. The NCEES has adopted the *Steel Construction Manual*, 13th ed. 2005, published by the American Institute of Steel Construction (AISC). Chapter 4 of this book conforms to the 2005 edition of the *AISC Steel Construction Manual*.

The American Forest and Paper Association has published the *National Design Specification for Wood Construction*, *ANSI/NF & PA NDS-2005*, and this edition of the code has been adopted by the NCEES for their examinations. Chapter 5 of this book conforms to the 2005 edition of the NDS code.

The American Concrete Institute has published the *Building Code Requirements for Masonry Structures*: *ACI 530-08/ASCE 5-08/TMS 402-08*, and this edition of the code has been adopted by the NCEES for their examinations. Chapter 6 of this book conforms to the 2008 edition of the ACI code.

The NCEES has also adopted the *International Building Code*, *IBC-2009*, published by the International Code Council, and *Minimum Design Loads for Buildings and Other Structures*, *ASCE 7-05*, published by the American Society of Civil Engineers, and all chapters of this book conform to these codes.

Abbreviations are used throughout this book to refer to common reference sources. These sources are listed in the References and Codes section, with their abbreviations in brackets. This book also cites other publications that discuss current design procedures. These other publications are also listed in the References and Codes section.

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# References and Codes Used to Prepare This Book

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The information that was used to write and update this book was based on the exam specifications at the time of publication. However, as with engineering practice itself, the PE examination is not always based on the most current codes or cutting-edge technology. Similarly, codes, standards, and regulations adopted by state and local agencies often lag issuance by several years. It is likely that the codes that are most current, the codes that you use in practice, and the codes that are the basis of your exam will all be different. PPI lists on its website the dates and editions of the codes, standards, and regulations on which NCEES has announced the PE exams are based. It is your responsibility to find out which codes are relevant to your exam. In the meantime, the following codes have been incorporated into this edition. The list of references are those that were used in the writing of this book, and may prove helpful in your own exam preparation.

## CODES

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American Concrete Institute. *Building Code Requirements for Structural Concrete*, (ACI 318-08). 2008. Farmington Hills, MI. [ACI]

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# 1 Reinforced Concrete Design

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## 1. STRUT-AND-TIE MODELS

### Nomenclature

|          |                                                                                                                  |                      |
|----------|------------------------------------------------------------------------------------------------------------------|----------------------|
| $a$      | shear span, distance between concentrated load and face of support                                               | ft                   |
| $A_{cs}$ | effective cross-sectional area of a strut in a strut-and-tie model, taken perpendicular to the axis of the strut | in <sup>2</sup>      |
| $A_{nz}$ | effective cross-sectional area of the face of a nodal zone                                                       | in <sup>2</sup>      |
| $A_{si}$ | area of reinforcement in the $i^{\text{th}}$ layer of reinforcement crossing the strut                           | in <sup>2</sup>      |
| $A_{ts}$ | total area of nonprestressed reinforcement in a tie                                                              | in <sup>2</sup>      |
| $A_v$    | area of shear reinforcement perpendicular to tension reinforcement within a distance $s$                         | in <sup>2</sup>      |
| $A_{vh}$ | area of shear reinforcement parallel to tension reinforcement within a distance $s$                              | in <sup>2</sup>      |
| $b$      | width of member                                                                                                  | in                   |
| $b_w$    | web width                                                                                                        | in                   |
| $c$      | clear cover to reinforcement                                                                                     | in                   |
| $C$      | compressive force acting on a nodal zone                                                                         | kips                 |
| $d$      | distance from extreme compression fiber to centroid of longitudinal tension reinforcement                        | in                   |
| $d_b$    | nominal diameter of bar, wire, or prestressing strand                                                            | in                   |
| $f'_c$   | compressive strength of concrete                                                                                 | lb/in <sup>2</sup>   |
| $f_{ce}$ | effective compressive strength of the concrete in a strut                                                        | kips/in <sup>2</sup> |
| $F_{ns}$ | nominal compressive strength of a strut                                                                          | kips                 |
| $F_{nt}$ | nominal tensile strength of a tie                                                                                | kips                 |
| $h$      | depth of member                                                                                                  | in                   |
| $l_a$    | anchorage length of a reinforcing bar                                                                            | in                   |
| $l_b$    | width of bearing plate                                                                                           | in                   |
| $l_{dh}$ | development length in tension of a hooked bar                                                                    | in                   |
| $R$      | support reaction acting on a nodal zone                                                                          | kips                 |
| $s$      | stirrup spacing                                                                                                  | in                   |
| $s_i$    | spacing of the $i^{\text{th}}$ layer of reinforcement                                                            | in                   |
| $T$      | tension force acting on a nodal zone                                                                             | kips                 |

|       |                                                                 |    |
|-------|-----------------------------------------------------------------|----|
| $w_s$ | effective width of strut perpendicular to the axis of the strut | in |
| $w_t$ | effective width of concrete concentric with a tie               | in |

### Symbols

|            |                                                                                                                                           |     |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------|-----|
| $\beta_n$  | factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone                           | —   |
| $\beta_s$  | factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut | —   |
| $\alpha_i$ | angle between the strut and the bars in the $i^{\text{th}}$ layer of reinforcement crossing the strut                                     | deg |
| $\theta$   | angle between the axis of a strut and a tension chord                                                                                     | deg |
| $\lambda$  | correction factor related to unit weight of concrete                                                                                      | —   |
| $\rho$     | reinforcement ratio                                                                                                                       | —   |
| $\phi$     | strength reduction factors                                                                                                                | —   |

A strut-and-tie model consists of the application of an analogous truss model to regions of a member in which the stress distribution is nonuniform and the normal beam theory does not apply. An example of this situation is a deep beam, which may be designed using nonlinear analysis methods or by the strut-and-tie analogy method given in ACI 318. A deep beam is defined in ACI Sec. 11.7.1 as a beam in which the ratio of clear span to overall depth does not exceed four, or in which the shear span to depth ratio does not exceed two.

As shown in Fig. 1.1, a discontinuity, or D region, occurs at a change in the geometry of a member and at concentrated loads and reactions. The D region extends a distance equal to the overall depth of the member from the location of the change in geometry or the point of application of a load or reaction. Outside of the D region, beam theory is applicable, and this region is known as a beam, or B, region.

Typical examples of strut-and-tie models are shown in Fig. 1.2. Several possible solutions may exist for a given example. Compressive forces are resisted by concrete struts and tensile forces are resisted by steel ties. Struts, ties, and external forces meet at nodes. Tensile stresses in the concrete are neglected, and struts should be oriented parallel to the directions of anticipated cracking. Forces in struts and ties are uniaxial, and equilibrium must be maintained at the nodes and in the truss



Figure 1.1 Discontinuity Regions

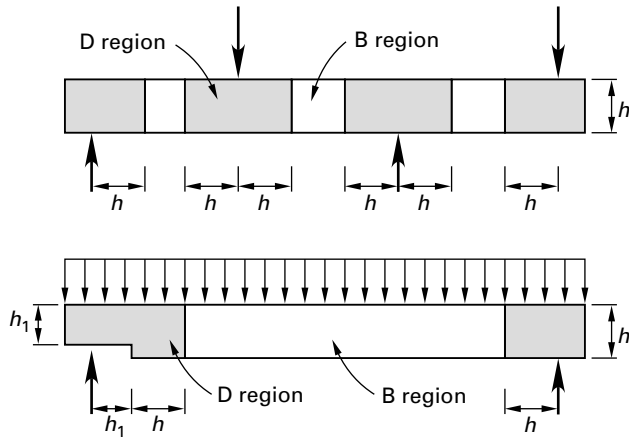
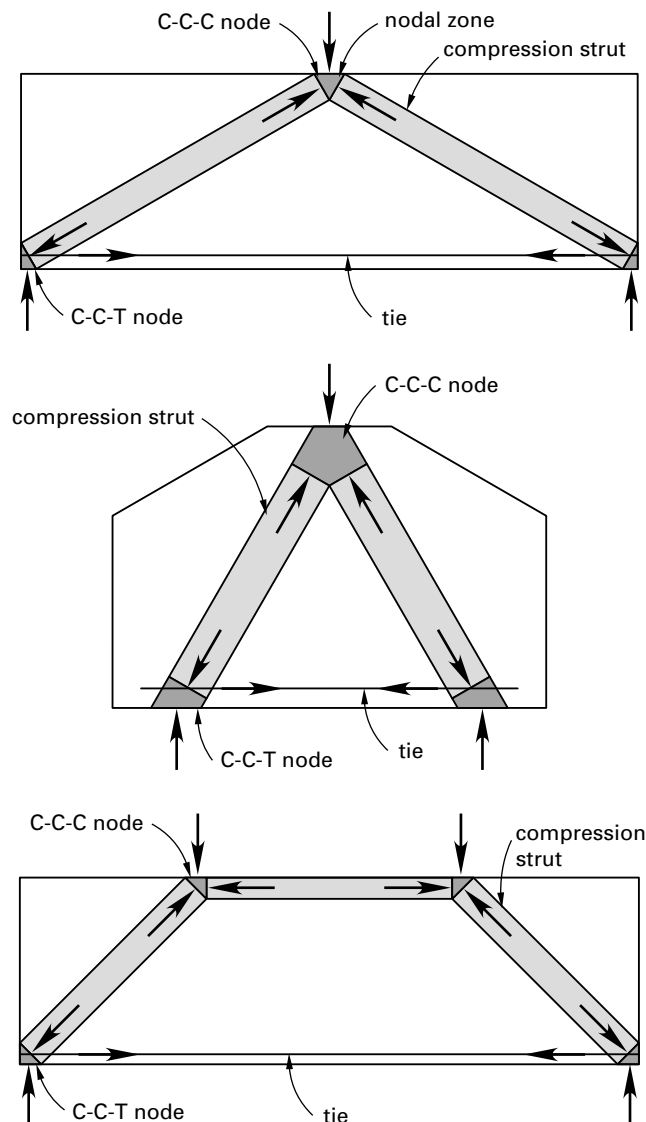


Figure 1.2 Typical Strut-and-Tie Model Examples



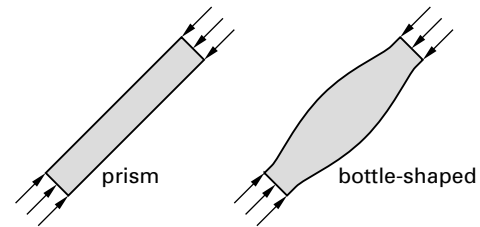
as a whole. In setting up the strut-and-tie model, the following procedure may be followed.

- Determine the reactions on the model.
- Select the location of the members by aligning the direction of struts in the direction of the anticipated cracking.
- Determine the areas of struts, ties, and nodes necessary to provide the required strength.
- Provide anchorage for the ties.
- Provide crack control reinforcement.

The two types of concrete struts (see Fig. 1.3) that may form in the model are the prism strut and the bottle-shaped strut. The prism strut has a uniform cross section and forms in the compression zone of a beam. A bottle-shaped strut expands and contracts along its length. The strength of this strut is governed by the transverse tension developed by the lateral spread of the applied compression force. As specified in ACI Sec. A.3.3, using two orthogonal layers of confinement reinforcement near each face to resist the transverse tension increases the strength of the strut. The required reinforcement ratio is

$$\sum \frac{A_{si}}{bs_i} \sin \alpha_i \geq 0.003 \quad [\text{ACI A-4}]$$

Figure 1.3 Types of Concrete Struts



The effective compressive strength of the concrete in a strut is specified in ACI Sec. A.3.2 as

$$f_{ce} = 0.85\beta_s f'_c \quad [\text{ACI A-3}]$$

$\beta_s$  is the factor, given in Table 1.1, that accounts for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut.

In accordance with ACI Sec. A.3.1, the nominal compressive strength of a strut is

$$\begin{aligned} F_{ns} &= f_{ce} A_{cs} \quad [\text{ACI A-2}] \\ &= f_{ce} w_s b \end{aligned}$$

Ties are the tension members of the strut-and-tie model that are formed by reinforcing or prestressing steel. They may consist of longitudinal chord reinforcement

**Table 1.1** Values of  $\beta_s$  and  $\lambda$ 

| $\beta_s$ | type of strut                                                          |
|-----------|------------------------------------------------------------------------|
| 1.0       | strut of uniform cross section                                         |
| 0.75      | bottle-shaped strut, with reinforcement as specified in ACI Sec. A.3.3 |
| 0.60      | unreinforced, bottle-shaped strut                                      |
| 0.40      | strut in a tension member or the tension flange of a member            |
| 0.60      | all other cases                                                        |
| $\lambda$ | weight of concrete                                                     |
| 1.0       | normal                                                                 |
| 0.85      | sand-lightweight                                                       |
| 0.75      | all lightweight                                                        |

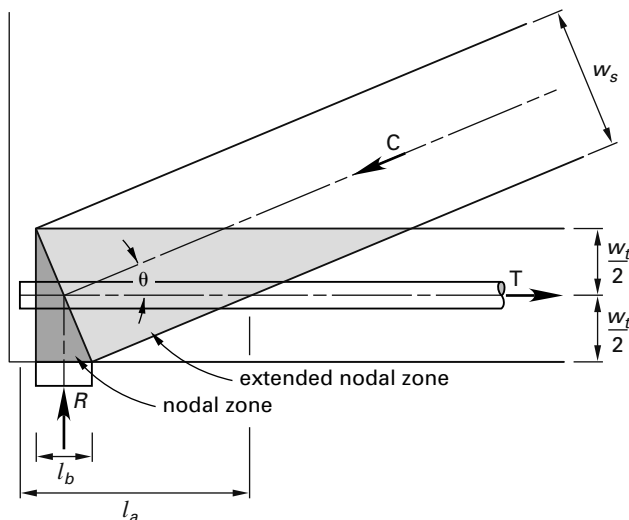
or vertical stirrups. Adequate anchorage must be provided to the reinforcement by means of end plates or hooks, or by a straight development length. In a reinforced concrete beam, the nominal strength of a reinforcing bar acting as a tie is given by ACI Sec. A.4.1 as

$$F_{nt} = A_{ts}f_y$$

If the bars in a tie are in one layer, as shown in Fig. 1.4, the width of the tie may be taken as the diameter of the bars, plus twice the cover to the surface of the bars.

A node is the location at which struts, ties, and external loads meet. As shown in Fig. 1.2, a node may be classified as C-C-T, with two of the members acting on the node in compression and the third member in tension. Similarly, when all three members acting on the node are in compression, the node is classified as C-C-C.

A nodal zone, as defined in ACI Sec. A.1 and shown in Fig. 1.4, is the volume of concrete surrounding a node that is assumed to transfer strut-and-tie forces through

**Figure 1.4** Nodal Zones


the node. The effective compressive strength of the concrete in a nodal zone is specified in ACI Sec. A.5.2 as

$$f_{ce} = 0.85\beta_n f'_c \quad [\text{ACI A-8}]$$

$\beta_n$  is the factor, given in Table 1.2, that accounts for the effect of anchoring ties on the effective compressive strength of a nodal zone.

**Table 1.2** Values of  $\beta_n$ 

| $\beta_n$ | type of nodal zone                                        |
|-----------|-----------------------------------------------------------|
| 1.0       | bounded on all sides by struts, or bearing areas, or both |
| 0.80      | anchoring one tie                                         |
| 0.60      | anchoring two or more ties                                |

In accordance with ACI Sec. A.5.1, the nominal compressive strength of a nodal zone is

$$\begin{aligned} F_{nn} &= f_{ce}A_{nz} \quad [\text{ACI A-7}] \\ &= f_{ce}w_s b \end{aligned}$$

The faces of the nodal zone shown in Fig. 1.4 are perpendicular to the axes of the strut, tie, and bearing plate. The lengths of the faces are in direct proportion to the forces acting. Hence, the node has equal stresses on all faces and is termed a *hydrostatic nodal zone*. The effective width of the strut shown in Fig. 1.4 is

$$w_s = w_t \cos \theta + l_b \sin \theta$$

The extended nodal zone shown in Fig. 1.4 is that portion of the member bounded by the intersection of the effective strut width and the effective tie width. As specified in ACI Sec. A.4.3.2, the anchorage length of the reinforcement is measured from the point of intersection of the bar and the extended nodal zone. The reinforcement may be anchored by a plate, by hooks, or by a straight development length.

To control cracking in a deep beam, ACI Secs. 11.7.4 and 11.7.5 require the provision of two orthogonal layers of confinement reinforcement near each face. The maximum reinforcement spacing in each layer is

$$s \leq \frac{d}{5} \leq 12 \text{ in}$$

The area of horizontal reinforcement must not be less than

$$A_{vh} = 0.0015b_w s$$

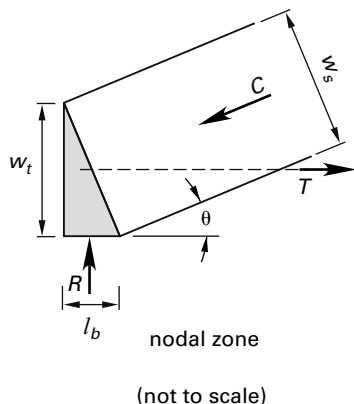
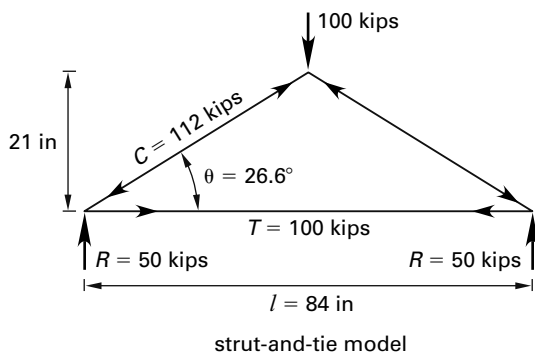
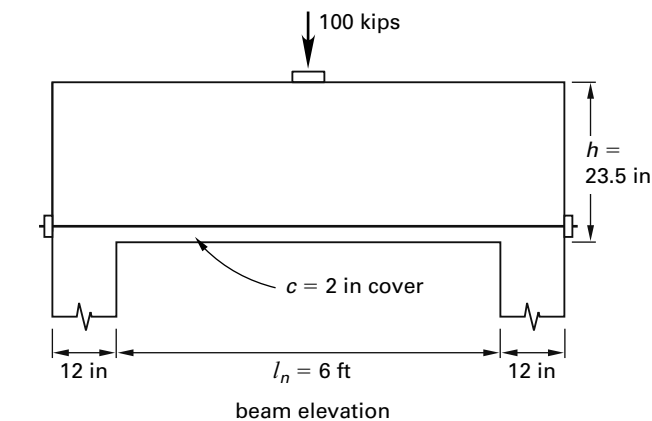
The area of vertical reinforcement must not be less than

$$A_v = 0.0025b_w s$$

Alternatively, the reinforcement specified in ACI Sec. A.3.3 may be provided.

### Example 1.1

A reinforced concrete beam with a clear span of 6 ft, an effective depth of 21 in, and a width of 12 in, as shown in the illustration, has a concrete compressive strength of 5000 lbf/in<sup>2</sup>. The factored applied force of 100 kips includes an allowance for the self-weight of the beam. Determine the area of grade 60 tension reinforcement required, and check that the equivalent concrete strut and nodal zone at the left support comply with the requirements of ACI App. A.



### Solution

The clear span to depth ratio is

$$\frac{l_n}{h} = \frac{(6 \text{ ft}) \left( \frac{12 \text{ in}}{\text{ft}} \right)}{23.5 \text{ in}} = 3.1$$

$$< 4 \quad [\text{satisfies ACI Sec. 11.7.1 as a deep beam}]$$

The idealized strut-and-tie model is shown in the illustration. The angle between the struts and the tie is

$$\theta = \tan^{-1} \left( \frac{21 \text{ in}}{42 \text{ in}} \right)$$

$$= 26.6^\circ$$

$$> 25^\circ \quad [\text{satisfies ACI Sec. A.2.5}]$$

The equivalent tie force is determined from the strut-and-tie model as

$$T = \frac{(50 \text{ kips})(42 \text{ in})}{21 \text{ in}}$$

$$= 100 \text{ kips}$$

The strength reduction factor is given by ACI Sec. 9.3.2.6 as

$$\phi = 0.75$$

The necessary reinforcement area is

$$A_{ts} = \frac{T}{\phi f_y}$$

$$= \frac{100 \text{ kips}}{(0.75) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)}$$

$$= 2.22 \text{ in}^2$$

Use three no. 8 bars, which gives

$$A = 2.37 \text{ in}^2$$

$$> A_{ts} \quad [\text{satisfactory}]$$

As shown in the illustration, the dimensions of the nodal zone are  $w_t$ ,  $l_b$ , and  $w_s$ . The equivalent tie width is

$$w_t = d_b + 2c$$

$$= 1 \text{ in} + (2)(2 \text{ in})$$

$$= 5 \text{ in}$$

The width of the equivalent support strut is

$$l_b = w_t \tan \theta$$

$$= (5 \text{ in}) \tan 26.6^\circ$$

$$= 2.5 \text{ in}$$

The width of the equivalent concrete strut is

$$\begin{aligned} w_s &= \frac{w_t}{\cos \theta} \\ &= \frac{5 \text{ in}}{\cos 26.6^\circ} \\ &= 5.59 \text{ in} \end{aligned}$$

The stress in the equivalent tie is

$$\begin{aligned} f_T &= \frac{T}{bw_t} \\ &= \frac{100 \text{ kips}}{(12 \text{ in})(5 \text{ in})} \\ &= 1.67 \text{ kips/in}^2 \end{aligned}$$

For a hydrostatic nodal zone, the stress in the equivalent concrete strut is

$$\begin{aligned} f_C &= f_T \\ &= 1.67 \text{ kips/in}^2 \end{aligned}$$

The stress in the equivalent support strut is

$$\begin{aligned} f_R &= f_T \\ &= 1.67 \text{ kips/in}^2 \end{aligned}$$

For normal weight concrete and an unreinforced bottle-shaped strut, the design compressive strength of the concrete in the strut is given by ACI Eq. (A-3) as

$$\begin{aligned} \phi f_{ce} &= (0.75)(0.85\beta_s f'_c) \\ &= (0.75)(0.85)(0.6)(1.0) \left( 5 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 1.91 \text{ kips/in}^2 \\ &> f_C \quad [\text{satisfactory}] \end{aligned}$$

The design compressive strength of a nodal zone anchoring one layer of reinforcing bars without confining reinforcement is given by ACI Eq. (A-8) as

$$\begin{aligned} \phi f_{ce} &= (0.75)(0.85\beta_n f'_c) \\ &= (0.75)(0.85)(0.8) \left( 5 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 2.55 \text{ kips/in}^2 \\ &> f_C \quad [\text{satisfactory}] \end{aligned}$$

The anchorage length available for the tie reinforcement, using 2 in end cover, is

$$\begin{aligned} l_a &= \frac{w_t}{2 \tan \theta} + \frac{l_b}{2} + 6 \text{ in} - 2 \text{ in} \\ &= \frac{5 \text{ in}}{2 \tan 26.6^\circ} + \frac{2.5 \text{ in}}{2} + 6 \text{ in} - 2 \text{ in} \\ &= 10.2 \text{ in} \end{aligned}$$

The development length for a grade 60, no. 8 bar with 2.5 in side cover and 2 in end cover, and with a standard 90° hook, is given by ACI Secs. 12.5.2 and 12.5.3 as

$$\begin{aligned} l_{dh} &= \frac{(0.7)(1200)d_b}{\sqrt{f'_c}} \\ &= \frac{(0.7) \left( 1200 \frac{\text{lb}}{\text{in}^2} \right) (1.0 \text{ in})}{\sqrt{5000 \frac{\text{lb}}{\text{in}^2}}} \\ &= 11.9 \text{ in} \\ &> l_a \quad [\text{anchorage length is inadequate}] \end{aligned}$$

Use an end plate to anchor the bars.

## 2. CORBELS

### Nomenclature

|          |                                                                             |                      |
|----------|-----------------------------------------------------------------------------|----------------------|
| $a$      | shear span, distance between concentrated load and face of supports         | ft                   |
| $A_f$    | area of reinforcement in bracket or corbel resisting factored moment        | in <sup>2</sup>      |
| $A_h$    | area of closed stirrups parallel to primary tension reinforcement           | in <sup>2</sup>      |
| $A_n$    | area of reinforcement in bracket or corbel resisting tensile force $N_{uc}$ | in <sup>2</sup>      |
| $A_{sc}$ | area of primary tension reinforcement                                       | in <sup>2</sup>      |
| $A_{vf}$ | area of shear-friction reinforcement                                        | in <sup>2</sup>      |
| $b$      | width of compression face of member                                         | in                   |
| $h$      | overall thickness of member                                                 | in                   |
| $f_y$    | specified yield strength of reinforcement                                   | kips/in <sup>2</sup> |
| $M_u$    | factored moment at section                                                  | ft-kip               |
| $N_{uc}$ | factored tensile force applied at top of corbel                             | kips                 |
| $V_c$    | nominal shear strength provided by concrete                                 | kips                 |
| $V_s$    | nominal shear strength provided by shear reinforcement                      | kips                 |
| $V_u$    | factored shear force at section                                             | kips                 |

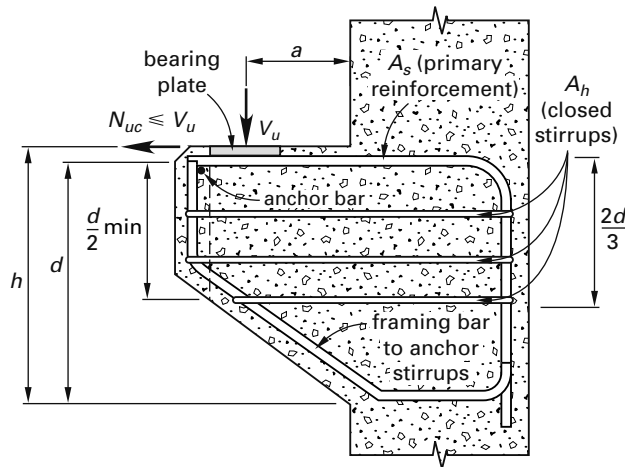
### Symbols

|           |                                                                                 |   |
|-----------|---------------------------------------------------------------------------------|---|
| $\lambda$ | correction factor related to unit weight of concrete as given in ACI Sec. 8.6.1 | — |
| $\mu$     | coefficient of friction                                                         | — |
| $\rho$    | reinforcement ratio                                                             | — |
| $\phi$    | strength reduction factor, 0.75 for shear and flexure                           | — |

As shown in Fig. 1.5, a corbel is a cantilever bracket supporting a load-bearing member. In accordance with ACI Sec. 11.8, the maximum allowable value of the ratio of the shear span to the effective depth is

$$\frac{a}{d} = 1$$

Figure 1.5 Corbel Details



Similarly, the ratio of the horizontal tensile force to the vertical force is limited to a maximum value of

$$\frac{N_{uc}}{V_u} = 1$$

As specified in ACI Sec. 11.8.2, the depth of the corbel at the outside edge of bearing area shall not be less than half the effective depth.

The forces acting on the corbel at the face of the support are the shear force  $V_u$ , the tensile force  $N_{uc}$ , and the bending moment  $M_u$ . The applied factored shear force is

$$\begin{aligned} V_u &\leq 0.2\phi f'_c b_w d \quad [\text{normal weight concrete}] \\ &\leq 0.8\phi b_w d \quad [\text{normal weight concrete}] \end{aligned}$$

From ACI Sec. 11.8.3.4, the horizontal tensile force caused by volume changes is

$$N_{uc} \geq 0.2 V_u$$

The bending moment caused by the applied shear force and the horizontal tensile force is

$$M_u = V_u a + N_{uc}(h - d)$$

The required area of shear friction reinforcement provided by the primary steel and horizontal closed stirrups is given by ACI Sec. R11.6.4.1 as

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

ACI Sec. 11.6.4.3 gives the value of the coefficient of friction at the face of the support as  $\mu = 1.4\lambda$  for concrete placed monolithically. ACI Sec. 11.6.4.3 defines the correction factor related to the unit weight of concrete as  $\lambda = 1.0$  for normal weight concrete. ACI Sec. 11.8.3.1 gives the value of the strength reduction factor as  $\phi = 0.75$  for all design calculations.

The required area of reinforcement to resist  $N_{uc}$  is given by ACI Sec. 11.8.3.4 as

$$A_n = \frac{N_{uc}}{\phi f_y}$$

The required area of flexural reinforcement,  $A_f$ , to resist the moment  $M_u$  is derived by the normal flexural theory given by ACI Sec. 10.3, using a strength reduction factor of  $\phi = 0.75$  as specified by ACI Sec. 11.8.3.1.

The required total area of primary tension reinforcement is given by ACI Sec. 11.8.3.5 as

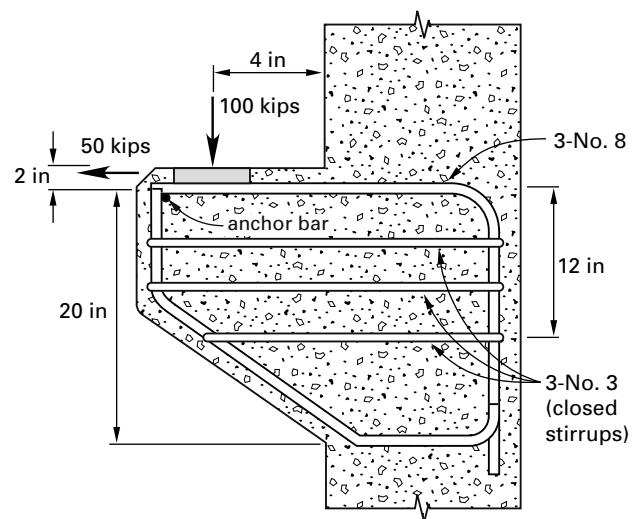
$$\begin{aligned} A_{sc} &= A_f + A_n \\ &\geq \frac{2A_{vf}}{3} + A_n \\ &\geq \frac{0.04b_w d f'_c}{f_y} \end{aligned}$$

The minimum required area of closed ties, distributed over a depth of two-thirds of the effective depth, is given by ACI Sec. 11.8.4 as

$$A_h = \frac{A_{sc} - A_n}{2}$$

### Example 1.2

The reinforced concrete corbel shown in the illustration, with a width of 15 in, is reinforced with grade 60 bars. This corbel has a concrete compressive strength of 3000 lbf/in<sup>2</sup>. Determine whether the corbel is adequate for the applied factored loads indicated.



*Solution*

$$\begin{aligned}
 0.2\phi f'_c b_w d &= (0.2)(0.75) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) (15 \text{ in})(20 \text{ in}) \\
 &= 135 \text{ kips} \\
 &> V_u \quad [\text{satisfies ACI Sec. 11.8.3.2.1}] \\
 0.8\phi b_w d &= (0.8)(0.75)(15 \text{ in})(20 \text{ in}) \\
 &= 180 \text{ kips} \\
 &> V_u \quad [\text{satisfies ACI Sec. 11.8.3.2.1}]
 \end{aligned}$$

The shear friction reinforcement area is given by ACI Sec. R11.6.4.1 as

$$\begin{aligned}
 A_{vf} &= \frac{V_u}{\phi f_y \mu} = \frac{100 \text{ kips}}{(0.75) \left( 60 \frac{\text{kips}}{\text{in}^2} \right) (1.4)} \\
 &= 1.59 \text{ in}^2
 \end{aligned}$$

The tension reinforcement area is given by ACI Sec. 11.8.3.4 as

$$\begin{aligned}
 A_n &= \frac{N_{uc}}{\phi f_y} \\
 &= \frac{50 \text{ kips}}{(0.75) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)} \\
 &= 1.111 \text{ in}^2
 \end{aligned}$$

The factored moment acting on the corbel is

$$\begin{aligned}
 M_u &= V_u a + N_{uc}(h - d) \\
 &= (100 \text{ kips})(4 \text{ in}) + (50 \text{ kips})(22 \text{ in} - 20 \text{ in}) \\
 &= 500 \text{ in-kips}
 \end{aligned}$$

For a strength reduction factor of  $\phi = 0.75$ , the area of required flexural reinforcement is given by ACI Sec. 10.2 as

$$\begin{aligned}
 A_f &= \frac{0.85 b_w d f'_c \left( 1 - \sqrt{1 - \frac{M_u}{0.319 b d^2 f'_c}} \right)}{f_y} \\
 &= \frac{(0.85)(15 \text{ in})(20 \text{ in}) \left( 3 \frac{\text{kips}}{\text{in}^2} \right)}{60 \frac{\text{kips}}{\text{in}^2}} \\
 &\quad \times \left( 1 - \sqrt{1 - \frac{500 \text{ in-kips}}{(0.319)(15 \text{ in})(20 \text{ in})^2 \left( 3 \frac{\text{kips}}{\text{in}^2} \right)}} \right) \\
 &= 0.568 \text{ in}^2
 \end{aligned}$$

The primary reinforcement area required is given by ACI Sec. 11.8.3.5 as

$$\begin{aligned}
 A_{sc} &= A_f + A_n \\
 &= 0.568 \text{ in}^2 + 1.111 \text{ in}^2 \\
 &= 1.68 \text{ in}^2
 \end{aligned}$$

Three no. 8 bars are provided, giving an area of

$$\begin{aligned}
 A_{s, \text{prov}} &= 2.37 \text{ in}^2 \\
 &> A_s \quad [\text{satisfactory}]
 \end{aligned}$$

Also, from ACI Sec. 11.8.3.5, the area of primary reinforcement must not be less than

$$\begin{aligned}
 \frac{2A_{vf}}{3} + A_n &= \frac{(2)(1.59 \text{ in}^2)}{3} + 1.111 \text{ in}^2 \\
 &= 2.17 \text{ in}^2 \\
 &< A_{s, \text{prov}} \quad [\text{satisfactory}]
 \end{aligned}$$

The required area of closed stirrups is given by ACI Sec. 11.8.4 as

$$\begin{aligned}
 A_h &= \frac{A_{sc} - A_n}{2} \\
 &= \frac{2.17 \text{ in}^2 - 1.111 \text{ in}^2}{2} \\
 &= 0.53 \text{ in}^2
 \end{aligned}$$

Three no. 3 closed stirrups are provided, giving an area of

$$\begin{aligned}
 A_{h, \text{prov}} &= 0.66 \text{ in}^2 \\
 &> A_h \quad [\text{satisfactory}]
 \end{aligned}$$

### 3. DESIGN FOR TORSION

#### Nomenclature

|           |                                                                                              |               |
|-----------|----------------------------------------------------------------------------------------------|---------------|
| $A_{cp}$  | area enclosed by outside perimeter of concrete cross section                                 | $\text{in}^2$ |
| $A_l$     | total area of longitudinal reinforcement to resist torsion                                   | $\text{in}^2$ |
| $A_o$     | gross area enclosed by shear flow                                                            | $\text{in}^2$ |
| $A_{oh}$  | gross area enclosed by centerline of the outermost closed transverse torsional reinforcement | $\text{in}^2$ |
| $A_t$     | area of one leg of a closed stirrup resisting torsion within a distance $s$                  | $\text{in}^2$ |
| $A_v$     | area of shear reinforcement perpendicular to flexural tension reinforcement                  | $\text{in}^2$ |
| $A_{v+t}$ | sum of areas of shear and torsion reinforcement                                              | $\text{in}^2$ |
| $b_w$     | web width                                                                                    | $\text{in}$   |
| $c$       | clear cover to tension reinforcement                                                         | $\text{in}$   |



|            |                                                                                |                      |
|------------|--------------------------------------------------------------------------------|----------------------|
| $d$        | distance from extreme compression fiber to centroid of tension reinforcement   | in                   |
| $d_b$      | diameter of reinforcement                                                      | in                   |
| $f_y$      | yield strength of reinforcement                                                | kips/in <sup>2</sup> |
| $f_{yt}$   | yield strength of transverse reinforcement                                     | kips/in <sup>2</sup> |
| $h$        | overall thickness of member                                                    | in                   |
| $M_{\max}$ | maximum factored moment at section caused by externally applied loads          | in-kips              |
| $M_u$      | factored moment at section                                                     | in-kips              |
| $p_{cp}$   | outside perimeter of the concrete cross section                                | in                   |
| $p_h$      | perimeter of centerline of outermost closed transverse torsional reinforcement | in                   |
| $s$        | spacing of shear or torsion reinforcement                                      | in                   |
| $S_b$      | section modulus of the section referred to the bottom fiber                    | in <sup>3</sup>      |
| $t$        | wall thickness before cracking                                                 | in                   |
| $T_n$      | nominal torsional moment strength                                              | in-kips              |
| $T_u$      | factored torsional moment at section                                           | in-kips              |
| $V_c$      | nominal shear strength provided by concrete                                    | kips                 |
| $V_s$      | nominal shear strength provided by shear reinforcement                         | kips                 |
| $V_u$      | factored shear force at section                                                | kips                 |

#### Symbols

|           |                                                                                                    |                     |
|-----------|----------------------------------------------------------------------------------------------------|---------------------|
| $\tau$    | shear stress in walls                                                                              | lbf/in <sup>2</sup> |
| $\theta$  | angle of compression diagonals in truss analogy for torsion, 45° for a reinforced concrete section | deg                 |
| $\lambda$ | correction factor related to unit weight of concrete                                               | —                   |
| $\phi$    | strength reduction factor, 0.75 for shear and torsion                                              | —                   |

After torsional cracking occurs, the central core of a reinforced concrete member is largely ineffectual in resisting applied torsion and can be neglected. Hence, it is assumed in ACI Sec. R11.5 that a member behaves as a thin-walled tube when subjected to torsion. To maintain a consistent approach, before cracking, a member is also analyzed as a thin-walled tube. As shown in Fig. 1.6, the shear stress in the tube walls produces a uniform shear flow, acting at the midpoint of the walls, with a magnitude of

$$q = \tau t$$

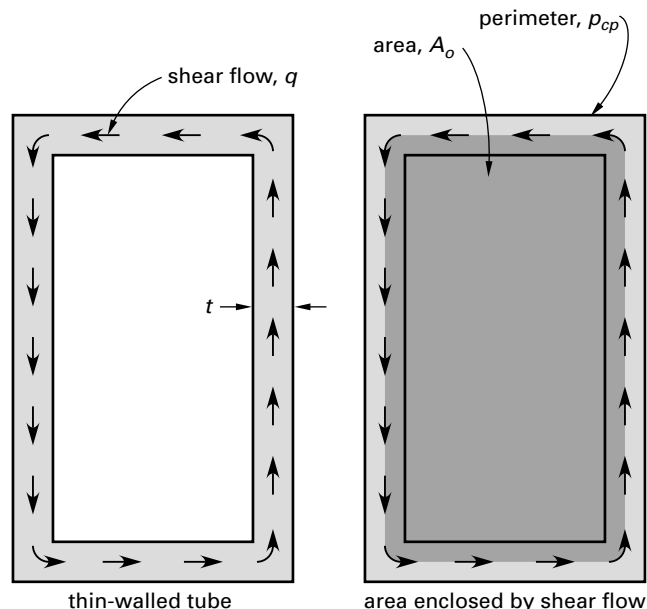
The applied torsion is resisted by the moment of the shear flow in the walls about the centroid of the section and is

$$T = 2A_o q$$

The gross area enclosed by shear flow,  $A_o$ , is the area enclosed by the center line of the walls.

$$A_o = \frac{2A_{cp}}{3}$$

Figure 1.6 Torsion in a Rectangular Beam



The shear stress in the walls is

$$\tau = \frac{T}{2A_o t}$$

In accordance with ACI Sec. 11.5.2.4, the critical section for the calculation of torsion in a reinforced concrete beam is located at a distance from the support equal to the effective depth. When a concentrated torsion occurs within this distance, the critical section for design shall be at the face of the support.

In accordance with ACI Sec. R11.5.1, cracking is assumed to occur in a member when the principal tensile stress reaches a value of

$$p_t = 4\lambda\sqrt{f'_c} = \tau$$

The cracking torsion is

$$T_{cr} = 4\lambda\sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

ACI Sec. 11.5.1 assumes that torsional effects may be neglected, and that closed stirrups and longitudinal torsional reinforcement are not required when the factored torque does not exceed the threshold torsion given by

$$T_u = \frac{\phi T_{cr}}{4} = \phi\lambda\sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

When the threshold value is exceeded, reinforcement must be provided to resist the full torsion, and the

concrete is considered ineffective. When both shear and torsion reinforcements are required, the sum of the individual areas must be provided, as specified in ACI Sec. R11.5.3.8. Because the stirrup area  $A_v$  for shear is defined in terms of both legs of a stirrup, while the stirrup area  $A_t$  for torsion is defined in terms of one leg only, the summation of the areas is

$$\sum \left( \frac{A_{v+t}}{s} \right) = \frac{A_v}{s} + \frac{2A_t}{s}$$

The spacing of the reinforcement is limited by the minimum required spacing of either the shear or torsion reinforcement. As specified in ACI Sec. 11.4.5, the spacing of shear reinforcement must not exceed

$$s = \frac{d}{2} \quad [V_s \leq 4\sqrt{f'_c}b_wd]$$

$$s = \frac{d}{4} \quad [V_s > 4\sqrt{f'_c}b_wd]$$

As specified in ACI Sec. 11.5.6, the spacing of torsion reinforcement must not exceed

$$s = \frac{ph}{8}$$

$$\leq 12 \text{ in}$$

After torsional cracking occurs, cracks are produced diagonally on all four faces of a member, forming a continuous spiral failure surface around the perimeter. For a reinforced concrete beam subjected to pure torsion, the principal tensile stresses and the cracks are produced at an angle  $\theta$ . ACI Sec. 11.5.3.6 specifies that the angle  $\theta$  may be assumed to be  $45^\circ$ . After cracking, the beam is idealized as a tubular space truss consisting of closed stirrups, longitudinal reinforcement, and concrete compression struts between the torsion cracks. To resist the applied torque, it is necessary to provide transverse and longitudinal reinforcement on each face. Closed stirrups are required to resist the vertical component, and longitudinal reinforcement is required to resist the horizontal component of the torsional stresses. After torsional cracking, the concrete outside the closed stirrups is ineffective in resisting the applied torsion, and the gross area enclosed by shear flow is redefined by ACI Sec. 11.5.3.6 as

$$A_o = 0.85A_{oh}$$

In an indeterminate structure, redistribution of internal forces after cracking results in a reduction in torsional moment with a compensating redistribution of internal forces. This redistribution of forces is referred to as *compatibility torsion*. Non-prestressed members may be designed for a maximum factored torsional moment given by ACI Sec. 11.5.2.2 as

$$T_u = \phi 4\lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right)$$

When the torsional moment cannot be reduced by redistribution of internal forces after cracking, the member must be designed for the full applied factored torque. This is referred to as *equilibrium torsion*.

When the threshold torsion value is exceeded, closed stirrups and longitudinal reinforcement must be provided to resist the appropriate value of the torque, and the concrete is considered ineffective.

ACI Sec. 11.5.3.6 specifies the required area of one leg of a closed stirrup as

$$\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_{yt} \cot \theta} = \frac{T_u}{2\phi A_o f_{yt} \cot 45^\circ}$$

$$= \frac{T_u}{1.7\phi A_{oh} f_{yt}}$$

The minimum combined area of stirrups for shear and torsion is given by ACI Sec. 11.5.5.2 as

$$\frac{A_v + 2A_t}{s} = 0.75 \frac{\sqrt{f'_c} b_w}{f_{yt}} \quad [\text{ACI 11-23}]$$

$$\geq \frac{50b_w}{f_{yt}}$$

The corresponding required area of longitudinal reinforcement is specified in ACI Secs. 11.5.3.7 and R11.5.3.10 as

$$A_l = \left( \frac{A_t}{s} \right) \left( \frac{p_h f_{yt}}{f_{yl}} \right) \cot^2 \theta \quad [\text{ACI 11-22}]$$

$$= \left( \frac{A_t}{s} \right) \left( \frac{p_h f_{yt}}{f_{yl}} \right) \cot^2 45^\circ$$

$$= \left( \frac{A_t}{s} \right) \left( \frac{p_h f_{yt}}{f_{yl}} \right)$$

When the threshold torsional moment is exceeded, the minimum permissible area of longitudinal reinforcement is then given by ACI Sec. 11.5.5.3 as

$$A_{l,\min} = \frac{5A_{cp}\sqrt{f'_c}}{f_y} - \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) \quad [\text{ACI 11-24}]$$

In ACI Eq. (11-24),  $A_t/s$  must not be less than  $25b_w/f_{yt}$ .

The bars are distributed around the inside perimeter of the closed stirrups at a maximum spacing of 12 in. A longitudinal bar is required in each corner of a closed stirrup.

The minimum diameter of longitudinal reinforcement is given by ACI Sec. 11.5.6.2 as

$$d_b = 0.042s$$

$$\geq 3/8 \text{ in}$$

To prevent crushing of the concrete compression diagonals, the combined stress caused by the factored torsion and shear forces is limited by ACI Sec. 11.5.3.1. For solid sections, the dimensions of the section must be such that

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \quad [\text{ACI 11-18}]$$

For hollow sections,

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7 A_{oh}^2} \leq \phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \quad [\text{ACI 11-19}]$$

### Example 1.3

A simply supported reinforced concrete beam of normal weight concrete with an overall depth of  $h = 23$  in, an effective depth of  $d = 20$  in, and a width of  $b_w = 14$  in, is reinforced with grade 60 bars, and has a concrete compressive strength of  $3000 \text{ lbf/in}^2$ . Determine the combined shear and torsion reinforcement required when the factored shear force is  $V_u = 10$  kips and the factored torsion is  $T_u = 200$  in-kips.

#### Solution

The design shear strength provided by the concrete is given by ACI Eq. (11-3) as

$$\begin{aligned} \phi V_c &= 2\phi b_w d \lambda \sqrt{f'_c} \\ &= \frac{(2)(0.75)(14 \text{ in})(20 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 23 \text{ kips} \\ &> 2V_u \end{aligned}$$

In accordance with ACI Sec. 11.4.6.1, shear reinforcement is not required.

The area enclosed by the outside perimeter of the beam is

$$\begin{aligned} A_{cp} &= h b_w \\ &= (23 \text{ in})(14 \text{ in}) \\ &= 322 \text{ in}^2 \end{aligned}$$

The length of the outside perimeter of the beam is

$$\begin{aligned} p_{cp} &= 2(h + b_w) \\ &= (2)(23 \text{ in} + 14 \text{ in}) \\ &= 74 \text{ in} \end{aligned}$$

Torsional reinforcement is not required in accordance with ACI Sec. 11.5.1 when the factored torque does not exceed the threshold torsion given by

$$\begin{aligned} T_u &= \phi \lambda \sqrt{f'_c} \left(\frac{A_{cp}^2}{p_{cp}}\right) = \frac{(0.75)(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} (322 \text{ in}^2)^2}{(74 \text{ in}) \left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 58 \text{ in-kips} \end{aligned}$$

Because the applied torsion of 200 in-kips exceeds the threshold torsion, torsion reinforcement consisting of closed stirrups and longitudinal reinforcement is necessary. Using no. 3 stirrups with 1.5 in cover, the area enclosed by the center line of the stirrups is

$$\begin{aligned} A_{oh} &= (23 \text{ in} - 3 \text{ in} - 0.375 \text{ in})(14 \text{ in} - 3 \text{ in} - 0.375 \text{ in}) \\ &= 208.52 \text{ in}^2 \end{aligned}$$

From ACI Eq. (11-21), the required area of one arm of a closed stirrup is

$$\begin{aligned} \frac{A_t}{s} &= \frac{T_u}{1.7\phi A_{oh} f_{yt}} \\ &= \frac{200 \text{ in-kips}}{(1.7)(0.75)(208.52 \text{ in}^2) \left(60 \frac{\text{kips}}{\text{in}^2}\right)} \\ &= \left(0.0125 \frac{\text{in}^2}{\text{in-arm}}\right) \left(12 \frac{\text{in}}{\text{ft}}\right) \\ &= 0.15 \text{ in}^2/\text{ft-arm} \end{aligned}$$

From ACI Eq. (11-23) the minimum area of closed stirrups is given by the lesser of

$$\begin{aligned} \frac{A_v + 2A_t}{s} &= \frac{50b_w}{f_{yt}} \\ &= \frac{(50)(14 \text{ in}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{60,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.14 \frac{\text{in}^2}{\text{ft}} \\ &< 0.15 \text{ in}^2/\text{ft-arm} \quad [\text{does not govern}] \end{aligned}$$

$$\begin{aligned} \frac{A_v + 2A_t}{s} &= \frac{0.75\sqrt{f'_c} b_w}{f_{yt}} \\ &= \frac{0.75\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} (14 \text{ in}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{60,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.12 \text{ in}^2/\text{ft-arm} \\ &< 0.15 \text{ in}^2/\text{ft-arm} \quad [\text{does not govern}] \end{aligned}$$

The perimeter of the center line of the closed stirrups is

$$\begin{aligned} p_h &= (2)(23 \text{ in} + 14 \text{ in} - (2)(3.375 \text{ in})) \\ &= 60.5 \text{ in} \end{aligned}$$

The maximum permissible spacing of the closed stirrups is specified in ACI Sec. 11.5.6.1 as

$$\begin{aligned} s_{\max} &= \frac{p_h}{8} \\ &= \frac{60.5 \text{ in}}{8} \\ &= 7.6 \text{ in} \end{aligned}$$

Closed stirrups consisting of two arms of no. 3 bars at 6 in spacing provides an area of

$$\begin{aligned} \frac{A}{s} &= 0.44 \frac{\text{in}^2}{\text{ft}} \\ &> 0.15 \text{ in}^2/\text{ft} \quad [\text{satisfactory}] \end{aligned}$$

The required area of the longitudinal reinforcement is given by ACI Eq. (11-22) as

$$\begin{aligned} A_l &= \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) \\ &= \frac{\left( 0.15 \frac{\text{in}^2}{\text{ft-arm}} \right) (60.5 \text{ in})}{12 \frac{\text{in}}{\text{ft}}} \\ &= 0.76 \text{ in}^2/\text{arm} \end{aligned}$$

Because the required value of  $A_t/s = 0.0125 \text{ in}^2/\text{in-arm}$  is greater than  $25b_w/f_{yt} = 0.00583$ , the minimum permissible area of longitudinal reinforcement is

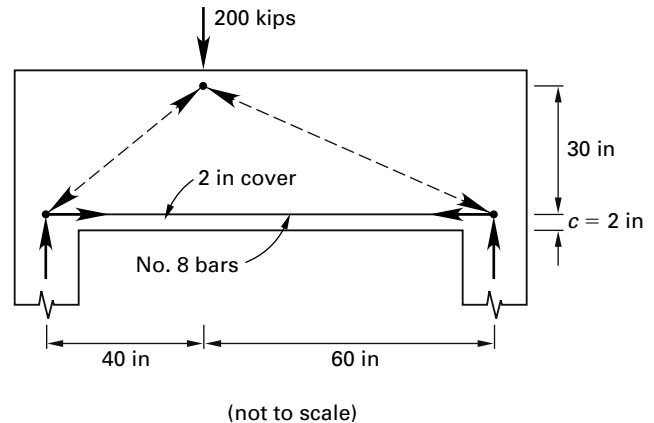
$$\begin{aligned} A_{l,\min} &= \frac{5A_{cp}\sqrt{f'_c}}{f_y} - \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) \\ &= \frac{(5)(322 \text{ in}^2)\sqrt{3000} \frac{\text{lb}}{\text{in}^2}}{60,000 \frac{\text{lb}}{\text{in}^2}} \\ &\quad - \left( 0.0125 \frac{\text{in}^2}{\text{in-arm}} \right) (60.5 \text{ in}) \\ &= 0.714 \text{ in}^2 \quad [\text{does not govern}] \\ &< 0.76 \text{ in}^2/\text{arm} \end{aligned}$$

Using eight no. 3 bars distributed around the perimeter of the closed stirrups gives a longitudinal steel area of

$$\begin{aligned} A_l &= 0.88 \text{ in}^2 \\ &> 0.76 \text{ in}^2 \quad [\text{satisfactory}] \end{aligned}$$

## PRACTICE PROBLEMS

For Probs. 1–3, refer to the illustration shown. The reinforced concrete beam has a width of 14 in and a concrete compressive strength of 5000 lbf/in<sup>2</sup>. The factored applied force of 200 kips includes an allowance for the self-weight of the beam.



1. Using the equivalent strut-and-tie model indicated, what is most nearly the area of grade 60 tension reinforcement required?

- (A) 1.78 in<sup>2</sup>
- (B) 2.67 in<sup>2</sup>
- (C) 3.33 in<sup>2</sup>
- (D) 3.56 in<sup>2</sup>

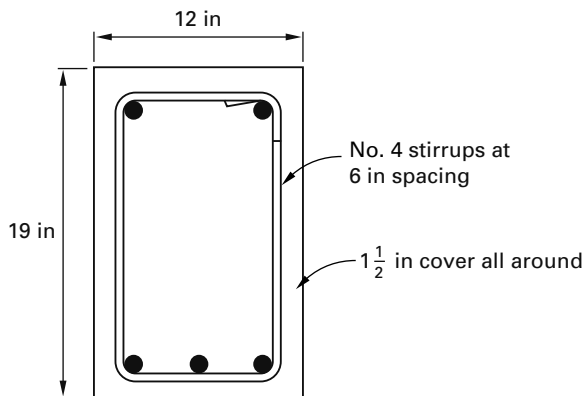
2. Is the equivalent, unreinforced bottle-shaped concrete strut at the right support compliant with the requirements of ACI App. A?

- (A) No, the design compressive strength of the concrete is 1.91 kips/in<sup>2</sup>.
- (B) No, the design compressive strength of the concrete is 2.55 kips/in<sup>2</sup>.
- (C) Yes, the design compressive strength of the concrete is 2.29 kips/in<sup>2</sup>.
- (D) Yes, the design compressive strength of the concrete is 2.55 kips/in<sup>2</sup>.

3. Is the equivalent nodal zone at the right support compliant with the requirements of ACI App. A?

- (A) No, the design compressive strength is 1.90 kips/in<sup>2</sup>
- (B) No, the design compressive strength is 2.10 kips/in<sup>2</sup>
- (C) Yes, the design compressive strength is 2.55 kips/in<sup>2</sup>
- (D) Yes, the design compressive strength is 3.40 kips/in<sup>2</sup>

Problems 4 and 5 refer to the reinforced concrete beam shown. The concrete strength is 3000 lbf/in<sup>2</sup>, and all reinforcement is grade 60. The concrete section is adequate to support the applied shear force.



4. What is most nearly the maximum factored torsion that the beam can support?

- (A) 340 in-kips
- (B) 370 in-kips
- (C) 420 in-kips
- (D) 560 in-kips

5. What is most nearly the required area of longitudinal reinforcement for the beam?

- (A) 1.40 in<sup>2</sup>/arm
- (B) 1.60 in<sup>2</sup>/arm
- (C) 1.90 in<sup>2</sup>/arm
- (D) 2.10 in<sup>2</sup>/arm

## SOLUTIONS

1. For the idealized strut-and-tie model shown in the illustration, the angle between the strut and tie at the right support is

$$\begin{aligned}\theta &= \tan^{-1} \left( \frac{30 \text{ in}}{60 \text{ in}} \right) \\ &= 26.6^\circ \\ &> 25^\circ \quad [\text{satisfies ACI Sec. A.2.5}]\end{aligned}$$

The reaction at the right support is

$$R = \frac{(200 \text{ kips})(40 \text{ in})}{100 \text{ in}} = 80 \text{ kips}$$

The equivalent tie force is determined from the strut-and-tie model as

$$T = \frac{(80 \text{ kips})(60 \text{ in})}{30 \text{ in}} = 160 \text{ kips}$$

The strength reduction factor is given by ACI Sec. 9.3.2.6 as

$$\phi = 0.75$$

The necessary reinforcement area is

$$\begin{aligned}A_{ts} &= \frac{T}{\phi f_y} = \frac{160 \text{ kips}}{(0.75) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)} \\ &= 3.56 \text{ in}^2\end{aligned}$$

Use five no. 8 bars, which gives

$$\begin{aligned}A &= 3.95 \text{ in}^2 \\ &> A_{ts} \quad [\text{satisfactory}]\end{aligned}$$

**The answer is (D).**

2. The dimensions of the nodal zone are the equivalent tie width,  $w_t$ , and the width of the equivalent concrete strut,  $w_s$ .

$$\begin{aligned}w_t &= d_b + 2c \\ &= 1 \text{ in} + (2)(2 \text{ in}) \\ &= 5 \text{ in} \\ w_s &= \frac{w_t}{\cos \theta} \\ &= \frac{5 \text{ in}}{\cos 26.6^\circ} \\ &= 5.59 \text{ in}\end{aligned}$$

The stress in the equivalent tie is

$$\begin{aligned} f_T &= \frac{T}{bw_t} = \frac{160 \text{ kips}}{(14 \text{ in})(5 \text{ in})} \\ &= 2.29 \text{ kips/in}^2 \end{aligned}$$

For a hydrostatic nodal zone, the stress in the equivalent concrete strut is

$$f_C = f_T = 2.29 \text{ kips/in}^2$$

For normal weight concrete and an unreinforced, bottle-shaped strut, the effective compressive strength of the concrete in the strut is given by ACI Eq. (A-3) as

$$\begin{aligned} f_{ce} &= 0.85\beta_s f'_c = (0.85)(0.6)(1.0) \left( 5 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 2.55 \text{ kips/in}^2 \end{aligned}$$

The design compressive stress is

$$\begin{aligned} \phi f_{ce} &= (0.75) \left( 2.55 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 1.91 \text{ kips/in}^2 \\ &< f_C \quad [\text{unsatisfactory}] \end{aligned}$$

**The answer is (A).**

**3.** The nominal compressive strength of a nodal zone anchoring one layer of reinforcing bars without confining reinforcement is given by ACI Eq. (A-8) as

$$\begin{aligned} f_{ce} &= 0.85\beta_n f'_c \\ &= (0.85)(0.8) \left( 5 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 3.40 \text{ kips/in}^2 \end{aligned}$$

The design compressive stress is

$$\begin{aligned} \phi f_{ce} &= (0.75) \left( 3.40 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 2.55 \text{ kips/in}^2 \\ &> f_C \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (C).**

**4.** The area enclosed by the center line of the stirrups is

$$\begin{aligned} A_{oh} &= (19 \text{ in} - 3 \text{ in} - 0.5 \text{ in})(12 \text{ in} - 3 \text{ in} - 0.5 \text{ in}) \\ &= 131.75 \text{ in}^2 \end{aligned}$$

The area of one arm of the no. 4 closed stirrups provided at 6 in centers is

$$\begin{aligned} \frac{A_t}{s} &= \frac{0.20 \text{ in}^2}{6 \text{ in-arm}} \\ &= 0.033 \text{ in}^2/\text{in-arm} \end{aligned}$$

From ACI Eq. (11-21) the maximum factored torsional moment that may be applied to the section is

$$\begin{aligned} T_u &= \frac{1.7\phi A_{oh} f_{yt} A_t}{s} \\ &= \frac{(1.7)(0.75)(131.75 \text{ in}^2) \left( 60 \frac{\text{kips}}{\text{in}^2} \right) (0.20 \text{ in}^2)}{6 \text{ in}} \\ &= 336 \text{ in-kips} \quad (340 \text{ in-kips}) \end{aligned}$$

**The answer is (A).**

**5.** The perimeter of the center line of the closed stirrups is

$$\begin{aligned} p_h &= (2)(19 \text{ in} + 12 \text{ in} - (2)(3.5 \text{ in})) \\ &= 48 \text{ in} \end{aligned}$$

The required area of the longitudinal reinforcement is given by ACI Eq. (11-22) as

$$\begin{aligned} A_l &= \left( \frac{A_t}{s} \right) p_h \left( \frac{f_{yt}}{f_y} \right) \\ &= \left( 0.033 \frac{\text{in}^2}{\text{in-arm}} \right) (48 \text{ in}) \\ &= 1.60 \text{ in}^2/\text{arm} \end{aligned}$$

Using six no. 5 bars distributed around the perimeter of the closed stirrups gives a longitudinal steel area of

$$\begin{aligned} A_l &= 1.86 \text{ in}^2 \\ &> 1.60 \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (B).**





# 2 Foundations

|                                                |      |
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| 2. Combined Footings . . . . .                 | 2-10 |
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## 1. ECCENTRICALLY LOADED COLUMN BASES

### Nomenclature

|          |                                                                                                        |                    |
|----------|--------------------------------------------------------------------------------------------------------|--------------------|
| $A_b$    | area of reinforcement in central band                                                                  | in <sup>2</sup>    |
| $A_s$    | total required reinforcement area                                                                      | in <sup>2</sup>    |
| $A_1$    | loaded area at base of column                                                                          | ft <sup>2</sup>    |
| $A_2$    | area of the base of the pyramid, with side slopes of 1:2, formed within the footing by the loaded area | ft <sup>2</sup>    |
| $b_o$    | perimeter of critical section for punching shear                                                       | in                 |
| $b_1$    | width of the critical perimeter measured in the direction of $M_u$                                     | in                 |
| $b_2$    | width of the critical perimeter measured perpendicular to $b_1$                                        | in                 |
| $B$      | length of short side of a rectangular footing                                                          | ft                 |
| $c$      | length of side of column                                                                               | in                 |
| $c_1$    | length of short side of a rectangular column                                                           | in                 |
| $c_2$    | length of long side of a rectangular column                                                            | in                 |
| $d$      | average effective depth of reinforcement in the footing                                                | in                 |
| $d_b$    | bar diameter                                                                                           | in                 |
| $e$      | eccentricity with respect to center of footing                                                         | in                 |
| $e'$     | eccentricity with respect to edge of footing                                                           | in                 |
| $f'_c$   | specified compressive strength of concrete                                                             | lb/in <sup>2</sup> |
| $f_y$    | yield strength of reinforcement                                                                        | lb/in <sup>2</sup> |
| $h$      | depth of footing                                                                                       | in                 |
| $K_u$    | design moment factor                                                                                   | lb/in <sup>2</sup> |
| $L$      | length of long side of a rectangular footing, length of side of a square footing                       | ft                 |
| $L$      | live load                                                                                              | kips               |
| $M$      | maximum unfactored moment caused by service loads                                                      | in-lbf             |
| $M_u$    | factored moment                                                                                        | ft-kips            |
| $P$      | column axial service load                                                                              | kips               |
| $P_{bn}$ | nominal bearing strength                                                                               | kips               |

|       |                                                               |                    |
|-------|---------------------------------------------------------------|--------------------|
| $P_T$ | total service load on the soil                                | kips               |
| $P_u$ | column axial factored load                                    | kips               |
| $q$   | soil bearing pressure caused by service loads                 | lb/ft <sup>2</sup> |
| $q_u$ | net factored pressure acting on footing                       | lb/ft <sup>2</sup> |
| $S$   | section modulus of footing                                    | in <sup>3</sup>    |
| $V_c$ | nominal shear strength                                        | kips               |
| $V_u$ | factored shear force at section                               | kips               |
| $W_b$ | weight of footing                                             | kips               |
| $w_c$ | unit weight of concrete                                       | lb/ft <sup>3</sup> |
| $x$   | distance from edge of footing to critical section             | ft                 |
| $y$   | longer overall dimension of rectangular part of cross section | in                 |

### Symbols

|           |                                                      |   |
|-----------|------------------------------------------------------|---|
| $\alpha$  | reinforcement location factor                        | — |
| $\beta$   | coating factor                                       | — |
| $\beta$   | ratio of the long side to the short side of footing  | — |
| $\beta_c$ | ratio of long side to short side of the column       | — |
| $\beta_1$ | compression zone factor given in ACI Sec. 10.2.7.3   | — |
| $\lambda$ | lightweight aggregate concrete factor                | — |
| $\rho$    | reinforcement ratio                                  | — |
| $\rho_t$  | reinforcement ratio for a tension-controlled section | — |
| $\phi$    | strength reduction factor                            | — |

### Soil Pressure Distribution

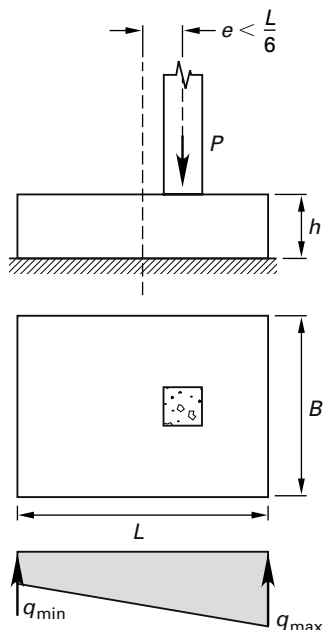
The eccentricity of a column on its footing determines the shape of the soil pressure distribution. As shown in Fig. 2.1, when the eccentricity of the column on the footing is less than  $L/6$ , the resulting soil pressure distribution is trapezoidal. The bending moment acting on the footing is

$$M = Pe$$

The maximum and minimum values of the soil pressure are

$$q_{\max} = \frac{P}{BL} + \frac{M}{S}$$

$$q_{\min} = \frac{P}{BL} - \frac{M}{S}$$

Figure 2.1 Eccentricity Less than  $L/6$ 


For a rectangular footing, the length of the short side is  $B$  and the length of the long side is  $L$ . The section modulus of the footing is

$$S = \frac{BL^2}{6}$$

The maximum and minimum values of the soil pressure are

$$q_{\max} = \left(\frac{P}{BL}\right) \left(1 + \frac{6e}{L}\right)$$

$$q_{\min} = \left(\frac{P}{BL}\right) \left(1 - \frac{6e}{L}\right)$$

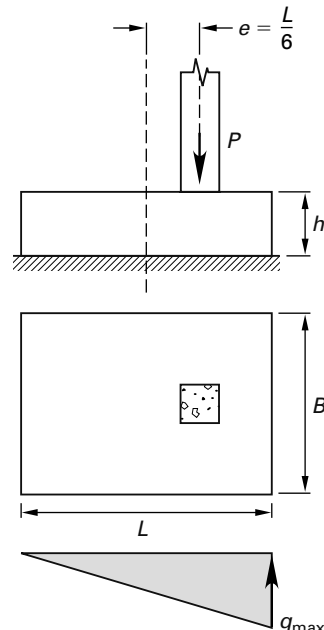
When the eccentricity of the column on the footing equals one-sixth of the longest side,  $L/6$ , as shown in Fig. 2.2, the resulting soil pressure distribution is triangular. The bending moment acting on the footing is

$$M = \frac{PL}{6}$$

The maximum and minimum values of the soil pressure are

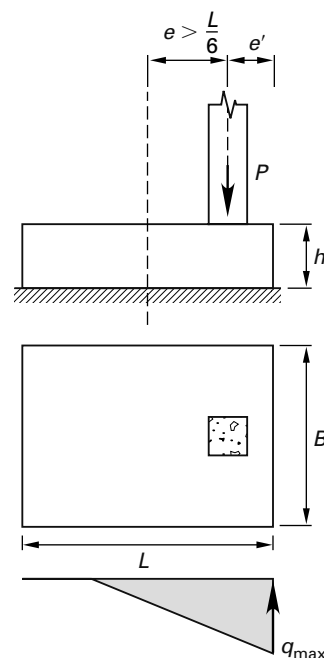
$$\begin{aligned} q_{\max} &= \frac{P}{BL} + \left(\frac{PL}{6}\right) \left(\frac{6}{BL^2}\right) \\ &= \frac{2P}{BL} \\ q_{\min} &= \frac{P}{BL} - \left(\frac{PL}{6}\right) \left(\frac{6}{BL^2}\right) \\ &= 0 \end{aligned}$$

It is not possible to develop tensile stress at the interface of the soil and the soffit of the footing. Hence, when the

Figure 2.2 Eccentricity Equal to  $L/6$ 


eccentricity of the column on the footing exceeds one-sixth of the longest side,  $L/6$ , as shown in Fig. 2.3, the triangular soil pressure distribution extends over only a portion of the base. For equilibrium, the centroid of the pressure distribution must coincide with the line of action of the column axial service load,  $P$ . The length of the interface in compression is

$$\begin{aligned} x &= 3e' \\ e' &= \frac{L}{2} - e \end{aligned}$$

Figure 2.3 Eccentricity Greater than  $L/6$ 


Equating vertical forces gives

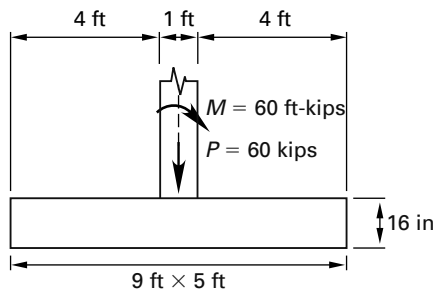
$$P = \frac{Bxq_{\max}}{2}$$

The maximum value of the soil pressure is

$$\begin{aligned} q_{\max} &= \frac{2P}{Bx} \\ &= \frac{2P}{3Be'} \end{aligned}$$

### Example 2.1

The 9 ft × 5 ft reinforced normal weight concrete footing shown in the illustration supports a column with an axial load of  $P=60$  kips and a bending moment of  $M=60$  ft-kips. Determine the maximum pressure in the soil.



*Solution*

The weight of the footing is

$$\begin{aligned} W_b &= w_c LBh \\ &= \frac{\left(150 \frac{\text{lb}}{\text{ft}^3}\right)(9 \text{ ft})(5 \text{ ft})(16 \text{ in})}{\left(1000 \frac{\text{lb}}{\text{kip}}\right)\left(12 \frac{\text{in}}{\text{ft}}\right)} \\ &= 9 \text{ kips} \end{aligned}$$

The total load on the soil is

$$\begin{aligned} P_T &= P + W_b \\ &= 60 \text{ kips} + 9 \text{ kips} \\ &= 69 \text{ kips} \end{aligned}$$

The equivalent eccentricity of the total load is

$$\begin{aligned} e &= \frac{M}{P_T} \\ &= \frac{60 \text{ ft-kips}}{69 \text{ kips}} \\ &= 0.87 \text{ ft} \\ &< L/6 \quad [\text{within middle third of the base}] \end{aligned}$$

The maximum soil pressure is

$$\begin{aligned} q_{\max} &= \left(\frac{P_T}{BL}\right)\left(1 + \frac{6e}{L}\right) \\ &= \left(\frac{69 \text{ kips}}{(5 \text{ ft})(9 \text{ ft})}\right)\left(1 + \frac{(6)(0.87 \text{ ft})}{9 \text{ ft}}\right) \\ &= 2.4 \text{ kips/ft}^2 \end{aligned}$$

### Factored Soil Pressure

A reinforced concrete footing must be designed for punching shear, flexural shear, and flexure. The critical section for each of these effects is located at a different position in the footing, and each must be designed for the applied factored loads. Hence, the soil pressure distribution caused by factored loads must be determined. Because the self-weight of the footing produces an equal and opposite pressure in the soil, the footing is designed for the net pressure from the column load only, and the weight of the footing is not included.

### Example 2.2

The 9 ft × 5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load of  $P_u=100$  kips. It has a factored bending moment of  $M_u=100$  ft-kips. Determine the factored pressure distribution in the soil. The effective depth is  $d=12$  in.

*Solution*

The equivalent eccentricity of the factored column loads is

$$\begin{aligned} e &= \frac{M_u}{P_u} \\ &= \frac{100 \text{ ft-kips}}{100 \text{ kips}} \\ &= 1 \text{ ft} \\ &< L/6 \quad [\text{within middle third of the base}] \end{aligned}$$

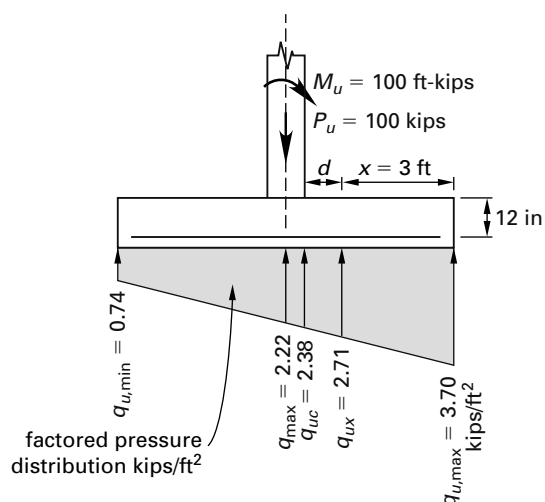
The maximum factored soil pressure is

$$\begin{aligned} q_{u,\max} &= \left(\frac{P_u}{BL}\right)\left(1 + \frac{6e}{L}\right) \\ &= \left(\frac{100 \text{ kips}}{(5 \text{ ft})(9 \text{ ft})}\right)\left(1 + \frac{(6)(1.0 \text{ ft})}{9 \text{ ft}}\right) \\ &= 3.70 \text{ kips/ft}^2 \end{aligned}$$

The minimum factored soil pressure is

$$\begin{aligned} q_{u,\min} &= \left(\frac{P_u}{BL}\right)\left(1 - \frac{6e}{L}\right) \\ &= \left(\frac{100 \text{ kips}}{(5 \text{ ft})(9 \text{ ft})}\right)\left(1 - \frac{(6)(1.0 \text{ ft})}{9 \text{ ft}}\right) \\ &= 0.74 \text{ kips/ft}^2 \end{aligned}$$

The relevant values at critical sections are shown in the illustration.

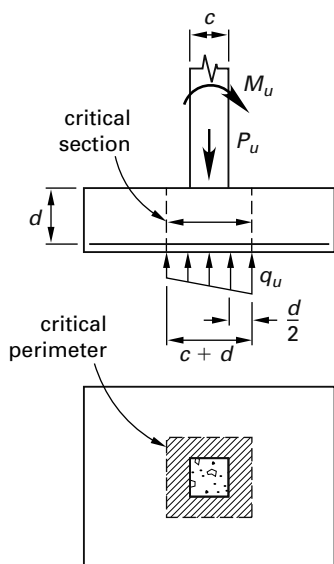


### Design for Punching Shear

The depth of footing is usually governed by the punching shear capacity. The critical perimeter for punching shear is specified in ACI Secs. 15.5.2 and 11.11.1.2 and illustrated in Fig. 2.4. For a concrete column, the critical perimeter is a distance from the face of the column equal to half the effective depth. The length of the critical perimeter is

$$\begin{aligned} b_o &= 2(b_1 + b_2) \\ &= 2(c_1 + c_2) + 4d \\ &= 4(c + d) \quad [c_1 = c_2 = c] \end{aligned}$$

Figure 2.4 Critical Perimeter for Punching Shear



The width of the critical perimeter measured in the direction of  $M_u$  is

$$b_1 = c_1 + d$$

The width of the critical perimeter measured perpendicular to  $b_1$  is

$$b_2 = c_2 + d$$

The value of  $c_1$  is the width of the column measured in the direction of  $M_u$ , and  $c_2$  is the width of the column measured perpendicular to  $c_1$ .

When the column supports only an axial load,  $P_u$ , shear stress at the critical perimeter is uniformly distributed around the critical perimeter, and is

$$v_u = \frac{V_u}{db_o}$$

The factored shear force acting on the critical perimeter is

$$V_u = P_u \left( 1 - \frac{b_1 b_2}{BL} \right)$$

$L$  is the length of the footing measured in the direction of  $M_u$ .  $B$  is the length of the footing measured perpendicular to  $L$ .

When, in addition to the axial load, a bending moment,  $M_u$ , is applied to the column, an eccentric shear stress is also introduced into the critical section with the maximum value occurring on the face nearest the largest bearing pressure. The maximum value of this shear stress is given by ACI Sec. R11.11.7.2 as

$$v_u = \frac{\gamma_v M_u y}{J_c}$$

The distance from the centroid of the critical perimeter to edge of the critical perimeter is

$$y = \frac{b_1}{2} \quad [\text{footing with central column}]$$

The fraction of the applied moment transferred by shear, as specified by ACI Secs. 11.11.7.1 and 13.5.3.2, is

$$\gamma_v = 1 - \frac{1}{1 + \left( \frac{2}{3} \right) \sqrt{\frac{b_1}{b_2}}}$$

$J_c$  is the polar moment of inertia of the critical perimeter, and

$$\frac{J_c}{y} = \frac{b_1 d (b_1 + 3b_2) + d^3}{3}$$

[ footing with central column  
as specified by ACI Sec. R11.11.7.2 ]

When both axial load and bending moment occur, the shear stresses caused by both conditions are combined,

as specified in ACI Sec. R11.11.7.2, to give a maximum value of

$$v_u = \frac{V_u}{db_o} + \frac{\gamma_v M_u y}{J_c}$$

The design punching shear strength of the footing is determined by ACI Sec 11.11.2.1 as

$$\phi V_c = 4\phi db_o \lambda \sqrt{f'_c} \quad [\text{ACI 11-33}]$$

When  $\beta_c > 2$ , the design punching shear strength is

$$\phi V_c = \phi db_o \lambda \left( 2 + \frac{4}{\beta_c} \right) \sqrt{f'_c} \quad [\text{ACI 11-31}]$$

$$\phi = 0.75$$

### Example 2.3

The 9 ft  $\times$  5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load of  $P_u = 100$  kips and a bending moment of  $M_u = 100$  ft-kips. For a concrete strength of 3000 lbf/in<sup>2</sup>, determine whether the punching shear capacity of the footing is satisfactory. The effective depth is  $d = 12$  in.

*Solution*

The length of the critical perimeter is

$$\begin{aligned} b_o &= 4(c + d) \\ &= (4)(12 \text{ in} + 12 \text{ in}) \\ &= \frac{96 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \\ &= 8 \text{ ft} \end{aligned}$$

The average factored soil pressure acting on the area bounded by the critical perimeter is obtained from Ex. 2.2 as

$$q_{u,ave} = 2.22 \text{ kips/ft}^2$$

Shear at the critical perimeter caused by the axial load is

$$\begin{aligned} V_u &= P_u - (q_{u,ave})(c + d)^2 \\ &= 100 \text{ kips} - \left( 2.22 \frac{\text{kips}}{\text{ft}^2} \right) \\ &\quad \times \left( (12 \text{ in} + 12 \text{ in}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) \right)^2 \\ &= 91 \text{ kips} \end{aligned}$$

The polar moment of inertia of the critical perimeter is

$$\begin{aligned} \frac{J_c}{y} &= \frac{b_1 d(b_1 + 3b_2) + d^3}{3} \quad \left[ \begin{array}{l} \text{footing with a} \\ \text{central column} \end{array} \right] \\ &= \frac{(24 \text{ in})(12 \text{ in}) \left( 24 \text{ in} + (3)(24 \text{ in}) \right) + (12 \text{ in})^3}{3} \\ &= 9792 \text{ in}^3 \end{aligned}$$

The fraction of the column moment transferred by shear is

$$\begin{aligned} \gamma_v &= 1 - \frac{1}{1 + \left( \frac{2}{3} \right) \sqrt{\frac{b_1}{b_2}}} \\ &= 1 - \frac{1}{1 + \left( \frac{2}{3} \right) \sqrt{\frac{12 \text{ in}}{12 \text{ in}}}} \\ &= 0.40 \end{aligned}$$

The combined shear stress from the applied axial load and the column moment is

$$\begin{aligned} v_u &= \frac{V_u}{db_o} + \frac{\gamma_v M_u y}{J_c} \\ &= \frac{(91 \text{ kips}) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)}{(12 \text{ in})(96 \text{ in})} \\ &\quad + \frac{(0.4)(1200 \text{ in-kips}) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)}{9792 \text{ in}^3} \\ &= 79 \frac{\text{lbf}}{\text{in}^2} + 49 \frac{\text{lbf}}{\text{in}^2} \\ &= 128 \text{ lbf/in}^2 \end{aligned}$$

The ratio of the long side to the short side of the column is

$$\begin{aligned} \beta_c &= \frac{c_2}{c_1} \\ &= \frac{12 \text{ in}}{12 \text{ in}} \\ &= 1.00 \\ &< 2 \end{aligned}$$

The allowable shear stress for two-way action is given by ACI Eq. (11-33) as

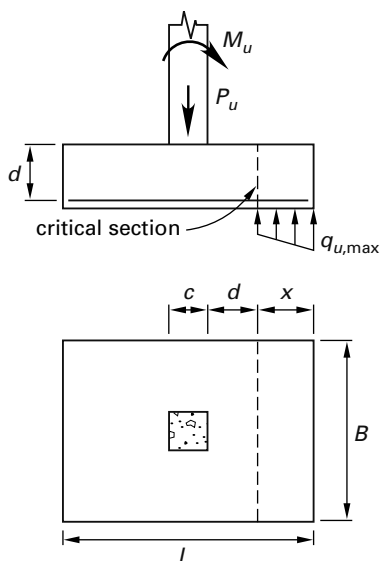
$$\begin{aligned} \phi v_c &= 4\phi \lambda \sqrt{f'_c} \\ &= (4)(0.75)(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 164 \text{ lbf/in}^2 \\ &> v_u \quad [\text{satisfactory}] \end{aligned}$$

## Design for Flexural Shear

For concrete columns, the location of the critical section for flexural shear is defined in ACI Secs. 15.5.2 and 11.1.3.1 as being located a distance,  $d$ , from the face of the concrete column, as shown in Fig. 2.5. The design flexural shear strength of the footing is given by ACI Sec. 11.2.1.1 as

$$\phi V_c = 2\phi bd\lambda\sqrt{f'_c} \quad [\text{ACI 11-3}]$$

Figure 2.5 Critical Section for Flexural Shear



### Example 2.4

The 9 ft  $\times$  5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load of  $P_u = 100$  kips and a bending moment of  $M_u = 100$  ft-kips. For a concrete strength is 3000 lbf/in<sup>2</sup>, determine whether the flexural shear capacity of the footing is satisfactory. The effective depth is  $d = 12$  in.

#### Solution

The distance of the critical section for flexural shear from the edge of the footing is

$$\begin{aligned} x &= \frac{L}{2} - \frac{c}{2} - d \\ &= \frac{9 \text{ ft}}{2} - \frac{1 \text{ ft}}{2} - 1 \text{ ft} \\ &= 3 \text{ ft} \end{aligned}$$

The net factored pressure on the footing at this section is obtained in Ex. 2.2 as

$$q_{ux} = 2.71 \text{ kips/ft}^2$$

The factored shear force at the critical section is

$$\begin{aligned} V_u &= \frac{Bx(q_{u,\max} + q_{ux})}{2} \\ &= (5 \text{ ft})(3 \text{ ft}) \left( \frac{3.70 \frac{\text{kips}}{\text{ft}^2} + 2.71 \frac{\text{kips}}{\text{ft}^2}}{2} \right) \\ &= 48 \text{ kips} \end{aligned}$$

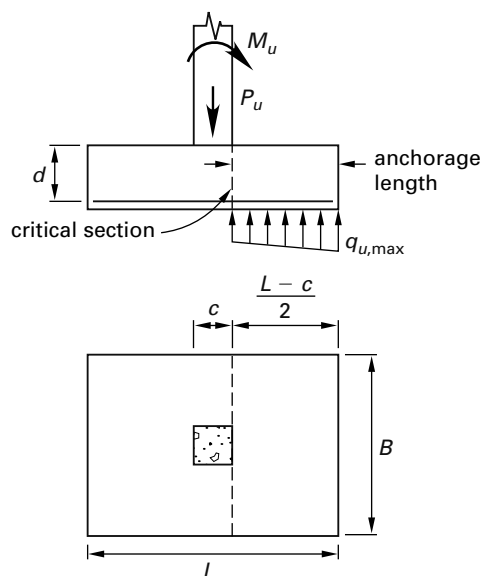
The design flexural shear capacity of the footing is given by ACI Eq. (11-3) as

$$\begin{aligned} \phi V_c &= 2\phi Bd\lambda\sqrt{f'_c} \\ &= \frac{(2)(0.75)(60 \text{ in})(12 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 59 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

## Design for Flexure

ACI Sec. 15.4.2 defines the critical section for flexure to be located at the face of a concrete column, as shown in Fig. 2.6. The required reinforcement area is determined in accordance with ACI Sec. 10.2. The minimum ratio,  $\rho_{\min}$ , of reinforcement area to gross concrete area is specified in ACI Sec. 7.12.2, for both main reinforcement and distribution reinforcement, as 0.0018 for grade 60 bars. As specified in ACI Sec. 10.5.4, the maximum spacing of the main reinforcement shall not exceed 18 in, or three times the footing depth. The diameter of bar provided must be such that the development length does not

Figure 2.6 Critical Section for Flexure





exceed the available anchorage length. Distribution reinforcement may be spaced at a maximum of 18 in, or five times the footing depth.

### Example 2.5

The 9 ft × 5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load of  $P_u = 100$  kips and a bending moment of  $M_u = 100$  ft-kips. For a concrete strength of 3000 lbf/in<sup>2</sup>, determine the area of grade 60 reinforcement required in the longitudinal direction. The effective depth is  $d = 12$  in.

#### Solution

The net factored pressure on the footing at the face of the column is obtained in Ex. 2.2 as

$$q_{uc} = 2.38 \text{ kips/ft}^2$$

The factored moment at the face of the column is

$$\begin{aligned} M_u &= \frac{B\left(\frac{L}{2} - \frac{c}{2}\right)^2 (2q_{u,\max} + q_{uc})}{6} \\ &= \frac{(5 \text{ ft})\left(\frac{9 \text{ ft}}{2} - \frac{1 \text{ ft}}{2}\right)^2}{6} \\ &\quad \times \left( (2)\left(3.70 \frac{\text{kips}}{\text{ft}^2}\right) + 2.38 \frac{\text{kips}}{\text{ft}^2} \right) \\ &= 130.40 \text{ ft-kips} \end{aligned}$$

The design moment factor is

$$\begin{aligned} K_u &= \frac{M_u}{Bd^2} \\ &= \frac{(130.40 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(60 \text{ in})(12 \text{ in})^2} \\ &= 181.11 \text{ lbf/in}^2 \end{aligned}$$

For a tension-controlled section the required reinforcement ratio is

$$\begin{aligned} \rho &= 0.85f'_c \frac{1 - \sqrt{1 - \frac{K_u}{0.383f'_c}}}{f_y} \\ &= (0.85)\left(3 \frac{\text{kips}}{\text{in}^2}\right) \\ &\quad \times \left( \frac{1 - \sqrt{1 - \frac{181.11 \frac{\text{lbf}}{\text{in}^2}}{(0.383)\left(3000 \frac{\text{lbf}}{\text{in}^2}\right)}}}{60 \frac{\text{kips}}{\text{in}^2}} \right) \\ &= 0.00349 \end{aligned}$$

The compression zone factor is given by ACI Sec. 10.2.7.3 as

$$\beta_1 = 0.85$$

The maximum allowable reinforcement ratio for a tension-controlled section is obtained from ACI Sec. 10.3.4 as

$$\begin{aligned} \rho_t &= \frac{0.319\beta_1 f'_c}{f_y} \\ &= \frac{(0.319)(0.85)\left(3 \frac{\text{kips}}{\text{in}^2}\right)}{60 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.014 \\ &> \rho \quad [\text{satisfactory, the section is tension controlled}] \end{aligned}$$

The required area of reinforcement is

$$\begin{aligned} A_s &= \rho B d \\ &= (0.00349)(60 \text{ in})(12 \text{ in}) \\ &= 2.51 \text{ in}^2 \end{aligned}$$

The minimum allowable reinforcement area for a footing is given by ACI Sec. 7.12.2 as

$$\begin{aligned} A_{s,\min} &= 0.0018 B h \\ &= (0.0018)(60 \text{ in})(16 \text{ in}) \\ &= 1.73 \text{ in}^2 \quad [\text{does not govern}] \end{aligned}$$

Providing six no. 6 bars gives an area of

$$\begin{aligned} A_{s,\text{prov}} &= 2.64 \text{ in}^2 \\ &> A_s \quad [\text{satisfactory}] \end{aligned}$$

The anchorage length provided to the reinforcement is

$$\begin{aligned} l_a &= \frac{L}{2} - \frac{c}{2} - \text{end cover} \\ &= \left(\frac{9 \text{ ft}}{2} - \frac{1 \text{ ft}}{2}\right)\left(12 \frac{\text{in}}{\text{ft}}\right) - 3 \text{ in} \\ &= 45 \text{ in} \end{aligned}$$

The clear spacing of the bars exceeds twice the bar diameter, the clear cover exceeds the bar diameter, and the development length of the no. 6 bars is given by ACI Secs. 12.2.2 and 12.2.4 as

$$\begin{aligned} l_d &= \frac{\psi_t \psi_e d_b f_y}{25 \lambda \sqrt{f'_c}} \\ \psi_t &= 1.0 \\ \psi_e &= 1.0 \\ \lambda &= 1.0 \end{aligned}$$

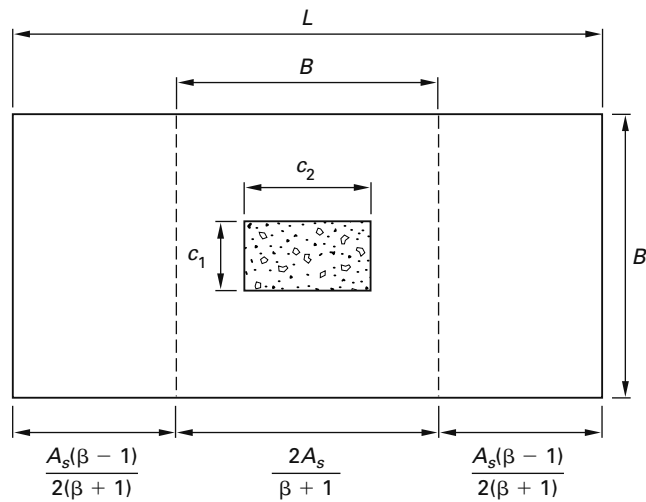
The development length of the no. 6 bars is

$$\begin{aligned}
 l_d &= \frac{d_b f_y}{25 \sqrt{f'_c}} \\
 &= \frac{(0.75) \left( 60,000 \frac{\text{lb}}{\text{in}^2} \right)}{25 \sqrt{3000 \frac{\text{lb}}{\text{in}^2}}} \\
 &= 33 \text{ in} \quad [\text{for no. 6 bar}] \\
 &< 45 \text{ in anchorage length provided} \quad [\text{satisfactory}]
 \end{aligned}$$

### Transverse Reinforcement Band Width

Bending moments are calculated at the critical sections in both the longitudinal and transverse directions. The reinforcement required in the longitudinal direction is distributed uniformly across the width of the footing. Part of the reinforcement required in the transverse direction is concentrated in a central band width equal to the length of the short side of the footing, as shown in Fig. 2.7.

Figure 2.7 Transverse Reinforcement Band Width and Areas



The area of reinforcement required in the central band is given by ACI Sec. 15.4.4.2 as

$$A_b = \frac{2A_s}{\beta + 1}$$

The remainder of the reinforcement required in the transverse direction is

$$A_r = \frac{A_s(\beta - 1)}{\beta + 1}$$

This is distributed uniformly on each side of the center band.

### Example 2.6

The 9 ft  $\times$  5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load of  $P_u = 100$  kips and a bending moment of  $M_u = 100$  ft-kips. For a concrete strength of 3000 lbf/in<sup>2</sup>, determine the area of grade 60 reinforcement required in the transverse direction. The effective depth is  $d = 12$  in.

#### Solution

The factored moment in the transverse direction at the critical section, which is at the face of the column, is

$$\begin{aligned}
 M_u &= \frac{L q_{u,ave} \left( \frac{B}{2} - \frac{c_1}{2} \right)^2}{2} \\
 &= \frac{(9 \text{ ft}) \left( 2.22 \frac{\text{kips}}{\text{ft}^2} \right) \left( \frac{5 \text{ ft}}{2} - \frac{1 \text{ ft}}{2} \right)^2}{2} \\
 &= 39.96 \text{ ft-kips}
 \end{aligned}$$

The design moment factor is

$$\begin{aligned}
 K_u &= \frac{M_u}{L d^2} \\
 &= \frac{(39.96 \text{ ft-kips}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lb}}{\text{kip}} \right)}{(108 \text{ in})(12 \text{ in})^2} \\
 &= 30.83 \text{ lbf/in}^2
 \end{aligned}$$

For a tension-controlled section the required reinforcement ratio is

$$\begin{aligned}
 \rho &= 0.85 f'_c \frac{1 - \sqrt{1 - \frac{K_u}{0.383 f'_c}}}{f_y} \\
 &= (0.85) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) \frac{1 - \sqrt{1 - \frac{30.83 \frac{\text{lbf}}{\text{in}^2}}{(0.383) \left( 3000 \frac{\text{lbf}}{\text{in}^2} \right)}}}{60 \frac{\text{kips}}{\text{in}^2}} \\
 &= 0.00057
 \end{aligned}$$

The required area of reinforcement is

$$\begin{aligned}
 A_s &= \rho L d \\
 &= (0.00057)(108 \text{ in})(12 \text{ in}) \\
 &= 0.74 \text{ in}^2
 \end{aligned}$$

The reinforcement required in the central 5 ft band width is

$$\begin{aligned} A_b &= \frac{2A_s}{\beta + 1} \\ &= \frac{(2)(0.74 \text{ in}^2)}{\frac{9 \text{ ft}}{5 \text{ ft}} + 1} \\ &= 0.53 \text{ in}^2 \end{aligned}$$

The minimum allowable reinforcement area is given by ACI Sec. 7.12.2 as

$$\begin{aligned} A_{s,\min} &= 0.0018Bh \\ &= (0.0018)(60 \text{ in})(16 \text{ in}) \\ &= 1.73 \text{ in}^2 \\ &> A_b \end{aligned}$$

Minimum reinforcement governs, and will be equally spaced along the length of the footing. The total required reinforcement is

$$\begin{aligned} A_{s,\min} &= 0.0018Lh \\ &= (0.0018)(108 \text{ in})(16 \text{ in}) \\ &= 3.11 \text{ in}^2 \end{aligned}$$

Providing 29 no. 3 bars gives an area of

$$\begin{aligned} A_{s,\text{prov}} &= 3.19 \text{ in}^2 \\ &> A_{s,\min} \quad [\text{satisfactory}] \end{aligned}$$

The anchorage length provided to the reinforcement is

$$\begin{aligned} l_a &= \frac{B}{2} - \frac{c}{2} - \text{end cover} \\ &= 30 \text{ in} - 6 \text{ in} - 3 \text{ in} \\ &= 21 \text{ in} \end{aligned}$$

The clear spacing of the bars exceeds twice the bar diameter, the clear cover exceeds the bar diameter, and the development length of the no. 3 bars is obtained from Ex. 2.5 as

$$\begin{aligned} l_d &= 43.8d_b = (43.8)(0.375 \text{ in}) \\ &= 16.4 \text{ in} \quad [\text{no. 3 bar}] \\ &< 21 \text{ in anchorage length provided} \quad [\text{satisfactory}] \end{aligned}$$

### Transfer of Force at Base of Column

Load transfer between a reinforced concrete column and a footing may be effected by bearing on concrete and by reinforcement. The design bearing capacity of the

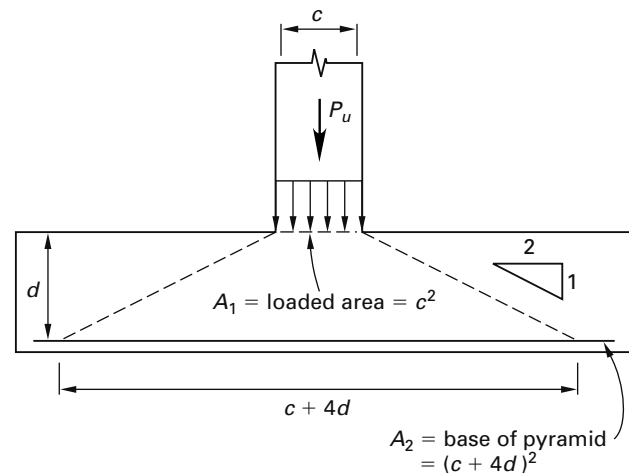
column concrete at the interface is given by ACI Sec. 10.14.1 as

$$\begin{aligned} \phi P_{bn} &= 0.85\phi f'_c A_1 \\ &= 0.553f'_c A_1 \quad [\text{for } \phi = 0.65] \end{aligned}$$

As shown in Fig. 2.8, the bearing capacity of the footing concrete at the interface is given by ACI Sec. 10.14.1 as

$$\begin{aligned} \phi P_{bn} &= 0.85\phi f'_c A_1 \sqrt{\frac{A_2}{A_1}} \\ &\leq (2)(0.85\phi f'_c A_1) \end{aligned}$$

Figure 2.8 Transfer of Load to Footing



In accordance with ACI Sec. R15.8.1.1, when the bearing strength at the base of the column or at the top of the footing is exceeded, reinforcement must be provided to carry the excess load. This reinforcement may be provided by dowels or extended longitudinal bars, and the capacity of this reinforcement is

$$\phi P_s = \phi A_s f_y$$

A minimum area of reinforcement is required across the interface, and this is given by ACI Sec. 15.8.2.1 as

$$A_{s,\min} = 0.005A_1$$

### Example 2.7

The 9 ft × 5 ft reinforced concrete footing described in Ex. 2.1 supports a column with a factored axial load only of  $P_u = 100$  kips. The concrete strength in both the column and the footing is  $f'_c = 3000$  lbf/in<sup>2</sup>. Determine whether the bearing capacity at the interface is adequate.

### Solution

The bearing capacity of the column concrete is given by ACI Sec. 10.14.1 as

$$\begin{aligned}\phi P_{bn} &= 0.553 f'_c A_1 \\ &= (0.553) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) (144 \text{ in}^2) \\ &= 239 \text{ kips} \\ &> P_u \quad [\text{satisfactory}]\end{aligned}$$

The minimum dowel area required at the interface is given by ACI Sec. 15.8.2.1 as

$$\begin{aligned}A_{s,\min} &= 0.005 A_1 \\ &= (0.005) (144 \text{ in}^2) \\ &= 0.72 \text{ in}^2\end{aligned}$$

Provide four no. 4 bars to give an area of

$$\begin{aligned}A_s &= 0.80 \text{ in}^2 \\ &> A_{s,\min} \quad [\text{satisfactory}]\end{aligned}$$

The excess reinforcement factor for the dowel bars is

$$\begin{aligned}\epsilon &= \frac{A_{s,\min}}{A_s} \\ &= \frac{0.72 \text{ in}^2}{0.80 \text{ in}^2} \\ &= 0.90\end{aligned}$$

The development length of the dowels in the column and in the footing is given by ACI Sec. 12.3.2 as the larger of

$$\begin{aligned}l_{dc} &= \epsilon 0.0003 d_b f_y \\ &= (0.90) \left( 0.0003 \frac{\text{in}^2}{\text{lbf}} \right) (0.50 \text{ in}) \left( 60,000 \frac{\text{lbf}}{\text{in}^2} \right) \\ &= 8.1 \text{ in} \\ l_{dc} &= \frac{\epsilon 0.02 d_b f_y}{\lambda \sqrt{f'_c}} \\ &= \frac{(0.90)(0.02)(0.50 \text{ in}) \left( 60,000 \frac{\text{lbf}}{\text{in}^2} \right)}{(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}} \\ &= 9.9 \text{ in}\end{aligned}$$

Use a length of 10 in.

The length of the base of the pyramid with side slopes of 1:2, formed within the footing by the loaded area, is

$$\begin{aligned}L_p &= c + 4d = 12 \text{ in} + (4)(12 \text{ in}) \\ &= 60 \text{ in} \\ &= B \quad [\text{satisfactory}]\end{aligned}$$

The area of the base of the pyramid is

$$\begin{aligned}A_2 &= L_p^2 = (60 \text{ in})^2 \\ &= 3600 \text{ in}^2 \\ \sqrt{\frac{A_2}{A_1}} &= \sqrt{\frac{3600 \text{ in}^2}{144 \text{ in}^2}} \\ &= 5\end{aligned}$$

Use a maximum value of 2, as specified in ACI Sec. 10.14.1.

The bearing capacity of the footing concrete is given by ACI Sec. 10.14.1 as

$$\begin{aligned}\phi P_{bn} &= (2)(0.553 f'_c A_1) \\ &= (2)(0.553) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) (144 \text{ in}^2) \\ &= 480 \text{ kips} \\ &> P_u \quad [\text{satisfactory}]\end{aligned}$$

## 2. COMBINED FOOTINGS

### Nomenclature

|        |                                                         |                     |
|--------|---------------------------------------------------------|---------------------|
| $A_b$  | area of reinforcement in central band                   | in <sup>2</sup>     |
| $A_s$  | total required reinforcement area                       | in <sup>2</sup>     |
| $b_o$  | perimeter of critical section for punching shear        | in                  |
| $B$    | length of short side of a footing                       | ft                  |
| $c$    | length of side of column                                | in                  |
| $d$    | average effective depth of reinforcement in the footing | in                  |
| $d_b$  | bar diameter                                            | in                  |
| $f'_c$ | specified compressive strength of concrete              | lbf/in <sup>2</sup> |
| $h$    | depth of footing                                        | in                  |
| $K_u$  | design moment factor                                    | lbf/in <sup>2</sup> |
| $l$    | distance between column centers                         | ft                  |
| $L$    | length of long side of a footing                        | ft                  |
| $M_u$  | factored moment                                         | ft-kips             |
| $P$    | column axial service load                               | kips                |
| $P_u$  | column axial factored load                              | kips                |
| $q$    | soil bearing pressure caused by service loads           | lbf/ft <sup>2</sup> |
| $q_b$  | soil bearing pressure caused by weight of footing       | lbf/ft <sup>2</sup> |
| $q_e$  | equivalent bearing pressure                             | lbf/ft <sup>2</sup> |
| $q_u$  | net factored pressure acting on footing                 | lbf/ft <sup>2</sup> |
| $V_c$  | nominal shear strength                                  | kips                |

|       |                                                                       |                    |
|-------|-----------------------------------------------------------------------|--------------------|
| $w_c$ | unit weight of concrete                                               | lb/ft <sup>3</sup> |
| $x$   | distance from edge of footing or center of column to critical section | ft                 |
| $x_o$ | distance from edge of property line to centroid of service loads      | ft                 |

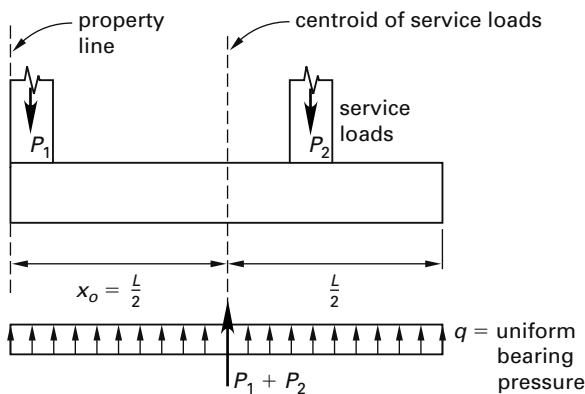
### Symbols

|           |                                                        |   |
|-----------|--------------------------------------------------------|---|
| $\beta$   | ratio of the long side to the short side of footing    | — |
| $\beta_c$ | ratio of the long side to the short side of the column | — |
| $\beta_1$ | compression zone factor given in ACI Sec. 10.2.7.3     | — |
| $\lambda$ | lightweight aggregate concrete factor                  | — |
| $\rho$    | reinforcement ratio                                    | — |
| $\rho_t$  | reinforcement ratio for a tension-controlled section   | — |
| $\phi$    | strength reduction factor                              | — |

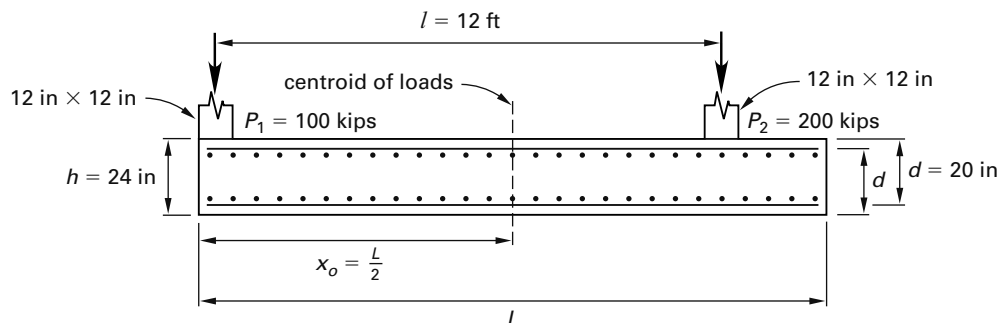
### Soil Pressure Distribution

A combined footing is used, as shown in Fig. 2.9, when a column must be located adjacent to the property line. A second column is placed on the combined footing, and the length of the footing is adjusted until its centroid coincides with the centroid of the service loads on the two columns. Hence, a uniformly distributed soil pressure is produced under the combined footing. The

**Figure 2.9** Soil Pressure for Service Loads



*Illustration for Example 2.8*



footing width is adjusted to ensure that the soil bearing pressure does not exceed the allowable pressure.

### Example 2.8

Determine the plan dimensions required to provide a uniform soil bearing pressure of  $q = 3000$  lb/ft<sup>2</sup> under the service loads indicated for the combined footing shown in the illustration.

### Solution

The weight of the footing produces a uniformly distributed pressure on the soil of

$$q_b = w_c h = \frac{\left(150 \frac{\text{lb}}{\text{ft}^3}\right)(2 \text{ ft})}{1000 \frac{\text{lb}}{\text{kip}}} = 0.3 \text{ kips/ft}^2$$

The maximum allowable equivalent soil bearing pressure is

$$q_e = q - q_b = 3 \frac{\text{kips}}{\text{ft}^2} - 0.3 \frac{\text{kips}}{\text{ft}^2} = 2.7 \text{ kips/ft}^2$$

The centroid of the column service loads is located a distance  $x_o$  from the property line. The distance is obtained by taking moments about the property line, and given by

$$x_o = \frac{0.5P_1 + 12.5P_2}{P_1 + P_2} = \frac{(0.5 \text{ ft})(100 \text{ kips}) + (12.5 \text{ ft})(200 \text{ kips})}{100 \text{ kips} + 200 \text{ kips}} = 8.5 \text{ ft}$$

Hence, the length of footing required to produce a uniform bearing pressure on the soil is

$$L = 2x_o = (2)(8.5 \text{ ft}) = 17 \text{ ft}$$

The width of footing required to produce a uniform pressure on the soil of 3000 lbf/ft<sup>2</sup> is

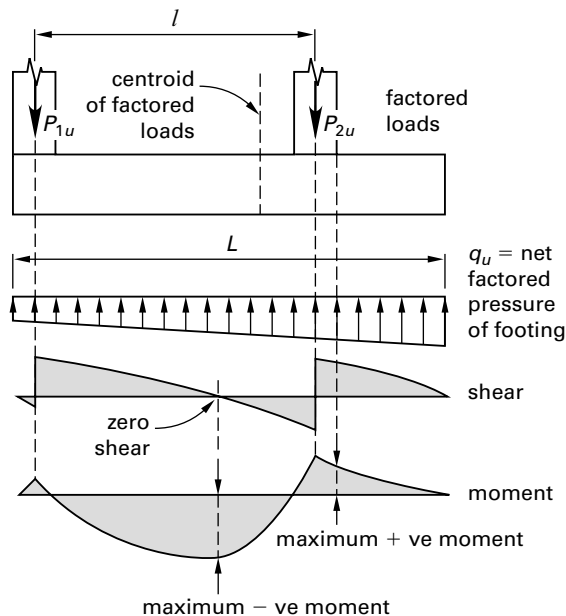
$$B = \frac{P_1 + P_2}{q_e L} = \frac{100 \text{ kips} + 200 \text{ kips}}{\left(2.7 \frac{\text{kips}}{\text{ft}^2}\right)(17 \text{ ft})}$$

$$= 6.5 \text{ ft}$$

### Factored Soil Pressure

The reinforced concrete footing must be designed for punching shear, flexural shear, and flexure. The critical section for each of these effects is located at a different position in the footing, and each must be designed for the applied factored loads. The soil pressure distribution due to factored loads must be determined as shown in Fig. 2.10. The soil pressure will not necessarily be uniformly distributed unless the ratios of the factored loads to service loads on both columns are identical. Because the self-weight of the footing produces an equal and opposite pressure in the soil, the footing is designed for the net pressure from the column loads only, and the weight of the footing is not included.

Figure 2.10 Soil Pressure for Factored Loads



### Example 2.9

The normal weight concrete combined footing described in Ex. 2.8 has a factored load on each column that is 1.5 times the service load. Determine the factored pressure distribution in the soil.

### Solution

The ratios of the factored loads to the service loads on both columns are identical. The net factored pressure on the footing is uniform, and has a value of

$$q_u = 1.5 q_e$$

$$= (1.5) \left(2.7 \frac{\text{kips}}{\text{ft}^2}\right)$$

$$= 4.05 \text{ kips/ft}^2$$

For column no. 1, the factored load is

$$P_{1u} = 1.5 P_1$$

$$= (1.5)(100 \text{ kips})$$

$$= 150 \text{ kips}$$

For column no. 2, the factored load is

$$P_{2u} = 1.5 P_2$$

$$= (1.5)(200 \text{ kips})$$

$$= 300 \text{ kips}$$

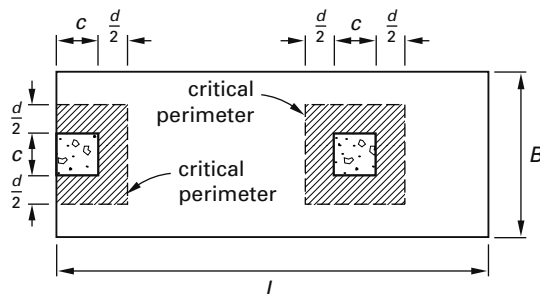
The shear force and bending moment diagrams for the combined footing are shown in the illustration.

### Design for Punching Shear

The depth of footing may be governed by the punching shear capacity. The critical perimeter for punching shear is specified in ACI Secs. 15.5.2 and 11.11.1.2 and illustrated in Fig. 2.11. For a concrete column, the critical perimeter is a distance from the face of the column equal to half the effective depth. For the interior column, the length of the critical perimeter is

$$b_o = 4(c + d)$$

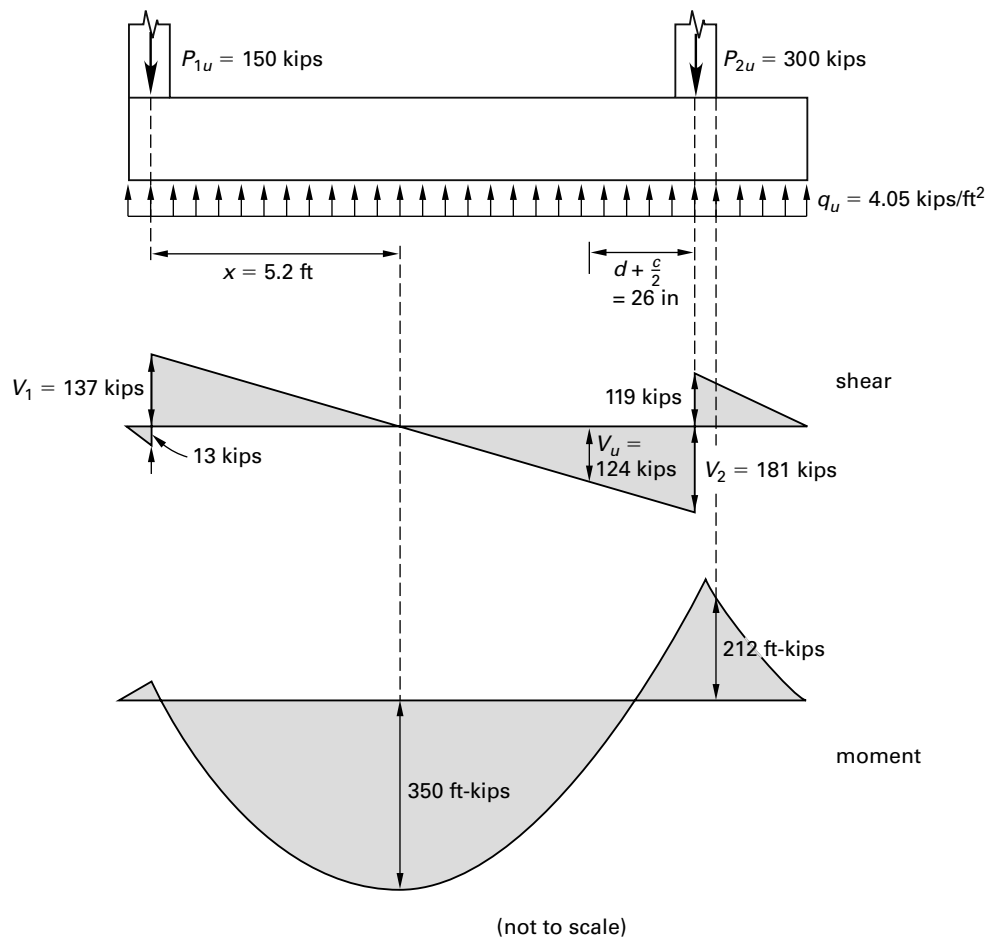
Figure 2.11 Critical Perimeter for Punching Shear



For the end column, the length of the critical perimeter is

$$b_o = (c + d) + 2 \left(c + \frac{d}{2}\right)$$

Illustration for Example 2.9



The design punching shear strength of the footing is determined by ACI Sec. 11.11.2.1 as

$$\phi V_c = 4\phi db_o \lambda \sqrt{f'_c} \quad [\text{ACI 11-33}]$$

When  $\beta_c > 2$ , the design punching shear strength is

$$\phi V_c = \phi db_o \lambda \left( 2 + \frac{4}{\beta_c} \right) \sqrt{f'_c} \quad [\text{ACI 11-31}]$$

$$\phi = 0.75$$

### Example 2.10

For the combined footing described in Ex. 2.9, determine whether the punching shear capacity is adequate. The concrete strength is 3000 lbf/in<sup>2</sup>, and the effective depth is  $d = 20$  in.

### Solution

For column no. 1, the length of the critical perimeter is

$$\begin{aligned} b_o &= (c + d) + 2 \left( c + \frac{d}{2} \right) \\ &= (12 \text{ in} + 20 \text{ in}) + (2) \left( 12 \text{ in} + \frac{20 \text{ in}}{2} \right) \\ &= 76 \text{ in} \end{aligned}$$

The punching shear force at the critical perimeter is

$$\begin{aligned} V_u &= P_{1u} - q_u (c + d) \left( c + \frac{d}{2} \right) \\ &= 150 \text{ kips} \\ &\quad - \frac{\left( 4.05 \frac{\text{kips}}{\text{ft}^2} \right) (12 \text{ in} + 20 \text{ in}) \left( 12 \text{ in} + \frac{20 \text{ in}}{2} \right)}{\left( 12 \frac{\text{in}}{\text{ft}} \right)^2} \\ &= 130 \text{ kips} \end{aligned}$$



The design punching shear capacity is given by ACI Eq. (11-33) as

$$\begin{aligned}\phi V_c &= 4\phi db_o \lambda \sqrt{f'_c} \\ &= \frac{(4)(0.75)(20 \text{ in})(76 \text{ in})(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 250 \text{ kips} \\ &> V_u \quad [\text{satisfactory}]\end{aligned}$$

For column no. 2 the length of the critical perimeter is

$$\begin{aligned}b_o &= 4(c + d) \\ &= (4)(12 \text{ in} + 20 \text{ in}) \\ &= 128 \text{ in}\end{aligned}$$

The punching shear force at the critical perimeter is

$$\begin{aligned}V_u &= P_{2u} - q_u(c + d)^2 \\ &= 300 \text{ kips} - \frac{\left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(12 \text{ in} + 20 \text{ in})^2}{\left(12 \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 271 \text{ kips}\end{aligned}$$

The design punching shear capacity is given by ACI Eq. (11-33) as

$$\begin{aligned}\phi V_c &= 4\phi db_o \lambda \sqrt{f'_c} \\ &= \frac{(4)(0.75)(20 \text{ in})(128 \text{ in})(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 421 \text{ kips} \\ &> V_u \quad [\text{satisfactory}]\end{aligned}$$

### Design for Flexural Shear

For combined footings, the location of the critical section for flexural shear is defined in ACI Secs. 15.5.2 and 11.1.3.1 as being located a distance,  $d$ , from the face of the concrete column, as shown in Fig. 2.5. The shear force at the critical section is obtained from the shear force diagram for the footing. The design flexural shear strength of the footing is given by ACI Sec. 11.2.1.1 as

$$\phi V_c = 2\phi bd \lambda \sqrt{f'_c} \quad [\text{ACI 11-3}]$$

### Example 2.11

For the combined footing described in Ex. 2.9, determine whether the flexural shear capacity is adequate. The concrete strength is  $3000 \text{ lbf/in}^2$ , and the effective depth is  $d = 20 \text{ in}$ .

*Solution*

At the center of column no. 1 the shear force is

$$\begin{aligned}V_1 &= P_{1u} - \frac{q_u Bc}{2} \\ &= 150 \text{ kips} - \frac{\left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})(1.0 \text{ ft})}{2} \\ &= 137 \text{ kips}\end{aligned}$$

At the center of column no. 2 the shear force is

$$\begin{aligned}V_2 &= V_1 - q_u B l \\ &= 137 \text{ kips} - \left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})(12 \text{ ft}) \\ &= -181 \text{ kips}\end{aligned}$$

The shear force diagram is shown in Ex. 2.9. The critical flexural shear is at a distance from the center of column no. 2 given by

$$d + \frac{c}{2} = \frac{20 \text{ in} + \frac{12 \text{ in}}{2}}{12 \frac{\text{in}}{\text{ft}}} = 2.17 \text{ ft}$$

The critical flexural shear at this section is

$$\begin{aligned}V_u &= V_2 - q_u B \left(d + \frac{c}{2}\right) \\ &= -181 \text{ kips} + \left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})(2.17 \text{ ft}) \\ &= -124 \text{ kips}\end{aligned}$$

The flexural shear capacity of the footing is given by ACI Eq. (11-3) as

$$\begin{aligned}\phi V_c &= 2\phi bd \lambda \sqrt{f'_c} \\ &= (2)(0.75)(6.54 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right) (20 \text{ in}) \\ &\quad \times (1.0) \left(\frac{\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}\right) \\ &= 129 \text{ kips} \\ &> V_u \quad [\text{satisfactory}]\end{aligned}$$

### Design for Flexure

The footing is designed in the longitudinal direction as a beam continuous over two supports. As shown in Fig. 2.10, the maximum negative moment occurs at the section of zero shear. The maximum positive moment occurs at the outside face of column no. 2. In the transverse direction, it is assumed that the footing cantilevers about the face of both columns. The required

reinforcement is concentrated under each column in a band width equal to the length of the shorter side. The area of reinforcement required in the band width under column no. 1 is given by ACI Sec. 15.4.4.2 as

$$A_{1b} = \frac{2A_s P_{1u}}{(\beta + 1)(P_{1u} + P_{2u})}$$

### Example 2.12

Determine the longitudinal grade 60 reinforcement required for the combined footing of Ex. 2.9. The effective depth is  $d = 20$  in, and the concrete strength is 3000 lbf/in<sup>2</sup>.

#### Solution

From Ex. 2.9, the point of zero shear is a distance from the center of column no. 1 given by

$$x = \frac{V_1}{q_u B} = \frac{137 \text{ kips}}{\left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})} = 5.2 \text{ ft}$$

The maximum negative moment at this point is

$$\begin{aligned} M_u &= P_{1u}x - \frac{q_u B \left(x + \frac{c}{2}\right)^2}{2} \\ &= (150 \text{ kips})(5.2 \text{ ft}) \\ &\quad - \frac{\left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})\left(5.2 \text{ ft} + \frac{1 \text{ ft}}{2}\right)^2}{2} \\ &= 350 \text{ ft-kips} \end{aligned}$$

The design moment factor is

$$\begin{aligned} K_u &= \frac{M_u}{Bd^2} \\ &= \left(\frac{350 \text{ ft-kips}}{(78.5 \text{ in})(20 \text{ in})^2}\right)\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right) \\ &= 134 \text{ lbf/in}^2 \end{aligned}$$

For a tension-controlled section, the required reinforcement ratio is

$$\begin{aligned} \rho &= 0.85f'_c \left( \frac{1 - \sqrt{1 - \frac{K_u}{0.383f'_c}}}{f_y} \right) \\ &= (0.85) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) \left( \frac{1 - \sqrt{1 - \frac{134 \frac{\text{lbf}}{\text{in}^2}}{(0.383) \left( 3000 \frac{\text{lbf}}{\text{in}^2} \right)}}}{60 \frac{\text{kips}}{\text{in}^2}} \right) \\ &= 0.00256 \end{aligned}$$

The required area of reinforcement is

$$\begin{aligned} A_s &= \rho B d \\ &= (0.00256)(78.5 \text{ in})(20 \text{ in}) \\ &= 4.0 \text{ in}^2 \end{aligned}$$

The compression zone factor is given by ACI Sec. 10.2.7.3 as

$$\beta_1 = 0.85$$

The maximum allowable reinforcement ratio for a tension-controlled section is obtained from ACI Sec. 10.3.4 as

$$\begin{aligned} \rho_t &= \frac{0.319\beta_1 f'_c}{f_y} \\ &= \frac{(0.319)(0.85) \left( 3 \frac{\text{kips}}{\text{in}^2} \right)}{60 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.014 \\ &< \rho \quad [\text{satisfactory, the section is tension-controlled}] \end{aligned}$$

The minimum allowable reinforcement area for a footing is given by ACI Sec. 7.12.2 as

$$\begin{aligned} A_{s,\min} &= 0.0018 B h \\ &= (0.0018)(78.5 \text{ in})(24 \text{ in}) \\ &= 3.39 \text{ in}^2 \quad [\text{does not govern}] \\ &< A_s \end{aligned}$$

Providing 10 no. 6 bars gives an area of

$$\begin{aligned} A_{s,\text{prov}} &= 4.4 \text{ in}^2 \\ &> A_s \quad [\text{satisfactory}] \end{aligned}$$

The maximum positive moment at the outside face of column no. 2 is

$$\begin{aligned} M_u &= \frac{q_u B (L - l - c)^2}{2} \\ &= \frac{\left(4.05 \frac{\text{kips}}{\text{ft}^2}\right)(6.54 \text{ ft})(17 \text{ ft} - 12 \text{ ft} - 1 \text{ ft})^2}{2} \\ &= 212 \text{ ft-kips} \end{aligned}$$

The design moment factor is

$$\begin{aligned} K_u &= \frac{M_u}{Bd^2} \\ &= \left(\frac{212 \text{ ft-kips}}{(78.5 \text{ in})(20 \text{ in})^2}\right)\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right) \\ &= 81 \text{ lbf/in}^2 \end{aligned}$$

For a tension-controlled section the required reinforcement ratio is

$$\rho = 0.85f'_c \left( \frac{1 - \sqrt{1 - \frac{K_u}{0.383f'_c}}}{f_y} \right)$$

$$= (0.85) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) \left( \frac{1 - \sqrt{1 - \frac{81 \frac{\text{lbf}}{\text{in}^2}}{(0.383) \left( 3000 \frac{\text{lbf}}{\text{in}^2} \right)}}}{60 \frac{\text{kips}}{\text{in}^2}} \right)$$

$$= 0.00153$$

Hence, the required area of reinforcement is

$$A_s = \rho B d = (0.00153)(78.5 \text{ in})(20 \text{ in})$$

$$= 2.4 \text{ in}^2$$

Providing 12 no. 4 bars gives an area of

$$A_{s,\text{prov}} = 2.4 \text{ in}^2$$

$$= A_s \quad [\text{satisfactory}]$$

### 3. STRAP FOOTINGS

#### Nomenclature

|       |                                             |                   |
|-------|---------------------------------------------|-------------------|
| $A_1$ | base area of pad footing no. 1              | $\text{in}^2$     |
| $A_2$ | base area of pad footing no. 2              | $\text{in}^2$     |
| $B_S$ | length of short side of strap               | ft                |
| $B_1$ | length of short side of pad footing no. 1   | ft                |
| $B_2$ | length of short side of pad footing no. 2   | ft                |
| $h_S$ | depth of strap                              | in                |
| $h_1$ | depth of pad footing no. 1                  | in                |
| $h_2$ | depth of pad footing no. 2                  | in                |
| $l$   | distance between column centers             | ft                |
| $l_R$ | distance between soil reactions             | ft                |
| $L_S$ | length of long side of strap                | ft                |
| $L_1$ | length of long side of pad footing no. 1    | ft                |
| $L_2$ | length of long side of pad footing no. 2    | ft                |
| $q$   | soil pressure                               | $\text{lbf/ft}^2$ |
| $R_1$ | soil reaction under pad footing no. 1       | kips              |
| $R_2$ | soil reaction under pad footing no. 2       | kips              |
| $V_c$ | nominal shear strength provided by concrete | lbf               |
| $w_c$ | unit weight of concrete                     | $\text{lbf/ft}^3$ |
| $W_S$ | weight of strap beam                        | kips              |
| $W_1$ | weight of pad footing no. 1                 | kips              |
| $W_2$ | weight of pad footing no. 2                 | kips              |

#### Soil Pressure Distribution

As shown in Fig. 2.12, a strap footing consists of two pad footings connected by a strap beam. The strap

beam is underlaid by a layer of Styrofoam™ so that the soil pressure under the strap may be considered negligible. It is further assumed that the strap and pad footings act as a rigid body producing uniform soil pressure under the pad footings. The base areas of the two pad footings are adjusted to produce an identical soil pressure under both footings.

The total acting service load is obtained from Fig. 2.12 as

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_S$$

The soil pressure under the pad footings is

$$q = \frac{\sum P}{A_1 + A_2}$$

The soil reactions act at the center of the pad footings, and are

$$R_1 = qA_1$$

$$R_2 = qA_2$$

Pad footing no. 2 is located symmetrically with respect to column no. 2 so that the lines of action of  $P_2$  and  $R_2$  are coincident. The distance between soil reactions is

$$l_R = l + \frac{c_1}{2} - \frac{B_1}{2}$$

The total length of the strap is

$$L_S = l_R - \frac{B_1 + B_2}{2}$$

Equating vertical forces gives

$$R_2 = \sum P - R_1 \quad [\text{equilibrium equation no. 1}]$$

Taking moments about the center of pad footing no. 2 gives

$$R_1 = \frac{P_1 l + W_1 l_R + \frac{W_S(L_S + B_2)}{2}}{l_R}$$

[equilibrium equation no. 2]

To determine dimensions that will give a soil bearing pressure equal to the allowable pressure,  $q$ , suitable values are selected for  $h_1$ ,  $h_2$ ,  $h_S$ ,  $B_1$ ,  $B_2$ , and  $B_S$ . The dimensions  $l_R$  and  $L_S$  are determined, and

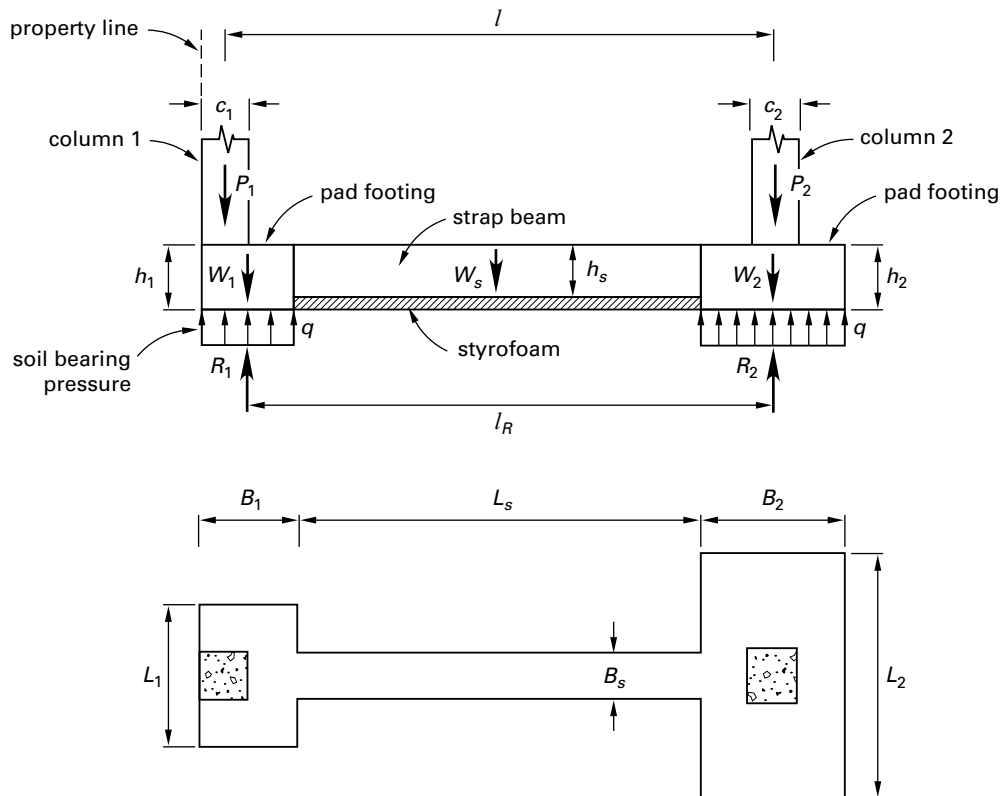
$$W_S = w_c L_S B_S h_S$$

An initial estimate is made of  $R_1$ , and

$$A_1 = \frac{R_1}{q}$$

$$W_1 = w_c A_1 h_1$$

Figure 2.12 Strap Footing With Applied Service Loads



An initial estimate is made of  $R_2$ , and

$$A_2 = \frac{R_2}{q}$$

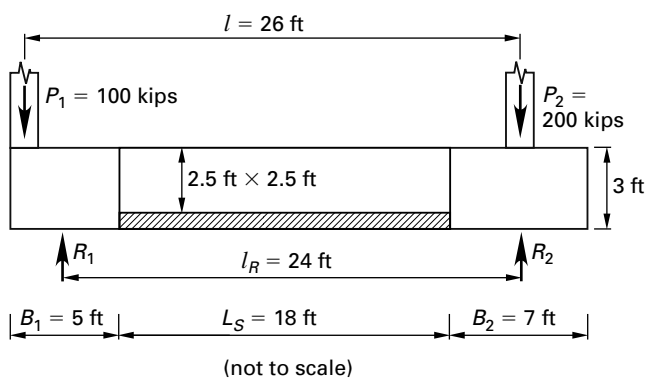
$$W_2 = w_c A_2 h_2$$

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_s$$

Substituting in the two equilibrium equations provides revised estimates of  $R_1$  and  $R_2$ , and the process is repeated until convergence is reached.

### Example 2.13

Determine the dimensions required for the strap footing shown in the illustration to provide a uniform bearing pressure of 4000 lbf/ft<sup>2</sup> under both pad footings for the service loads indicated.



### Solution

From the dimensions indicated in the illustration, the weight of the strap is

$$W_s = w_c L_s B_s h_s$$

$$= \left(0.15 \frac{\text{kips}}{\text{ft}^3}\right) (18 \text{ ft}) (2.5 \text{ ft}) (2.5 \text{ ft})$$

$$= 16.9 \text{ kips}$$

As an initial estimate, assume that soil reaction under pad footing no. 1 is

$$R_1 = 131 \text{ kips}$$

Then, the area and weight of pad footing no. 1 are

$$A_1 = \frac{R_1}{q} = \frac{131 \text{ kips}}{4 \frac{\text{kips}}{\text{ft}^2}}$$

$$= 32.8 \text{ ft}^2$$

$$W_1 = w_c A_1 h_1 = \left(0.15 \frac{\text{kips}}{\text{ft}^3}\right) (32.8 \text{ ft}^2) (3 \text{ ft})$$

$$= 14.8 \text{ kips}$$

As an initial estimate, assume that soil reaction under pad footing no. 2 is

$$R_2 = 225 \text{ kips}$$

Then, the area and weight of pad footing no. 2 are

$$A_2 = \frac{R_2}{q} = \frac{225 \text{ kips}}{4 \frac{\text{kips}}{\text{ft}^2}} = 56.3 \text{ ft}^2$$

$$W_2 = w_c A_2 h_2 = \left(0.15 \frac{\text{kips}}{\text{ft}^3}\right) (56.3 \text{ ft}^2) (3 \text{ ft}) = 25.3 \text{ kips}$$

The total applied load is

$$\begin{aligned} \sum P &= P_1 + P_2 + W_1 + W_2 + W_s \\ &= 100 \text{ kips} + 200 \text{ kips} + 14.7 \text{ kips} \\ &\quad + 25.3 \text{ kips} + 16.9 \text{ kips} \\ &= 356.9 \text{ kips} \end{aligned}$$

Taking moments about the center of pad footing no. 2 gives

$$R_1 = \frac{P_1 l + W_1 l_R + \frac{W_s (L_s + B_2)}{2}}{l_R}$$

$$= \frac{(100 \text{ kips})(26 \text{ ft}) + (14.7 \text{ kips})(24 \text{ ft}) + \frac{(16.9 \text{ ft})(18 \text{ ft} + 7 \text{ ft})}{2}}{24 \text{ ft}}$$

$$= 132 \text{ kips}$$

$$\approx 131 \text{ kips} \quad [\text{satisfactory}]$$

Equating vertical forces gives

$$R_2 = \sum P - R_1 = 356.9 \text{ kips} - 132 \text{ kips} = 224.9 \text{ kips}$$

$$\approx 225 \text{ kips} \quad [\text{satisfactory}]$$

The initial estimates are sufficiently accurate, and the required pad footing areas are

$$A_1 = 32.8 \text{ ft}^2$$

$$A_2 = 56.3 \text{ ft}^2$$

The required widths of the pad footings are

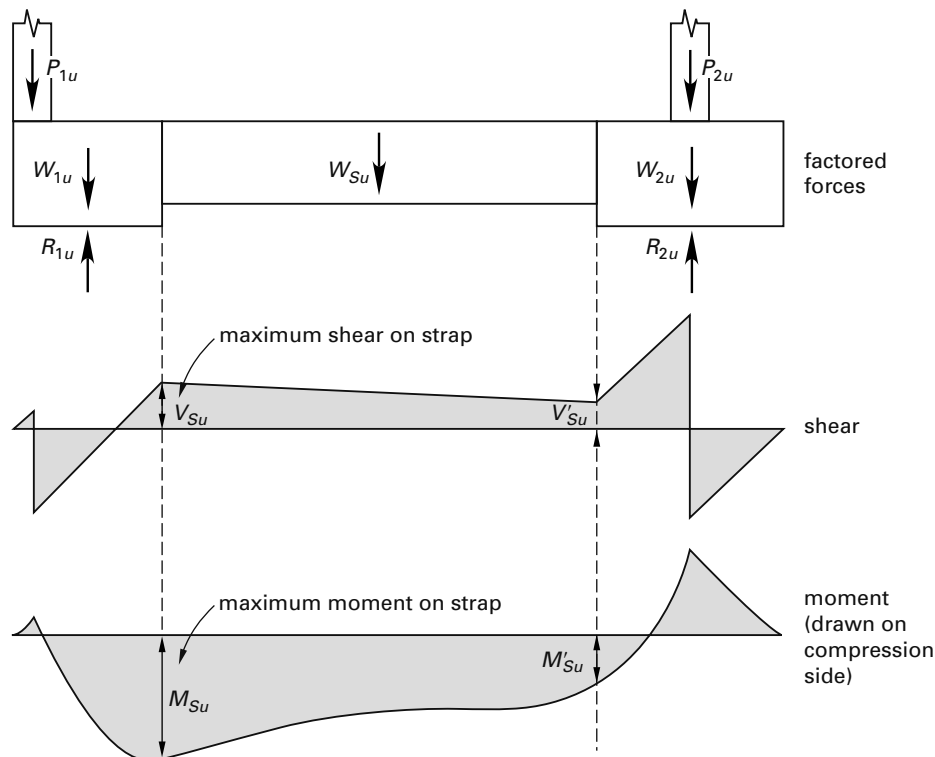
$$L_1 = \frac{A_1}{B_1} = \frac{32.8 \text{ ft}^2}{5 \text{ ft}} = 6.56 \text{ ft}$$

$$L_2 = \frac{A_2}{B_2} = \frac{56.3 \text{ ft}^2}{7 \text{ ft}} = 8.04 \text{ ft}$$

### Factored Soil Pressure

The reinforced concrete strap must be designed for flexural shear and flexure. The critical section for each of these effects is located at a different position in the strap, and each must be designed for the applied factored loads. The factored loads must be determined as shown in Fig. 2.13. The soil pressure will not necessarily be identical under both pad footings.

Figure 2.13 Soil Pressure for Factored Loads



**Example 2.14**

The strap footing described in Ex. 2.13 has a factored load on each column that is 1.5 times the service load. Determine the factored soil reaction under each pad footing.

*Solution*

For column no. 1, the factored load is

$$\begin{aligned} P_{1u} &= 1.5P_1 = (1.5)(100 \text{ kips}) \\ &= 150 \text{ kips} \end{aligned}$$

For column no. 2, the factored load is

$$\begin{aligned} P_{2u} &= 1.5P_2 = (1.5)(200 \text{ kips}) \\ &= 300 \text{ kips} \end{aligned}$$

The factored weights of the component parts are

$$\begin{aligned} W_{1u} &= 1.2W_1 = (1.2)(14.7 \text{ kips}) \\ &= 17.6 \text{ kips} \end{aligned}$$

$$\begin{aligned} W_{2u} &= 1.2W_2 = (1.2)(25.3 \text{ kips}) \\ &= 30.4 \text{ kips} \end{aligned}$$

$$\begin{aligned} W_{Su} &= 1.2W_S \\ &= (1.2)(16.9 \text{ kips}) \\ &= 20.3 \text{ kips} \end{aligned}$$

The total factored load on the footing is

$$\begin{aligned} \sum P_u &= P_{1u} + P_{2u} + W_{1u} + W_{2u} + W_{Su} \\ &= 150 \text{ kips} + 300 \text{ kips} + 17.6 \text{ kips} \\ &\quad + 30.4 \text{ kips} + 20.3 \text{ kips} \\ &= 518 \text{ kips} \end{aligned}$$

Taking moments about the center of pad footing no. 2 gives

$$\begin{aligned} R_{1u} &= \frac{\sum P_{1u}l + W_{1u}l_R + \frac{W_{Su}(L_S + B_2)}{2}}{l_R} \\ &= \frac{(150 \text{ kips})(26 \text{ ft}) + (17.6 \text{ kips})(24 \text{ ft})}{24 \text{ ft}} \\ &\quad + \frac{(20.3 \text{ kips})(18 \text{ ft} + 7 \text{ ft})}{2} \\ &= 191 \text{ kips} \end{aligned}$$

Equating vertical forces gives

$$\begin{aligned} R_{2u} &= \sum P_u - R_{1u} \\ &= 518 \text{ kips} - 191 \text{ kips} \\ &= 327 \text{ kips} \end{aligned}$$

**Design of Strap Beam for Shear**

The factored forces acting on the footing are shown in Fig. 2.13. The total factored load on the footing is

$$\sum P_u = P_{1u} + P_{2u} + W_{1u} + W_{2u} + W_{Su}$$

Taking moments about the center of pad footing no. 2 gives

$$R_{1u} = \frac{P_{1u}l + w_{1u}l_R + \frac{W_{Su}(L_S + B_2)}{2}}{l_R}$$

Equating vertical forces gives

$$R_{2u} = \sum P_u - R_{1u}$$

The shear at the left end of the strap is

$$V_{Su} = R_{1u} - P_{1u} - W_{1u}$$

The shear at the right end of the strap is

$$V'_{Su} = P_{2u} + W_{2u} - R_{2u}$$

The design flexural shear strength of the strap beam is

$$\phi V_c = 2\phi bd\lambda\sqrt{f'_c} \quad [\text{ACI 11-3}]$$

**Example 2.15**

The normal weight concrete strap footing for Ex. 2.13 has a concrete strength of 3000 lb/in<sup>2</sup> and a factored load on each column that is 1.5 times the service load. The strap beam has an effective depth of 27 in. Determine whether the shear capacity of the strap beam is adequate.

*Solution*

The factored soil reactions under the pad footings are obtained from Ex. 2.14 as

$$R_{1u} = 191 \text{ kips}$$

$$R_{2u} = 327 \text{ kips}$$

The factored shear force at the right end of the strap is

$$\begin{aligned} V'_{Su} &= P_{2u} + W_{2u} - R_{2u} \\ &= 300 \text{ kips} + 30.4 \text{ kips} - 327 \text{ kips} \\ &= 3.4 \text{ kips} \end{aligned}$$

The factored shear force at the left end of the strap is

$$\begin{aligned} V_{Su} &= R_{1u} - P_{1u} - W_{1u} \\ &= 191 \text{ kips} - 150 \text{ kips} - 17.6 \text{ kips} \\ &= 23.4 \text{ kips} \quad [\text{governs}] \end{aligned}$$

The design shear capacity of the strap beam is given by ACI Eq. (11-3) as

$$\begin{aligned}\phi V_c &= 2\phi b d \lambda \sqrt{f'_c} \\ &= \frac{(2)(0.75)(30 \text{ in})(27 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 67 \text{ kips} \\ &> 2 V_{Su} \quad [\text{no shear reinforcement is required}]\end{aligned}$$

### Design of Strap Beam for Flexure

From Fig. 2.13, the factored moment at the left end of the strap is

$$M_{Su} = P_{1u} \left( B_1 - \frac{c_1}{2} \right) - \frac{(R_{1u} - W_{1u})B_1}{2}$$

The factored moment at the right end of the strap is

$$M'_{Su} = \frac{(R_{2u} - W_{2u} - P_{2u})B_2}{2}$$

### Example 2.16

Determine the required grade 60 flexural reinforcement for the strap beam of Ex. 2.13. The strap beam has an effective depth of 27 in.

*Solution*

The factored moment at the right end of the strap is

$$\begin{aligned}M'_{Su} &= \frac{(R_{2u} - W_{2u} - P_{2u})B_2}{2} \\ &= \frac{(327 \text{ kips} - 30.4 \text{ kips} - 300 \text{ kips})(7 \text{ ft})}{2} \\ &= -11.9 \text{ ft-kips}\end{aligned}$$

The factored moment at the left end of the strap is

$$\begin{aligned}M_{Su} &= P_{1u} \left( B_1 - \frac{c_1}{2} \right) - \frac{(R_{1u} - W_{1u})B_1}{2} \\ &= (150 \text{ kips}) \left( 5 \text{ ft} - \frac{1 \text{ ft}}{2} \right) \\ &\quad - \frac{(191 \text{ kips} - 17.6 \text{ kips})(5 \text{ ft})}{2} \\ &= 242 \text{ ft-kips} \quad [\text{governs}]\end{aligned}$$

The design moment factor is

$$\begin{aligned}K_u &= \frac{M_{Su}}{b d^2} \\ &= \frac{(242 \text{ ft-kips}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)}{(30 \text{ in})(27 \text{ in})^2} \\ &= 133 \text{ lbf/in}^2\end{aligned}$$

For a tension-controlled section, the required reinforcement ratio is

$$\begin{aligned}\rho &= \frac{0.85 f'_c \left( 1 - \sqrt{1 - \frac{K_u}{0.383 f'_c}} \right)}{f_y} \\ &= \frac{(0.85) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) \left( 1 - \sqrt{1 - \frac{133 \frac{\text{lbf}}{\text{in}^2}}{(0.383) \left( 3000 \frac{\text{lbf}}{\text{in}^2} \right)}} \right)}{60 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.00254\end{aligned}$$

The controlling minimum reinforcement ratio is given by ACI Sec. 10.5.1 as the greater of

$$\begin{aligned}\rho_{\min} &= \frac{3\sqrt{f'_c}}{f_y} = \frac{3\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{60,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.00274\end{aligned}$$

$$\begin{aligned}\rho_{\min} &= \frac{200}{f_y} = \frac{200 \frac{\text{lbf}}{\text{in}^2}}{60,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.0033 \quad [\text{governs}]\end{aligned}$$

In accordance with ACI Sec. 10.5.3, the reinforcement ratio need not exceed

$$\begin{aligned}\rho_s &= \frac{4\rho}{3} = \frac{(4)(0.00254)}{3} \\ &= 0.0034 \quad [\text{does not control}]\end{aligned}$$

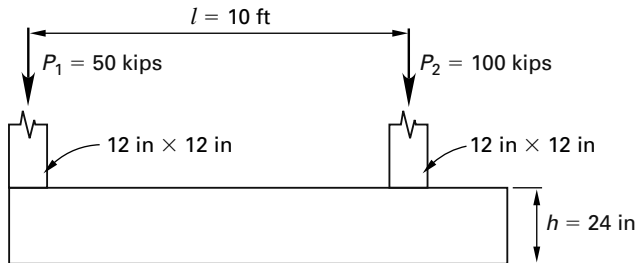
The required area of reinforcement in the top of the strap beam is

$$\begin{aligned}A_s &= \rho_{\min} b d = (0.0033)(30 \text{ in})(27 \text{ in}) \\ &= 2.67 \text{ in}^2\end{aligned}$$

Providing six no. 6 bars gives an area of 2.64 in<sup>2</sup>, which is satisfactory.

### PRACTICE PROBLEMS

Problems 1–8 refer to the combined footing shown. The factored load on each column is 1.4 times the service load, the effective depth is  $d = 20$  in, and the concrete strength is  $3000 \text{ lbf/in}^2$ .



1. What are most nearly the plan dimensions required to provide a uniform soil bearing pressure of  $q = 2000 \text{ lbf/ft}^2$  under the service loads indicated?
  - (A)  $14.3 \text{ ft} \times 5.25 \text{ ft}$
  - (B)  $14.3 \text{ ft} \times 6.17 \text{ ft}$
  - (C)  $21.5 \text{ ft} \times 3.49 \text{ ft}$
  - (D)  $21.5 \text{ ft} \times 4.10 \text{ ft}$
2. What is most nearly the factored pressure distribution in the soil?
  - (A)  $2.38 \text{ kips/ft}^2$
  - (B)  $2.80 \text{ kips/ft}^2$
  - (C)  $3.13 \text{ kips/ft}^2$
  - (D)  $4.37 \text{ kips/ft}^2$
3. What is most nearly the punching shear force for each column?
  - (A) column no. 1: 48.6 kips, column no. 2: 109 kips
  - (B) column no. 1: 56.3 kips, column no. 2: 120 kips
  - (C) column no. 1: 54.7 kips, column no. 2: 118 kips
  - (D) column no. 1: 58.4 kips, column no. 2: 123 kips
4. What is most nearly the punching shear capacity for each column?
  - (A) column no. 1: 200 kips, column no. 2: 400 kips
  - (B) column no. 1: 230 kips, column no. 2: 410 kips
  - (C) column no. 1: 250 kips, column no. 2: 421 kips
  - (D) column no. 1: 260 kips, column no. 2: 433 kips
5. What is most nearly the shear force at the center of each column?
  - (A) column no. 1: 60 kips, column no. 2:  $-80$  kips
  - (B) column no. 1: 63 kips, column no. 2:  $-84$  kips
  - (C) column no. 1: 66 kips, column no. 2:  $-88$  kips
  - (D) column no. 1: 70 kips, column no. 2:  $-95$  kips
6. What is most nearly the critical flexural shear?
  - (A)  $-52$  kips
  - (B)  $-54$  kips
  - (C)  $-58$  kips
  - (D)  $-62$  kips
7. What is most nearly the design flexural shear capacity?
  - (A) 120 kips
  - (B) 122 kips
  - (C) 124 kips
  - (D) 126 kips
8. What is most nearly the longitudinal grade 60 reinforcement required in the top of the combined footing?
  - (A)  $1.5 \text{ in}^2$
  - (B)  $2.0 \text{ in}^2$
  - (C)  $2.6 \text{ in}^2$
  - (D)  $3.2 \text{ in}^2$



**SOLUTIONS**

1. The weight of the footing produces a uniformly distributed pressure on the soil of

$$\begin{aligned} q_b &= w_c h \\ &= \frac{\left(150 \frac{\text{lb}}{\text{ft}^3}\right)(2 \text{ ft})}{1000 \frac{\text{lb}}{\text{kip}}} \\ &= 0.3 \text{ kips/ft}^2 \end{aligned}$$

The maximum allowable equivalent soil bearing pressure is

$$\begin{aligned} q_e &= q - q_b \\ &= 2 \frac{\text{kips}}{\text{ft}^2} - 0.3 \frac{\text{kips}}{\text{ft}^2} \\ &= 1.7 \text{ kips/ft}^2 \end{aligned}$$

The centroid of the column service loads is located a distance  $x_o$  from the property line. The distance  $x_o$  is obtained by taking moments about the property line, and is

$$\begin{aligned} x_o &= \frac{0.5P_1 + 10.5P_2}{P_1 + P_2} \\ &= \frac{(0.5 \text{ ft})(50 \text{ kips}) + (10.5 \text{ ft})(100 \text{ kips})}{50 \text{ kips} + 100 \text{ kips}} \\ &= 7.17 \text{ ft} \end{aligned}$$

The length of footing required to produce a uniform bearing pressure on the soil is

$$\begin{aligned} L &= 2x_o \\ &= (2)(7.17 \text{ ft}) \\ &= 14.3 \text{ ft} \end{aligned}$$

The width of footing required to produce a uniform pressure on the soil of 2000 lb/ft<sup>2</sup> is

$$\begin{aligned} B &= \frac{P_1 + P_2}{q_e L} \\ &= \frac{50 \text{ kips} + 100 \text{ kips}}{\left(1.7 \frac{\text{kips}}{\text{ft}^2}\right)(14.3 \text{ ft})} \\ &= 6.17 \text{ ft} \end{aligned}$$

The answer is (B).

2. Because the ratio of the factored loads to service loads on both columns is identical, the net factored pressure on the footing is uniform and has a value of

$$\begin{aligned} q_u &= 1.4q_e \\ &= (1.4)\left(1.7 \frac{\text{kips}}{\text{ft}^2}\right) \\ &= 2.38 \text{ kips/ft}^2 \end{aligned}$$

The answer is (A).

3. For column no. 1, the factored load is

$$\begin{aligned} P_{1u} &= 1.4P_1 \\ &= (1.4)(50 \text{ kips}) \\ &= 70 \text{ kips} \end{aligned}$$

For column no. 2, the factored load is

$$\begin{aligned} P_{2u} &= 1.4P_2 \\ &= (1.4)(100 \text{ kips}) \\ &= 140 \text{ kips} \end{aligned}$$

For column no. 1, the length of the critical perimeter is

$$\begin{aligned} b_o &= (c + d) + 2\left(c + \frac{d}{2}\right) \\ &= (12 \text{ in} + 20 \text{ in}) + (2)\left(12 \text{ in} + \frac{20 \text{ in}}{2}\right) \\ &= 76 \text{ in} \end{aligned}$$

The punching shear force at the critical perimeter is

$$\begin{aligned} V_u &= P_{1u} - q_u(c + d)\left(c + \frac{d}{2}\right) \\ &= 70 \text{ kips} \\ &\quad - \frac{\left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(12 \text{ in} + 20 \text{ in})\left(12 \text{ in} + \frac{20 \text{ in}}{2}\right)}{\left(12 \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 58.4 \text{ kips} \end{aligned}$$

For column no. 2, the length of the critical perimeter is

$$\begin{aligned} b_o &= 4(c + d) \\ &= (4)(12 \text{ in} + 20 \text{ in}) \\ &= 128 \text{ in} \end{aligned}$$

The punching shear force at the critical perimeter is

$$\begin{aligned} V_u &= P_{2u} - q_u(c + d)^2 \\ &= 140 \text{ kips} - \frac{\left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(12 \text{ in} + 20 \text{ in})^2}{\left(12 \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 123 \text{ kips} \end{aligned}$$

**The answer is (D).**

**4.** For column no. 1, the design punching shear capacity is given by ACI Eq. (11-33) as

$$\begin{aligned} \phi V_c &= 4\phi db_o \lambda \sqrt{f'_c} \\ &= \frac{(4)(0.75)(20 \text{ in})(76 \text{ in})(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 250 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

For column no. 2, the design punching shear capacity is given by ACI Eq. (11-33) as

$$\begin{aligned} \phi V_c &= 4\phi db_o \lambda \sqrt{f'_c} \\ &= \frac{(4)(0.75)(20 \text{ in})(128 \text{ in})(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 421 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (C).**

**5.** At the center of column no. 1, the shear force is

$$\begin{aligned} V_1 &= P_{1u} - \frac{q_u Bc}{2} \\ &= 70 \text{ kips} - \frac{\left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(6.17 \text{ ft})(1 \text{ ft})}{2} \\ &= 62.7 \text{ kips} \quad (63 \text{ kips}) \end{aligned}$$

At the center of column no. 2 the shear force is

$$\begin{aligned} V_2 &= V_1 - q_u B l \\ &= 62.7 \text{ kips} - \left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(6.17 \text{ ft})(10 \text{ ft}) \\ &= -84.1 \text{ kips} \quad (-84 \text{ kips}) \quad [\text{governs}] \end{aligned}$$

**The answer is (B).**

**6.** The critical flexural shear is at a distance from the center of column no. 2 given by

$$\begin{aligned} \left(d + \frac{c}{2}\right) &= 20 \text{ in} + \frac{12 \text{ in}}{2} \\ &= \frac{26 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} \\ &= 2.17 \text{ ft} \end{aligned}$$

The critical flexural shear at this section is

$$\begin{aligned} V_u &= V_2 - q_u B \left(d + \frac{c}{2}\right) \\ &= -84.1 \text{ kips} + \left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(6.17 \text{ ft})(2.17 \text{ ft}) \\ &= -52 \text{ kips} \end{aligned}$$

**The answer is (A).**

**7.** The design flexural shear capacity of the footing is given by ACI Eq. (11-3) as

$$\begin{aligned} \phi V_c &= 2\phi b d \lambda \sqrt{f'_c} \\ &= \frac{(2)(0.75)(6.17 \text{ ft})(20 \text{ in})(1.0) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} \left(12 \frac{\text{in}}{\text{ft}}\right)}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 122 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (B).**

8. From Prob. 4, the point of zero shear is a distance  $x$  from the center of column no. 1, given by

$$\begin{aligned} x &= \frac{V_1}{q_u B} \\ &= \frac{62.7 \text{ kips}}{\left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(6.17 \text{ ft})} \\ &= 4.3 \text{ ft} \end{aligned}$$

The maximum negative moment at this point is

$$\begin{aligned} M_u &= P_{1u}x - \frac{q_u B \left(x + \frac{c}{2}\right)^2}{2} \\ &= (70 \text{ kips})(4.3 \text{ ft}) \\ &\quad - \frac{\left(2.38 \frac{\text{kips}}{\text{ft}^2}\right)(6.17 \text{ ft})\left(4.3 \text{ ft} + \frac{1 \text{ ft}}{2}\right)^2}{2} \\ &= 132 \text{ ft-kips} \end{aligned}$$

The design moment factor is

$$\begin{aligned} K_u &= \frac{M_u}{Bd^2} \\ &= \frac{(132 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lb}}{\text{kip}}\right)}{(74.0 \text{ in})(20 \text{ in})^2} \\ &= 53.5 \text{ lb}/\text{in}^2 \end{aligned}$$

For a tension-controlled section the required reinforcement ratio is

$$\begin{aligned} \rho &= \frac{0.85f'_c \left(1 - \sqrt{1 - \frac{K_u}{0.383f'_c}}\right)}{f_y} \\ &= (0.85) \left(3 \frac{\text{kips}}{\text{in}^2}\right) \left( \frac{1 - \sqrt{1 - \frac{53.5 \frac{\text{lb}}{\text{in}^2}}{(0.383)(3000 \frac{\text{lb}}{\text{in}^2})}}}{60 \frac{\text{kips}}{\text{in}^2}} \right) \\ &= 0.00100 \end{aligned}$$

The required area of reinforcement is

$$\begin{aligned} A_s &= \rho B d \\ &= (0.00100)(74.0 \text{ in})(20 \text{ in}) \\ &= 1.48 \text{ in}^2 \end{aligned}$$

The minimum allowable reinforcement area is given by ACI Sec. 7.12.2 as

$$\begin{aligned} A_{s,\min} &= 0.0018 B h \\ &= (0.0018)(74.0 \text{ in})(24 \text{ in}) \\ &= 3.2 \text{ in}^2 \quad [\text{governs}] \\ &> A_s \end{aligned}$$

Providing eight no. 6 bars gives an area of

$$\begin{aligned} A_{s,\text{prov}} &= 3.52 \text{ in}^2 \\ &> A_{s,\min} \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (D).**

# 3

# Prestressed Concrete Design

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## 1. STRENGTH DESIGN OF FLEXURAL MEMBERS

### Nomenclature

|          |                                                                                                         |                      |
|----------|---------------------------------------------------------------------------------------------------------|----------------------|
| $a$      | depth of equivalent rectangular stress block                                                            | in                   |
| $A_{ct}$ | area of concrete section between the centroid and extreme tension fiber                                 | in <sup>2</sup>      |
| $A_g$    | area of concrete section                                                                                | in <sup>2</sup>      |
| $A_{ps}$ | area of prestressed reinforcement in tension zone                                                       | in <sup>2</sup>      |
| $A_s$    | area of nonprestressed tension reinforcement                                                            | in <sup>2</sup>      |
| $A'_s$   | area of compression reinforcement                                                                       | in <sup>2</sup>      |
| $b$      | width of compression face of member                                                                     | in                   |
| $c$      | distance from extreme compression fiber to neutral axis                                                 | in                   |
| $C_u$    | total compression force in equivalent rectangular stress block, $0.85abf'_c$                            | lbf                  |
| $d$      | distance from extreme compression fiber to centroid of nonprestressed reinforcement                     | in                   |
| $d'$     | distance from extreme compression fiber to centroid of compression reinforcement                        | in                   |
| $d_p$    | distance from extreme compression fiber to centroid of prestressed reinforcement as defined in Fig. 3.1 | in                   |
| $e$      | eccentricity of prestressing force                                                                      | in                   |
| $E_c$    | modulus of elasticity of concrete, $57\sqrt{f'_c}$                                                      | kips/in <sup>2</sup> |
| $E_p$    | modulus of elasticity of prestressing tendon                                                            | kips/in <sup>2</sup> |
| $f'_c$   | specified compressive strength of concrete                                                              | lbf/in <sup>2</sup>  |
| $f_{ps}$ | stress in prestressed reinforcement at nominal strength                                                 | kips/in <sup>2</sup> |
| $f_{pu}$ | specified tensile strength of prestressing tendons                                                      | kips/in <sup>2</sup> |
| $f_{py}$ | specified yield strength of prestressing tendons                                                        | kips/in <sup>2</sup> |

|          |                                                                                   |                      |
|----------|-----------------------------------------------------------------------------------|----------------------|
| $f_r$    | modulus of rupture of concrete, $7.5\sqrt{f'_c}$                                  | lbf/in <sup>2</sup>  |
| $f_{se}$ | effective stress in prestressed reinforcement after allowance for all losses      | kips/in <sup>2</sup> |
| $f_y$    | specified yield strength of nonprestressed reinforcement                          | kips/in <sup>2</sup> |
| $h$      | height of section                                                                 | in                   |
| $I_g$    | moment of inertia of gross concrete section                                       | in <sup>4</sup>      |
| $l$      | span length                                                                       | ft                   |
| $M_{cr}$ | cracking moment strength                                                          | in-kips              |
| $M_n$    | nominal flexural strength                                                         | in-kips              |
| $M_u$    | factored moment                                                                   | in-kips              |
| $N_c$    | tensile force in concrete caused by unfactored dead load plus live load           | lbf                  |
| $P_e$    | force in the prestressing tendons at service loads after allowance for all losses | kips                 |
| $S_b$    | section modulus of the concrete section referred to the bottom fiber              | in <sup>3</sup>      |
| $T_u$    | total tensile force in the prestressing tendons                                   | kips                 |

### Symbols

|              |                                                                                 |   |
|--------------|---------------------------------------------------------------------------------|---|
| $\beta_1$    | compression zone factor                                                         | — |
| $\gamma_p$   | factor for type of prestressing tendon                                          | — |
| $\epsilon_c$ | strain at extreme compression fiber at nominal strength, 0.003                  | — |
| $\epsilon_t$ | strain produced in prestressed reinforcement by the ultimate loading            | — |
| $\rho$       | ratio of nonprestressed tension reinforcement                                   | — |
| $\rho'$      | ratio of compression reinforcement                                              | — |
| $\rho_p$     | ratio of prestressed reinforcement                                              | — |
| $\lambda$    | correction factor related to unit weight of concrete as given in ACI Sec. 8.6.1 | — |
| $\phi$       | strength reduction factor                                                       | — |
| $\omega$     | reinforcement index of nonprestressed tension reinforcement                     | — |
| $\omega'$    | reinforcement index of compression reinforcement                                | — |
| $\omega_p$   | reinforcement index of prestressed reinforcement                                | — |

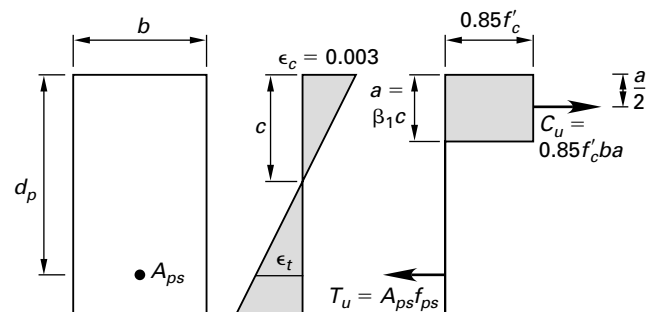
### Basic Principles

ACI Sec. 18.7.1 specifies that the design moment strength of a prestressed beam shall be computed by the strength design method. As shown in Fig. 3.1, ACI Sec. 10.2 assumes that the strain distribution over the depth of a prestressed beam is linear, and that a

rectangular stress block is formed in the concrete at the ultimate load. The maximum useable compressive strain in the concrete is specified as

$$\epsilon_c = 0.003$$

Figure 3.1 Prestressed Beam at Ultimate Load



The stress in the stress block is  $0.85f'_c$  as defined by ACI Sec. 10.2.7.1. The depth of the stress block is

$$a = \beta_1 c$$

The compression zone factor is defined in ACI Sec. 10.2.7.3 as

$$\begin{aligned} \beta_1 &= 0.85 \quad [f'_c \leq 4000 \text{ lbf/in}^2] \\ &= 0.85 - \frac{f'_c - 4000}{20,000} \quad \left[ \begin{array}{l} 4000 \text{ lbf/in}^2 < f'_c \\ \leq 8000 \text{ lbf/in}^2 \end{array} \right] \\ &= 0.65 \text{ minimum} \quad [f'_c > 8000 \text{ lbf/in}^2] \end{aligned}$$

Equating compressive and tensile forces on the section gives

$$0.85f'_c ab = A_{ps}f_{ps}$$

The depth of the stress block is derived as

$$a = \frac{A_{ps}f_{ps}}{0.85f'_c b}$$

The nominal flexural strength of the member is

$$\begin{aligned} M_n &= A_{ps}f_{ps} \left( d_p - \frac{a}{2} \right) \\ &= A_{ps}f_{ps} \left( d_p - \frac{0.59A_{ps}f_{ps}}{bf'_c} \right) \end{aligned}$$

Nonprestressed auxiliary reinforcement, conforming to ASTM A615, A706, and A996, may be added in the

tensile zone to increase the nominal strength. The nominal strength becomes

$$\begin{aligned} M_n &= A_{ps}f_{ps} \left( d_p - \frac{a}{2} \right) + A_s f_y \left( d - \frac{a}{2} \right) \\ a &= \frac{A_{ps}f_{ps} + A_s f_y}{0.85f'_c b} \end{aligned}$$

Compression reinforcement conforming to ASTM A615, A706, and A996, may also be added in the compression zone to increase the nominal strength. The nominal strength then becomes

$$\begin{aligned} M_n &= A_{ps}f_{ps} \left( d_p - \frac{a}{2} \right) + A_s f_y \left( d - \frac{a}{2} \right) \\ &\quad + A'_s f_y \left( \frac{a}{2} - d' \right) \\ a &= \frac{A_{ps}f_{ps} + A_s f_y - A'_s f_y}{0.85f'_c b} \end{aligned}$$

In ACI Sec. 10.3.4, a section is defined as tension controlled if the net tensile strain in the extreme tension steel,  $\epsilon_t$ , is not less than 0.005 when the concrete reaches its maximum useable compressive strain of 0.003. The strength reduction factor is then given by ACI Sec. 9.3.2 as

$$\phi = 0.9$$

The following relationships may be derived from Fig. 3.1.

$$\begin{aligned} \epsilon_t &= 0.005 \\ \frac{c}{d_p} &= 0.375 \\ a &= 0.375\beta_1 d_p \end{aligned}$$

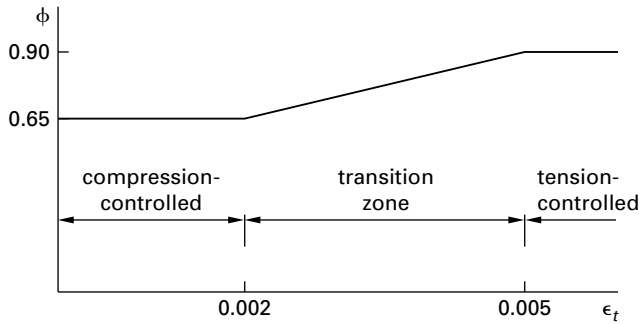
In ACI Sec. 10.3.3, a section is defined as compression-controlled if the net tensile strain in the extreme tension steel is not more than 0.002 when the concrete reaches its maximum useable compressive strain of 0.003. The strength reduction factor is then given by ACI Sec. 9.3.2 as

$$\phi = 0.65$$

The following relationships may then be derived from Fig. 3.1.

$$\begin{aligned} \epsilon_t &= 0.002 \\ \frac{c}{d_p} &= 0.600 \\ a &= 0.600\beta_1 d_p \end{aligned}$$

Members with a value of tensile strain between 0.002 and 0.005 are in the transition zone: The value of the strength reduction factor may be interpolated as shown in Fig. 3.2.

**Figure 3.2** Variation of  $\phi$  with  $\epsilon_t$ 

To prevent sudden failure when the modulus of rupture of the concrete is exceeded and the first flexural crack forms, ACI Sec. 18.8.2 requires the provision of sufficient prestressed and nonprestressed reinforcement to give a design flexural strength of

$$\phi M_n \geq 1.2 M_{cr}$$

The cracking moment of the member after all prestress losses have occurred is

$$M_{cr} = S_b(P_e R_b + f_r)$$

$$R_b = \frac{1}{A_g} + \frac{e}{S_b}$$

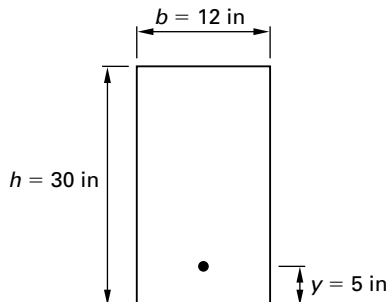
$$f_r = 7.5\lambda\sqrt{f'_c} \quad [\text{from ACI Sec. 9.5.2.3, Eq. (9-10)}]$$

This provision may be waived for

- two-way, unbonded post-tensioned slabs
- members with shear and flexural design strength at least twice the required strength

### Example 3.1

The normal weight concrete prestressed beam shown in the illustration is simply supported over a span of 30 ft. It has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low relaxation prestressing tendons provided is  $A_{ps} = 0.765$  in<sup>2</sup> with a specified tensile strength of  $f_{pu} = 270$  kips/in<sup>2</sup>, a yield strength of  $f_{py} = 243$  kips/in<sup>2</sup>, and an effective stress of  $f_{se} = 150$  kips/in<sup>2</sup> after all losses. Determine the cracking moment of the beam.



### Solution

The relevant properties of the beam are

$$\begin{aligned} A_g &= hb \\ &= (30 \text{ in})(12 \text{ in}) \\ &= 360 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} S_b &= \frac{A_g h}{6} \\ &= \frac{(360 \text{ in}^2)(30 \text{ in})}{6} \\ &= 1800 \text{ in}^3 \end{aligned}$$

$$\begin{aligned} e &= \frac{h}{2} - y \\ &= \frac{30 \text{ in}}{2} - 5 \text{ in} \\ &= 10 \text{ in} \end{aligned}$$

$$\begin{aligned} R_b &= \frac{1}{A_g} + \frac{e}{S_b} \\ &= \frac{1}{360 \text{ in}^2} + \frac{10 \text{ in}}{1800 \text{ in}^3} \\ &= 0.00833 \text{ in}^{-2} \end{aligned}$$

The force in the prestressing tendons at service loads after allowance for all losses is

$$\begin{aligned} P_e &= A_{ps} f_{se} \\ &= (0.765 \text{ in}^2) \left( 150 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 114.75 \text{ kips} \end{aligned}$$

The modulus of rupture of the concrete is given by ACI Eq. (9-10) as

$$\begin{aligned} f_r &= 7.5\lambda\sqrt{f'_c} \\ &= (7.5)(1.0)\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 581 \text{ lbf/in}^2 \end{aligned}$$

The cracking moment is

$$\begin{aligned} M_{cr} &= S_b(P_e R_b + f_r) \\ &= (1800 \text{ in}^3) \\ &\quad \times \left( (114.75 \text{ kips})(0.00833 \text{ in}^{-2}) + 0.581 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 2766 \text{ in-kips} \end{aligned}$$

# Flexural Strength of Members with Bonded Tendons

When the effective stress after all losses,  $f_{se}$ , is not less than half the tensile strength of the tendons,  $f_{pu}$ , ACI Sec. 18.7.2 gives the stress in the tendons at nominal strength as

$$f_{ps} = f_{pu} \left( 1 - \frac{\gamma_p}{\beta_1} \left( \rho_p \frac{f_{pu}}{f'_c} + \frac{d}{d_p} (\omega - \omega') \right) \right) \quad [\text{ACI 18-3}]$$

$$\begin{aligned} \gamma_p &= 0.55 && [\text{deformed bars with } f_{py}/f_{pu} \geq 0.80] \\ &= 0.40 && \left[ \begin{array}{l} \text{stress-relieved wire and strands,} \\ \text{and plain bars with } f_{py}/f_{pu} \geq 0.85 \end{array} \right] \\ &= 0.28 && \left[ \begin{array}{l} \text{low-relaxation wire and strands} \\ \text{with } f_{py}/f_{pu} \geq 0.90 \end{array} \right] \end{aligned}$$

The reinforcement index of nonprestressed tension reinforcement is

$$\omega = \frac{\rho f_y}{f'_c}$$

The reinforcement index of compression reinforcement is

$$\omega' = \frac{\rho' f_y}{f'_c}$$

The ratio of nonprestressed tension reinforcement is

$$\rho = \frac{A_s}{bd}$$

The ratio of compression reinforcement is

$$\rho' = \frac{A'_s}{bd}$$

The ratio of prestressed reinforcement is

$$\rho_p = \frac{A_{ps}}{bd_p}$$

When compression reinforcement is provided in the section

$$\begin{aligned} 0.17 &\geq \rho_p \left( \frac{f_{pu}}{f'_c} \right) + \frac{d(\omega - \omega')}{d_p} \\ d' &\leq 0.15 d_p \end{aligned}$$

When the section contains no auxiliary reinforcement, the value for the stress in prestressed reinforcement at nominal strength reduces to

$$f_{ps} = f_{pu} \left( 1 - \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f'_c} \right)$$

## Example 3.2

The fully bonded pretensioned beam described in Ex. 3.1 is simply supported over a span of 30 ft, and has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low relaxation prestressing tendons provided is 0.765 in<sup>2</sup>, with a specified tensile strength of 270 kips/in<sup>2</sup>, a yield strength of 243 kips/in<sup>2</sup>, and an effective stress of 150 kips/in<sup>2</sup> after all losses. Calculate the design flexural strength of the beam and determine whether the beam complies with ACI Sec. 18.8.2.

*Solution*

The compression zone factor for a specified concrete strength of  $f'_c = 6000$  lbf/in<sup>2</sup>, given by ACI Sec. 10.2.7.3, is

$$\beta_1 = 0.75$$

For a low-relaxation strand with a ratio of specified yield strength to specified tensile strength  $f_{py}/f_{pu} \geq 0.90$ , the tendon factor is given by ACI Sec. 18.7.2 as

$$\gamma_p = 0.28$$

The ratio of prestressed reinforcement is

$$\begin{aligned} \rho_p &= \frac{A_{ps}}{bd_p} \\ &= \frac{0.765 \text{ in}^2}{(12 \text{ in})(25 \text{ in})} \\ &= 0.00255 \\ \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f'_c} &= \frac{(0.28)(0.00255) \left( 270 \frac{\text{kips}}{\text{in}^2} \right)}{(0.75) \left( 6 \frac{\text{kips}}{\text{in}^2} \right)} \\ &= 0.043 \end{aligned}$$

The ratio of effective stress to tensile strength of the tendons is

$$\begin{aligned} \frac{f_{se}}{f_{pu}} &= \frac{150 \frac{\text{kips}}{\text{in}^2}}{270 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.56 \\ &> 0.5 \end{aligned}$$

The stress in the tendons at nominal strength, for a section without auxiliary reinforcement, may be derived from ACI Eq. (18-3) as

$$\begin{aligned} f_{ps} &= f_{pu} \left( 1 - \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f'_c} \right) \\ &= \left( 270 \frac{\text{kips}}{\text{in}^2} \right) \left( 1 - \frac{(0.28)(0.00255) \left( 270 \frac{\text{kips}}{\text{in}^2} \right)}{(0.75) \left( 6 \frac{\text{kips}}{\text{in}^2} \right)} \right) \\ &= 258 \text{ kips/in}^2 \end{aligned}$$

The depth of the stress block is

$$\begin{aligned} a &= \frac{A_{ps} f_{ps}}{0.85 f'_c b} = \frac{(0.765 \text{ in}^2) \left( 258 \frac{\text{kips}}{\text{in}^2} \right)}{(0.85) \left( 6 \frac{\text{kips}}{\text{in}^2} \right) (12 \text{ in})} \\ &= 3.23 \text{ in} \end{aligned}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$\begin{aligned} a_t &= 0.375 \beta_1 d_p \\ &= (0.375)(0.75)(25 \text{ in}) \\ &= 7.03 \text{ in} \\ &> a \end{aligned}$$

The section is tension-controlled and

$$\phi = 0.9$$

The nominal flexural strength of the section is

$$\begin{aligned} M_n &= A_{ps} f_{ps} \left( d_p - \frac{a}{2} \right) \\ &= (0.765 \text{ in}^2) \left( 258 \frac{\text{kips}}{\text{in}^2} \right) \left( 25 \text{ in} - \frac{3.23 \text{ in}}{2} \right) \\ &= 4615 \text{ in-kips} \end{aligned}$$

The design flexural strength of the beam is

$$\begin{aligned} \phi M_n &= (0.9)(4615 \text{ in-kips}) \\ &= 4154 \text{ in-kips} \end{aligned}$$

The cracking moment of the beam is determined in Ex. 3.1 as

$$\begin{aligned} M_{cr} &= 2766 \text{ in-kips} \\ \frac{\phi M_n}{M_{cr}} &= \frac{4154 \text{ in-kips}}{2766 \text{ in-kips}} \\ &= 1.5 \\ &> 1.2 \end{aligned}$$

The beam complies with ACI Sec. 18.8.2.

### Flexural Strength of Members with Unbonded Tendons

When the effective stress after all losses,  $f_{se}$ , is not less than half the tensile strength of the tendons,  $f_{pu}$ , ACI Sec. 18.7.2 permits the stress in the tendons at nominal strength to be calculated by the following methods.

For unbonded tendons and a span-to-depth ratio of 35 or less, ACI Sec. 18.7.2 gives the stress in the tendons at nominal strength as

$$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{100 \rho_p} \quad [\text{ACI 18-4}]$$

$$\begin{aligned} &\leq f_{py} \\ &\leq f_{se} + 60,000 \end{aligned}$$

$f_{se}$  and  $f'_c$  are in  $\text{lbf/in}^2$ .

For unbonded tendons and a span-to-depth ratio greater than 35, ACI Sec. 18.7.2 gives the stress in the tendons at nominal strength as

$$f_{ps} = f_{se} + 10,000 + \frac{f'_c}{300 \rho_p} \quad [\text{ACI 18-5}]$$

$$\begin{aligned} &\leq f_{py} \\ &\leq f_{se} + 30,000 \end{aligned}$$

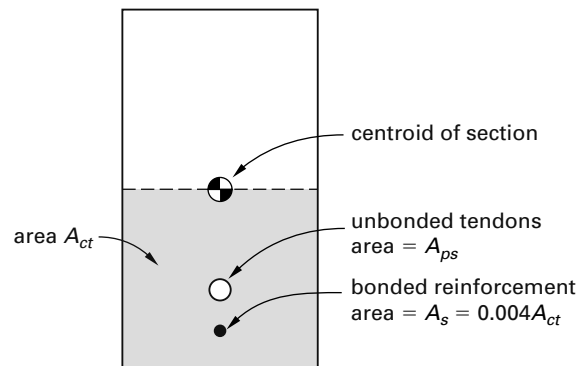
$f_{se}$  and  $f'_c$  are in  $\text{lbf/in}^2$ .

In accordance with ACI Sec. 18.9, auxiliary bonded reinforcement is required near the extreme tension fiber in all flexural members with unbonded tendons. The minimum required area is independent of the grade of steel, and is given by ACI Sec. 18.9.2 as

$$A_s = 0.004 A_{ct} \quad [\text{ACI 18-6}]$$

$A_{ct}$  is the area of the concrete section between the centroid of the section and the extreme tension fiber, as shown in Fig. 3.3.

**Figure 3.3** Bonded Reinforcement Area





In two-way slabs, when the tensile stress caused by dead load + live load exceeds  $2\sqrt{f'_c}$ , ACI Sec. 18.9.3.2 requires the provision of auxiliary reinforcement with a minimum area of

$$A_s = \frac{N_c}{0.5f_y} \quad [\text{ACI 18-7}]$$

### Example 3.3

The post-tensioned unbonded beam described in Ex. 3.1 is simply supported over a span of 30 ft, and has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low-relaxation prestressing tendons provided is 0.765 in<sup>2</sup>, with a specified tensile strength of 270 kips/in<sup>2</sup>, a yield strength of 243 kips/in<sup>2</sup>, and an effective stress of 150 kips/in<sup>2</sup> after all losses. An area of 0.8 in<sup>2</sup> of grade 60 auxiliary reinforcement is provided at a height of 3 in above the beam soffit. Calculate the design flexural strength of the beam, and determine whether the beam complies with ACI Sec. 18.8.2.

#### Solution

Because  $f_{se}/f_{pu} > 0.5$ , the method of ACI Sec. 18.7.2 may be used. The ratio of prestressed reinforcement is

$$\begin{aligned} \rho_p &= \frac{A_{ps}}{bd_p} \\ &= \frac{0.765 \text{ in}^2}{(12 \text{ in})(25 \text{ in})} \\ &= 0.00255 \end{aligned}$$

For unbonded tendons and a span-to-depth ratio of 35 or less, the stress in the unbonded tendons at nominal strength is given by ACI Sec. 18.7.2 as

$$\begin{aligned} f_{ps} &= f_{se} + 10 + \frac{f'_c}{100\rho_p} \quad [\text{ACI 18-4}] \\ &= 150 \frac{\text{kips}}{\text{in}^2} + 10 \frac{\text{kips}}{\text{in}^2} + \frac{6 \frac{\text{kips}}{\text{in}^2}}{(100)(0.00255)} \\ &= 184 \text{ kips/in}^2 \\ &< f_{py} \quad [\text{satisfactory}] \\ &< f_{se} + 60 \text{ kips/in}^2 \quad [\text{satisfactory}] \end{aligned}$$

The required minimum area of auxiliary reinforcement is specified by ACI Sec. 18.9.2 as

$$\begin{aligned} A_s &= 0.004A_{ct} \quad [\text{ACI 18-6}] \\ &= (0.004)(12 \text{ in})(15 \text{ in}) \\ &= 0.72 \text{ in}^2 \\ &< 0.80 \text{ in}^2 \text{ provided} \quad [\text{satisfactory}] \end{aligned}$$

Assuming full utilization of the auxiliary reinforcement, the depth of the stress block is

$$\begin{aligned} a &= \frac{A_{ps}f_{ps} + A_s f_y}{0.85f'_c b} \\ &= \frac{(0.765 \text{ in}^2)\left(184 \frac{\text{kips}}{\text{in}^2}\right) + (0.8 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(6 \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})} \\ &= 3.08 \text{ in} \end{aligned}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$\begin{aligned} a_t &= 0.375\beta_1 d_p \\ &= (0.375)(0.75)(25 \text{ in}) \\ &= 7.03 \text{ in} \\ &> a \end{aligned}$$

The section is tension-controlled and

$$\phi = 0.9$$

The nominal flexural strength of the section is

$$\begin{aligned} M_n &= A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right) \\ &= (0.765 \text{ in}^2)\left(184 \frac{\text{kips}}{\text{in}^2}\right)\left(25 \text{ in} - \frac{3.08 \text{ in}}{2}\right) \\ &\quad + (0.80 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(27 \text{ in} - \frac{3.08 \text{ in}}{2}\right) \\ &= 4524 \text{ in-kips} \end{aligned}$$

The design flexural strength of the beam is

$$\begin{aligned} \phi M_n &= (0.9)(4524 \text{ in-kips}) \\ &= 4072 \text{ in-kips} \end{aligned}$$

The cracking moment of the beam is determined in Ex. 3.1 as

$$\begin{aligned} M_{cr} &= 2766 \text{ in-kips} \\ \frac{\phi M_n}{M_{cr}} &= \frac{4072 \text{ in-kips}}{2766 \text{ in-kips}} \\ &= 1.47 \\ &> 1.2 \end{aligned}$$

The beam complies with ACI Sec. 18.8.2.

## 2. DESIGN FOR SHEAR AND TORSION

### Nomenclature

|            |                                                                                                                                                                                                   |                    |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| $A_{cp}$   | area enclosed by outside perimeter of concrete cross section                                                                                                                                      | $\text{in}^2$      |
| $A_l$      | total area of longitudinal reinforcement to resist torsion                                                                                                                                        | $\text{in}^2$      |
| $A_o$      | gross area enclosed by shear flow                                                                                                                                                                 | $\text{in}^2$      |
| $A_{oh}$   | gross area enclosed by the center line of the outermost closed transverse torsional reinforcement                                                                                                 | $\text{in}^2$      |
| $A_{ps}$   | area of prestressed reinforcement in tension zone                                                                                                                                                 | $\text{in}^2$      |
| $A_t$      | area of one leg of a closed stirrup resisting torsion within a spacing $s$                                                                                                                        | $\text{in}^2$      |
| $A_v$      | area of shear reinforcement within a spacing $s$                                                                                                                                                  | $\text{in}^2$      |
| $A_{v+t}$  | sum of areas of shear and torsion reinforcement                                                                                                                                                   | $\text{in}^2$      |
| $b_w$      | web width                                                                                                                                                                                         | in                 |
| $d$        | distance from extreme compression fiber to centroid of prestressed and nonprestressed tension reinforcement, as defined in ACI Sec. 11.4.1 (need not be less than $0.8h$ for prestressed members) | in                 |
| $d_{bl}$   | diameter of longitudinal torsional reinforcement                                                                                                                                                  | in                 |
| $d_p$      | actual distance from extreme compression fiber to centroid of prestressing tendons                                                                                                                | in                 |
| $f_{pc}$   | compressive stress in the concrete, caused by the final prestressing force at the centroid of the section                                                                                         | $\text{kips/in}^2$ |
| $f_{pu}$   | specified strength of prestressing tendons                                                                                                                                                        | $\text{kips/in}^2$ |
| $f_{se}$   | effective stress in prestressing reinforcement after allowance for all losses                                                                                                                     | $\text{kips/in}^2$ |
| $f_y$      | specified yield strength of reinforcement                                                                                                                                                         | $\text{kips/in}^2$ |
| $f_{yt}$   | yield strength of longitudinal torsional reinforcement                                                                                                                                            | $\text{kips/in}^2$ |
| $f_{yt}$   | yield strength of transverse reinforcement                                                                                                                                                        | $\text{kips/in}^2$ |
| $h$        | overall thickness of member                                                                                                                                                                       | in                 |
| $l$        | span length                                                                                                                                                                                       | in                 |
| $M_{\max}$ | maximum factored moment at section caused by externally applied loads                                                                                                                             | in-kips            |
| $M_u$      | factored moment at section                                                                                                                                                                        | in-kips            |
| $p_{cp}$   | outside perimeter of the concrete cross section                                                                                                                                                   | in                 |
| $p_h$      | perimeter of centerline of outermost closed transverse torsional reinforcement                                                                                                                    | in                 |
| $s$        | spacing of shear or torsion reinforcement                                                                                                                                                         | in                 |
| $S_b$      | section modulus of the section referred to the bottom fiber                                                                                                                                       | $\text{in}^3$      |
| $t$        | wall thickness                                                                                                                                                                                    | in                 |
| $T$        | applied torsion                                                                                                                                                                                   | in-kips            |
| $T_n$      | nominal torsional moment strength                                                                                                                                                                 | in-kips            |

|       |                                                                         |         |
|-------|-------------------------------------------------------------------------|---------|
| $T_u$ | factored torsional moment at section                                    | in-kips |
| $V_c$ | nominal shear strength provided by concrete                             | kips    |
| $V_n$ | nominal shear strength                                                  | kips    |
| $V_s$ | nominal shear strength provided by shear reinforcement                  | kips    |
| $V_u$ | factored shear force at section                                         | kips    |
| $w_u$ | factored load per unit length of beam or one way slab                   | kips/ft |
| $x$   | distance from the support                                               | in      |
| $x_o$ | distance between the center lines of vertical legs of closed stirrups   | in      |
| $y_o$ | distance between the center lines of horizontal legs of closed stirrups | in      |

### Symbols

|        |                                                       |                  |
|--------|-------------------------------------------------------|------------------|
| $\phi$ | strength reduction factor, 0.75 for shear and torsion | —                |
| $\tau$ | shear stress in the walls                             | $\text{lb/in}^2$ |

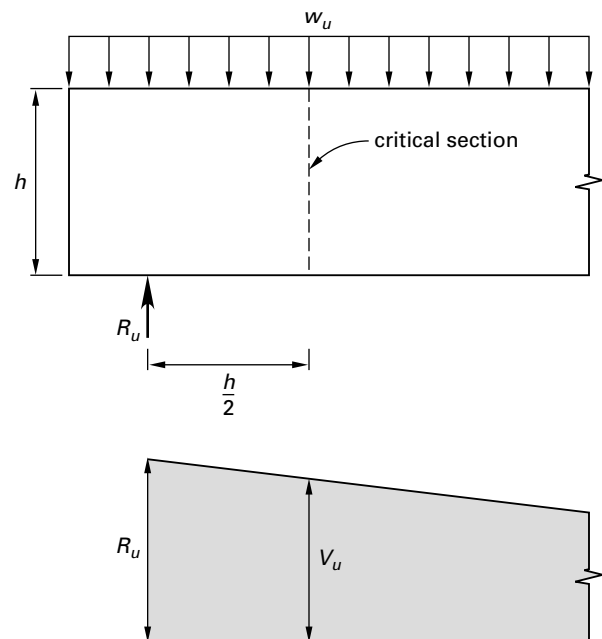
### Design for Shear

#### Critical Section for Shear

As shown in Fig. 3.4, the critical section for the calculation of shear in a prestressed beam is located at a distance from the support equal to half the overall depth. The maximum design factored shear force is

$$V_u = R_u - w_u \left( \frac{h}{2} \right)$$

Figure 3.4 Critical Section for Shear



The beam may be designed for this shear force at a distance from the face of the support equal to half the overall depth. As specified in ACI Sec. 11.1.3, this is permitted provided that

- the support reaction produces a compressive stress in the end of the beam
- loads are applied at or near the top of the beam
- concentrated loads are not located closer to the support than half the overall depth

### Shear Capacity of the Concrete Section

The nominal shear capacity provided by the concrete section may be obtained from ACI Sec. 11.3.2, provided that the effective prestressing force after all losses is not less than 40% of the tensile strength of the flexural reinforcement.

$$f_{se} \geq 0.4f_{pu}$$

This provides a simplified method for determining a nominal shear strength that is conservative and usually sufficiently accurate. This method gives a value for the nominal shear capacity of the concrete section of

$$\begin{aligned} V_c &= \left( 0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u} \right) b_w d \quad [\text{ACI 11-9}] \\ &\leq 5b_w d \lambda \sqrt{f'_c} \\ &\geq 2b_w d \lambda \sqrt{f'_c} \\ \frac{V_u d_p}{M_u} &\leq 1.0 \\ \sqrt{f'_c} &\leq 100 \frac{\text{lb}}{\text{in}^2} \quad [\text{from ACI Sec. 11.1.2}] \end{aligned}$$

The factored shear force at the section is  $V_u$ , and the factored moment occurring simultaneously with  $V_u$  at the section is  $M_u$ . The distance from the extreme compression fiber to the centroid of the prestressed and nonprestressed tension reinforcement is defined as  $d$  in ACI Sec. 11.3.1. However, it need not be less than  $0.8h$  for prestressed members. The actual distance from the extreme compression fiber to the centroid of the prestressing tendons is  $d_p$ .

For a simply supported beam with a uniformly distributed applied load, ACI Sec. R11.3.2 provides the expression

$$\frac{V_u d_p}{M_u} = \frac{d_p(l - 2x)}{x(l - x)}$$

As specified in ACI Sec. 11.4.6.1, a minimum area of shear reinforcement must be provided in all prestressed beams where the factored shear force exceeds one-half the design shear strength provided by the concrete section. This minimum area of shear reinforcement is

specified by ACI Secs. 11.4.6.3 and 11.4.6.4 as the least value given by

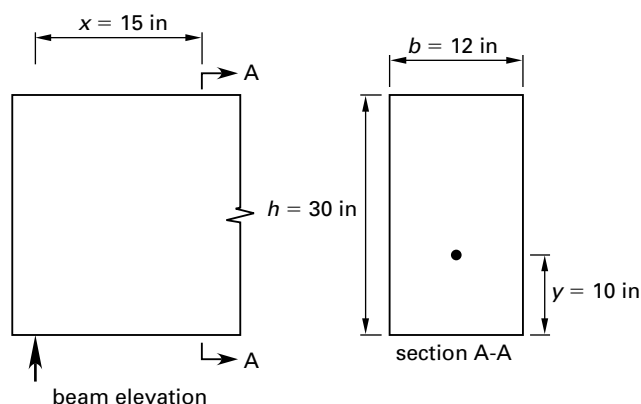
$$\begin{aligned} A_{v,\min} &= \frac{50b_w s}{f_{yt}} \\ &\geq \frac{A_{ps} f_{pu} s \sqrt{\frac{d}{b_w}}}{80f_{yt} d} \quad [\text{ACI 11-14 for } f_{se} > 0.4f_{pu}] \\ &\geq \frac{0.75\sqrt{f'_c} b_w s}{f_{yt}} \quad [\text{ACI 11-13}] \end{aligned}$$

$f_{yt}$ ,  $f_{pu}$ , and  $f'_c$  are in  $\text{lb}/\text{in}^2$ . As stated in ACI Sec. 11.4.3,  $d$  need not be taken less than  $0.8h$ .

ACI Sec. 11.4.5 limits the spacing of the stirrups to a maximum value of 0.75 times the overall thickness of the member,  $h$ , but not less than 24 in. When the value of the nominal shear strength,  $V_s$ , exceeds  $4b_w d \sqrt{f'_c}$ , the spacing is reduced to a maximum value of  $0.375h$ , or 12 in.

### Example 3.4

A normal weight concrete prestressed beam is simply supported over a span of 30 ft, and has a 28 day concrete strength of  $6000 \text{ lb}/\text{in}^2$ . The area of the low-relaxation prestressing tendons provided is  $A_{ps} = 0.765 \text{ in}^2$ , with a specified tensile strength of  $f_{pu} = 270 \text{ kips}/\text{in}^2$ , a yield strength of  $f_{py} = 243 \text{ kips}/\text{in}^2$ , and an effective stress of  $f_{se} = 150 \text{ kips}/\text{in}^2$  after all losses. As shown in the illustration, the centroid of the prestressing tendons is 10 in above the soffit of the beam at a distance of 15 in from the support. The beam supports a uniformly distributed factored load, including its self-weight, of 5 kips/ft. Determine whether the shear capacity of the beam is adequate.



### Solution

The critical section for shear occurs at a distance from the support given by

$$\begin{aligned} x &= \frac{h}{2} \\ &= 15 \text{ in} \end{aligned}$$

The actual depth of the cable at this location is

$$\begin{aligned}d_p &= h - y \\&= 30 \text{ in} - 10 \text{ in} \\&= 20 \text{ in}\end{aligned}$$

The effective depth of the section is

$$\begin{aligned}d &= 0.8h \\&= (0.8)(30 \text{ in}) \\&= 24 \text{ in} \quad [\text{governs}]\end{aligned}$$

The support reaction is

$$\begin{aligned}R_u &= \frac{w_u l}{2} \\&= \frac{\left(5 \frac{\text{kips}}{\text{ft}}\right)(30 \text{ ft})}{2} \\&= 75 \text{ kips}\end{aligned}$$

The maximum design factored shear force at the critical section is

$$\begin{aligned}V_u &= R_u - w_u \left(\frac{h}{2}\right) \\&= 75 \text{ kips} - \frac{\left(5 \frac{\text{kips}}{\text{ft}}\right)\left(\frac{30 \text{ in}}{2}\right)}{12 \frac{\text{in}}{\text{ft}}} \\&= 68.75 \text{ kips}\end{aligned}$$

From ACI Sec. R11.3.2,

$$\begin{aligned}\frac{V_u d_p}{M_u} &= \frac{d_p(l - 2x)}{x(l - x)} \\&= \frac{(20 \text{ in})(360 \text{ in} - (2)(15 \text{ in}))}{(15 \text{ in})(360 \text{ in} - 15 \text{ in})} \\&= 1.28\end{aligned}$$

From ACI Sec. 11.3.2, use a maximum value of

$$\frac{V_u d_p}{M_u} = 1.0$$

The ratio of the effective stress to the tensile strength of the tendons is

$$\begin{aligned}\frac{f_{se}}{f_{pu}} &= \frac{150 \frac{\text{kips}}{\text{in}^2}}{270 \frac{\text{kips}}{\text{in}^2}} \\&= 0.56 \\&> 0.4\end{aligned}$$

ACI Sec. 11.3.2 is applicable and the nominal shear capacity provided by the concrete is

$$\begin{aligned}V_c &= \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d_p}{M_u}\right)b_w d \quad [d = 0.8h = 24 \text{ in}] \\&= \left((0.6)(1.0)\sqrt{6000 \frac{\text{lb}}{\text{in}^2}} + (700)(1.0)\right) \\&\quad \times \left(\frac{(12 \text{ in})(24 \text{ in})}{1000 \frac{\text{lb}}{\text{kip}}}\right) \\&= 215 \text{ kips}\end{aligned}$$

From ACI Sec. 11.3.2, the maximum permitted value of the nominal shear capacity is

$$\begin{aligned}V_c &= 5b_w d \lambda \sqrt{f'_c} \\&= \frac{(5)(12 \text{ in})(24 \text{ in})(1.0)\sqrt{6000 \frac{\text{lb}}{\text{in}^2}}}{1000 \frac{\text{lb}}{\text{kip}}} \\&= 111.54 \text{ kips} \quad [\text{governs}]\end{aligned}$$

From ACI Sec. 9.3.2.3, the design shear capacity is

$$\begin{aligned}\phi V_c &= 0.75 V_c \\&= (0.75)(111.54 \text{ kips}) \\&= 83.66 \text{ kips} \\&> V_u \quad [\text{satisfactory}]\end{aligned}$$

Because the factored shear force exceeds one-half the design shear strength provided by the concrete section, a minimum area of shear reinforcement is required by ACI Sec. 11.4.6.1. The minimum area of shear reinforcement is given by the least value of

$$\begin{aligned}\frac{A_{v,\min}}{s} &= \frac{50b_w}{f_{yt}} \\&= \frac{(50)(12 \text{ in})\left(12 \frac{\text{in}}{\text{ft}}\right)}{60,000 \frac{\text{lb}}{\text{in}^2}} \\&= 0.12 \text{ in}^2/\text{ft}\end{aligned}$$

$$\begin{aligned}\frac{A_{v,\min}}{s} &= \frac{A_{ps} f_{pu} \sqrt{\frac{d}{b_w}}}{80 f_{yt} d} \quad [\text{ACI 11-14}] \\&= \frac{(0.765 \text{ in}^2)\left(270 \frac{\text{kips}}{\text{in}^2}\right)\sqrt{\frac{24 \text{ in}}{12 \text{ in}}}\left(12 \frac{\text{in}}{\text{ft}}\right)}{(80)\left(60 \frac{\text{kips}}{\text{in}^2}\right)(24 \text{ in})} \\&= 0.030 \text{ in}^2/\text{ft} \quad [\text{governs}]\end{aligned}$$

$$\begin{aligned}\frac{A_{v,\min}}{s} &= 0.75 \sqrt{f'_c} \left( \frac{b_w}{f_{yt}} \right) \\ &= (0.75) \sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} \left( \frac{(12 \text{ in}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{60,000 \frac{\text{lbf}}{\text{in}^2}} \right) \\ &= 0.14 \text{ in}^2/\text{ft}\end{aligned}$$

Provide shear reinforcement consisting of two vertical legs of no. 3 stirrups at 18 in spacing, which gives

$$\begin{aligned}\frac{A_v}{s} &= \frac{(0.22 \text{ in}^2) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{18 \text{ in}} \\ &= 0.15 \text{ in}^2/\text{ft} \\ &> 0.030 \text{ in}^2/\text{ft} \quad [\text{satisfactory}]\end{aligned}$$

The maximum spacing permitted by ACI Sec. 11.4.5.1 is the lesser of

$$\begin{aligned}s &= 24 \text{ in} \\ s &= 0.75h \\ &= (0.75)(30 \text{ in}) \\ &= 22.5 \text{ in} \\ &> 18 \text{ in} \quad [\text{satisfactory}]\end{aligned}$$

### Shear Capacity of Shear Reinforcement

The nominal shear capacity of shear reinforcement perpendicular to the member is given by ACI Sec. 11.4.7.2 as

$$\begin{aligned}V_s &= \frac{A_v f_{yt} d}{s} \quad [\text{ACI 11-15}] \\ &\leq 8b_w d \sqrt{f'_c}\end{aligned}$$

ACI Sec. 11.4.5.1 limits the spacing of the stirrups to a maximum value of three-quarters of  $h$  or 24 in. When the value of the nominal shear strength provided by the shear reinforcement,  $V_s$ , exceeds  $4b_w d \sqrt{f'_c}$ , the spacing is reduced to a maximum value of  $0.375d$ , or 12 in.

In accordance with ACI Sec. 11.1.1, the nominal shear strength of the concrete section and of the shear reinforcement are additive, and the combined nominal shear capacity of the concrete and the stirrups is

$$\begin{aligned}V_n &= V_c + V_s \\ \phi V_n &= \phi V_c + \phi V_s\end{aligned}$$

When the applied factored shear force  $V_u$  is less than  $\phi V_c/2$ , the concrete section is adequate to carry the shear without any shear reinforcement. Within the range  $\phi V_c/2 \leq V_u \leq \phi V_c$ , the minimum area of shear reinforcement is specified by ACI Secs. 11.4.6.3 and 11.4.6.4.

### Example 3.5

The prestressed beam described in Ex. 3.4 is simply supported over a span of 30 ft, and has a 28 day concrete strength of  $6000 \text{ lbf/in}^2$ . The area of the low-relaxation prestressing tendons provided is  $A_{ps} = 0.765 \text{ in}^2$ , with a specified tensile strength of  $f_{pu} = 270 \text{ kips/in}^2$ , a yield strength of  $f_{py} = 243 \text{ kips/in}^2$ , and an effective stress of  $f_{se} = 150 \text{ kips/in}^2$  after all losses. The centroid of the prestressing tendons is 10 in above the soffit of the beam, at a distance of 15 in from the support. The beam supports a uniformly distributed factored load, including its self-weight, of 8 kips/ft. Determine the shear reinforcement required.

#### Solution

From Ex. 3.4, the factored shear force at the critical section is

$$\begin{aligned}V_u &= \frac{(68.75 \text{ kips}) \left( 8 \frac{\text{kips}}{\text{ft}} \right)}{5 \frac{\text{kips}}{\text{ft}}} \\ &= 110 \text{ kips}\end{aligned}$$

The design shear strength of the concrete section is derived in Ex. 3.4 as

$$\phi V_c = 83.66 \text{ kips}$$

The design shear strength required from stirrups is

$$\begin{aligned}\phi V_s &= V_u - \phi V_c \\ &= 110 \text{ kips} - 83.66 \text{ kips} \\ &= 26.34 \text{ kips}\end{aligned}$$

The design shear strength of required shear reinforcement is limited by ACI Sec. 11.4.7.9 to a maximum value of

$$\begin{aligned}\phi V_s &= \phi 8b_w d \sqrt{f'_c} = \frac{(0.75)(8)(12 \text{ in})(24 \text{ in}) \sqrt{6000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kips}}} \\ &= 134 \text{ kips} \\ &> 26.35 \text{ kips} \quad [\text{satisfactory}]\end{aligned}$$

The area of shear reinforcement is given by ACI Sec. 11.4.7.2 as

$$\begin{aligned}\frac{A_v}{s} &= \frac{\phi V_s}{\phi f_{yt} d} = \frac{(26.34 \text{ kips}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{(0.75) \left( 60 \frac{\text{kips}}{\text{in}^2} \right) (24 \text{ in})} \\ &= 0.293 \text{ in}^2/\text{ft}\end{aligned}$$

Provide shear reinforcement consisting of two vertical legs of no. 3 stirrups at 9 in spacing, which gives

$$\frac{A_v}{s} = \frac{(0.22 \text{ in}^2) \left(12 \frac{\text{in}}{\text{ft}}\right)}{9 \text{ in}} = 0.293 \text{ in}^2/\text{ft} \quad [\text{satisfactory}]$$

## Design for Torsion

### General Principles

After torsional cracking occurs, the central core of a prestressed concrete member is largely ineffectual in resisting applied torsion and can be neglected. Hence, it is assumed in ACI Sec. R11.5 that a member behaves as a thin-walled tube when subjected to torsion. To maintain a consistent approach, a member is also analyzed as a thin-walled tube before cracking. As shown in Fig. 3.5, the shear stress in the tube walls produces a uniform shear flow, acting at the midpoint of the walls, with a magnitude of

$$q = \tau t$$

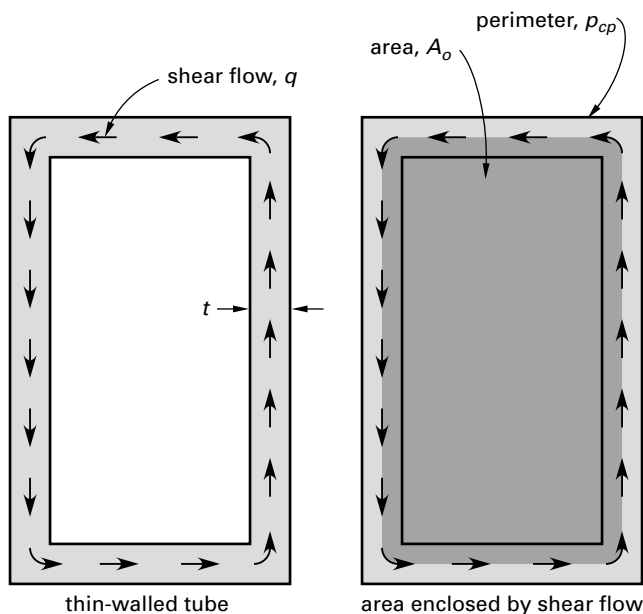
The applied torsion is resisted by the moment of the shear flow in the walls about the centroid of the section, and is

$$T = 2A_o q$$

The gross area enclosed by shear flow is the area  $A_o$  enclosed by the center line of the walls, given by

$$A_o = \frac{2A_{cp}}{3}$$

Figure 3.5 Thin-Walled Tube Analogy

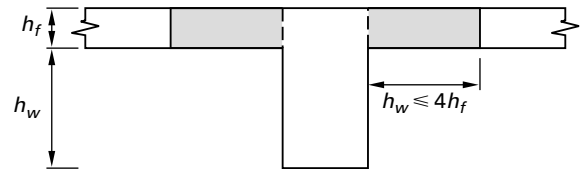


The shear stress in the walls is

$$\tau = \frac{T}{2A_o t}$$

For flanged sections, the value of the overhanging flange width (used in calculating the value of the area  $A_{cp}$  enclosed by the outside perimeter of the concrete cross section) and the value of  $p_{cp}$  (the outside perimeter of the concrete cross section) are determined as specified in ACI Secs. 11.5.1.1 and 13.2.4. The overhanging flange width is shown in Fig. 3.6.

Figure 3.6 Overhanging Flange Width



In accordance with ACI Sec. 11.5.2.5, the critical section for the calculation of torsion in a prestressed beam is located at a distance from the support equal to half the overall depth. When a concentrated torsion occurs within this distance, the critical section for design shall be at the face of the support.

Cracking is assumed to occur in a member when the principal tensile stress reaches a value of

$$p_t = 4\sqrt{f'_c} = \tau$$

The cracking torsion is

$$T_{cr} = 4\lambda\sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

It is assumed in ACI Sec. 11.5.1 that torsional effects may be neglected, and closed stirrups and longitudinal torsional reinforcement are not required when the factored torque does not exceed the threshold torsion given by

$$T_u = \frac{\phi T_{cr}}{4} = \phi\lambda\sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

When the threshold value is exceeded, reinforcement must be provided to resist the full torsion, and the concrete is considered ineffectual. When both shear and torsion reinforcements are required, the sum of the individual areas must be provided as specified in ACI Sec. 11.5.3.8. Because the stirrup area  $A_v$  for shear is defined in terms of both legs of a stirrup, while the

stirrup area  $A_t$  for torsion is defined in terms of one leg only, the summation of the areas is

$$\sum \frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

The spacing of the reinforcement is limited by the minimum required spacing of either the shear or torsion reinforcement.

### Example 3.6

The prestressed beam described in Ex. 3.4 is simply supported over a span of 30 ft, and has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low-relaxation prestressing tendons provided is  $A_{ps} = 0.765$  in<sup>2</sup>, with a specified tensile strength of  $f_{pu} = 270$  kips/in<sup>2</sup>, a yield strength of  $f_{py} = 243$  kips/in<sup>2</sup>, and an effective stress of  $f_{se} = 150$  kips/in<sup>2</sup> after all losses. The centroid of the prestressing tendons is 10 in above the soffit of the beam at a distance of 15 in from the support. The beam supports a uniformly distributed factored load, including its self-weight, of 8 kips/ft. Determine the threshold torsion of the section.

#### Solution

The compressive stress in the concrete, caused by the final prestressing force at the centroid of the section, is

$$\begin{aligned} f_{pc} &= \frac{P_e}{A_c} \\ &= \frac{A_{ps} f_{se}}{A_c} \\ &= \frac{(0.765 \text{ in}^2) \left( 150 \frac{\text{kips}}{\text{in}^2} \right)}{360 \text{ in}^2} \\ &= 0.319 \text{ kips/in}^2 \end{aligned}$$

The area enclosed by the outside perimeter of the beam is

$$\begin{aligned} A_{cp} &= hb \\ &= (30 \text{ in})(12 \text{ in}) \\ &= 360 \text{ in}^2 \end{aligned}$$

The length of the outside perimeter of the beam is

$$\begin{aligned} p_{cp} &= 2(h + b) \\ &= (2)(30 \text{ in} + 12 \text{ in}) \\ &= 84 \text{ in} \\ \frac{A_{cp}^2}{p_{cp}} &= \frac{(360 \text{ in}^2)^2}{84 \text{ in}} \\ &= 1543 \text{ in}^3 \end{aligned}$$

The threshold torsion is given by ACI Sec. 11.5.1 as

$$\begin{aligned} T_u &= \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4 \lambda \sqrt{f'_c}}} \\ &= (0.75)(1.0) \sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} (1543 \text{ in}^3) \\ &\quad \times \sqrt{1 + \frac{319 \frac{\text{lbf}}{\text{in}}}{(4)(1.0) \sqrt{6000 \frac{\text{lbf}}{\text{in}^2}}}} \\ &= \frac{1000 \frac{\text{lbf}}{\text{kip}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 128 \text{ in-kips} \end{aligned}$$

### Design for Torsion Reinforcement

After torsional cracking occurs, cracks are produced diagonally on all four faces of a member, forming a continuous spiral failure surface around the perimeter. For a prestressed concrete beam subjected to pure torsion, the principal tensile stresses and the cracks are produced at an angle  $\theta$ . ACI Sec. 11.5.3.6 specifies that the angle  $\theta$  may be assumed to be 37.5°. After cracking, the beam is idealized as a tubular space truss consisting of closed stirrups, longitudinal reinforcement, and concrete compression struts between the torsion cracks. To resist the applied torque, it is necessary to provide transverse and longitudinal reinforcement on each face. Closed stirrups are required to resist the vertical component, and longitudinal reinforcement is required to resist the horizontal component of the torsional stresses. After torsional cracking, the concrete outside the closed stirrups is ineffectual in resisting the applied torsion, and the gross area enclosed by shear flow is redefined by ACI Sec. 11.5.3.6 as

$$A_o = 0.85 A_{oh}$$

In an indeterminate structure, redistribution of internal forces after cracking results in a reduction in torsional moment with a compensating redistribution of internal forces. This is referred to as *compatibility torsion*, and the member may be designed for a maximum factored torsional moment of

$$T_u = 4 \phi \lambda \sqrt{f'_c} \left( \frac{A_{cp}^2}{p_{cp}} \right) \sqrt{1 + \frac{f_{pc}}{4 \lambda \sqrt{f'_c}}}$$

When the torsional moment cannot be reduced by redistribution of internal forces after cracking, the member must be designed for the full applied factored torque. This is referred to as *equilibrium torsion*.

When the threshold torsion value is exceeded, closed stirrups and longitudinal reinforcement must be provided to resist the appropriate value of the torque, and the concrete is considered ineffective.



ACI Sec. 11.5.3.6 specifies the required area of one leg of a closed stirrup as

$$\begin{aligned}\frac{A_t}{s} &= \frac{T_u}{2\phi A_o f_{yt} \cot \theta} \\ &= \frac{T_u}{2\phi A_o f_{yt} \cot 37.5^\circ} \\ &= \frac{T_u}{1.7\phi A_{oh} f_{yt} \cot 37.5^\circ}\end{aligned}$$

The minimum combined area of stirrups for shear and torsion is given by ACI Sec. 11.5.5.2 as

$$\begin{aligned}\frac{A_v + 2A_t}{s} &= 0.75 \sqrt{f'_c} \frac{b_w}{f_{yt}} \\ &\geq \frac{50b_w}{f_{yt}}\end{aligned}$$

The maximum spacing of closed stirrups is given by ACI Sec. 11.5.6.1 as

$$\begin{aligned}s &= \frac{p_h}{8} \\ &\leq 12 \text{ in}\end{aligned}$$

The corresponding required area of longitudinal reinforcement is specified in ACI Secs. 11.5.3.7 and R11.5.3.10 as

$$\begin{aligned}A_l &= \left(\frac{A_t}{s}\right) \left(\frac{p_h f_{yt}}{f_y}\right) \cot^2 \theta \\ &= \left(\frac{A_t}{s}\right) \left(\frac{p_h f_{yt}}{f_y}\right) \cot^2 37.5^\circ\end{aligned}$$

When the threshold torsional moment is exceeded, the minimum permissible area of longitudinal reinforcement is given by ACI Sec. 11.5.5.3 as

$$\begin{aligned}A_{l,\min} &= \frac{5\sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \quad [\text{ACI 11-24}] \\ \frac{A_t}{s} &\geq \frac{25b_w}{f_{yt}}\end{aligned}$$

The bars are distributed around the inside perimeter of the closed stirrups at a maximum spacing of 12 in. A longitudinal bar is required in each corner of a closed stirrup.

The minimum diameter of longitudinal reinforcement is given by ACI Sec. 11.5.6.2 as

$$\begin{aligned}d_{bl} &= \frac{s}{24} \\ &\geq \text{no. 3 bar}\end{aligned}$$

To prevent crushing of the concrete compression diagonals, the combined stress caused by the factored

torsion and shear forces is limited by ACI Sec. 11.5.3.1. The dimensions of the section must be such that

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

### Example 3.7

The prestressed beam described in Ex. 3.5 is simply supported over a span of 30 ft, and has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low relaxation prestressing tendons provided is  $A_{ps} = 0.765 \text{ in}^2$  with a specified tensile strength of  $f_{pu} = 270 \text{ kips/in}^2$ , a yield strength of  $f_{py} = 243 \text{ kips/in}^2$ , and an effective stress of  $f_{se} = 150 \text{ kips/in}^2$  after all losses. The centroid of the prestressing tendons is 10 in above the soffit of the beam at a distance of 15 in from the support. The beam supports a uniformly distributed factored load, including its self-weight, of 8 kips/ft, and an applied factored torsional moment of 150 in-kips. Determine the shear and torsional reinforcement required at the critical section, if the yield strength of the reinforcement is  $f_{yt} = f_y = 60 \text{ kips/in}^2$ .

### Solution

From Ex. 3.5, the required shear reinforcement at the critical section is

$$\frac{A_v}{s} = 0.293 \text{ in}^2/\text{ft}$$

The threshold torsion is obtained in Ex. 3.6 as

$$\begin{aligned}T_u &= 128 \text{ in-kips} \\ &< 150 \text{ in-kips}\end{aligned}$$

Torsional reinforcement is required.

With 1 in cover to  $1/2$  in stirrups, the distance between the center lines of the vertical legs of the closed stirrups is

$$\begin{aligned}x_o &= b_w - d_b - 2(\text{clear cover}) \\ &= 12 \text{ in} - 0.5 \text{ in} - (2)(1 \text{ in}) \\ &= 9.5 \text{ in}\end{aligned}$$

The distance between the center lines of the horizontal legs of the closed stirrups is

$$\begin{aligned}y_o &= h - d_b - 2(\text{clear cover}) \\ &= 30 \text{ in} - 0.5 \text{ in} - (2)(1 \text{ in}) \\ &= 27.5 \text{ in}\end{aligned}$$



The area enclosed by the center line of the closed stirrups is

$$\begin{aligned} A_{oh} &= x_o y_o \\ &= (9.5 \text{ in})(27.5 \text{ in}) \\ &= 261 \text{ in}^2 \end{aligned}$$

The gross area enclosed by shear flow is given by ACI Sec. 11.5.3.6 as

$$\begin{aligned} A_o &= 0.85 A_{oh} \\ &= (0.85)(261 \text{ in}^2) \\ &= 222 \text{ in}^2 \end{aligned}$$

The perimeter of the center line of the closed stirrups is

$$\begin{aligned} p_h &= 2(x_o + y_o) \\ &= (2)(9.5 \text{ in} + 27.5 \text{ in}) \\ &= 74 \text{ in} \end{aligned}$$

From ACI Eq. (11-21), assuming that  $\theta = 37.5^\circ$ , the required area of one leg of a closed stirrup is

$$\begin{aligned} \frac{A_t}{s} &= \frac{T_u}{2\phi A_o f_{yt} \cot 37.5^\circ} \\ &= \frac{(150 \text{ in-kips}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{(2)(0.75)(222 \text{ in}^2) \left(60 \frac{\text{kips}}{\text{in}^2}\right) (1.303)} \\ &= 0.069 \text{ in}^2/\text{ft}/\text{leg} \end{aligned}$$

The summation of the areas of required shear and torsion reinforcement is

$$\begin{aligned} \sum \frac{A_{v+t}}{s} &= \frac{A_v}{s} + \frac{2A_t}{s} = 0.293 \frac{\text{in}^2}{\text{ft}} + (2) \left(0.069 \frac{\text{in}^2}{\text{ft}}\right) \\ &= 0.43 \text{ in}^2/\text{ft} \end{aligned}$$

The minimum permissible combined area of stirrups for shear and torsion is given by ACI Sec. 11.5.5.2 as the greater of

$$\begin{aligned} \frac{A_v + 2A_t}{s} &= \frac{0.75 \sqrt{f'_c} b_w}{f_{yt}} \\ &= \frac{0.75 \sqrt{6000 \frac{\text{lb}}{\text{in}^2}} (12 \text{ in}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{60,000 \frac{\text{lb}}{\text{in}^2}} \\ &= 0.139 \text{ in}^2/\text{ft} \quad [\text{does not govern}] \end{aligned}$$

$$\begin{aligned} \frac{A_v + 2A_t}{s} &= \frac{50 b_w}{f_{yt}} \\ &= \frac{(50)(12 \text{ in}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{60,000 \frac{\text{lb}}{\text{in}^2}} \\ &= 0.12 \text{ in}^2/\text{ft} \quad [\text{does not govern}] \end{aligned}$$

The governing maximum permissible spacing of the closed stirrups is specified by ACI Sec. 11.5.6.1 as

$$\begin{aligned} s_{\max} &= \frac{p_h}{8} \\ &= \frac{74 \text{ in}}{8} \\ &= 9.25 \text{ in} \end{aligned}$$

Closed stirrups consisting of two arms of no. 3 bars at 6 in spacing provides an area of

$$\begin{aligned} \frac{A}{s} &= \left(2 \frac{\text{bars}}{\text{ft}}\right) (2) \left(0.11 \frac{\text{in}^2}{\text{bar}}\right) = 0.44 \frac{\text{in}^2}{\text{ft}} \\ &> 0.43 \text{ in}^2/\text{ft} \quad [\text{satisfactory}] \end{aligned}$$

The required area of the longitudinal reinforcement is given by ACI Eq. (11-22), assuming  $\theta = 37.5^\circ$ , as

$$\begin{aligned} A_t &= \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right) \cot^2 \theta \\ &= \frac{\left(0.069 \frac{\text{in}^2}{\text{ft}}\right) (74 \text{ in}) (\cot 37.5^\circ)^2}{12 \frac{\text{in}}{\text{ft}}} \\ &= 0.72 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} \frac{25 b_w}{f_{yt}} &= \frac{(25)(12 \text{ in})}{60,000 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.0050 \text{ in}^2/\text{in-leg} \\ &< A_t/s \quad [= 0.00576 \text{ in}^2/\text{in-leg}] \end{aligned}$$

Use  $A_t/s = 0.00576 \text{ in}^2/\text{in}/\text{leg}$  in ACI Eq. (11-24). The minimum permissible area of longitudinal reinforcement is given by ACI Eq. (11-24) as

$$\begin{aligned} A_{l,\min} &= \frac{5\sqrt{f'_c}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yt}}{f_y}\right) \\ &= \frac{5\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}}(360 \text{ in}^2)}{60,000 \frac{\text{lbf}}{\text{in}^2}} \\ &\quad - \left(0.00576 \frac{\text{in}^2}{\text{in}}\right)(74 \text{ in}) \\ &= 1.89 \text{ in}^2 \quad [\text{governs}] \end{aligned}$$

Use 10 no. 4 bars placed in the corners of the closed stirrups and distributed along the vertical legs to give a longitudinal steel area of

$$\begin{aligned} A_l &= 2.0 \text{ in}^2 \\ &> 1.89 \text{ in}^2 \quad [\text{satisfactory}] \end{aligned}$$

The minimum permitted diameter of longitudinal reinforcement is given by ACI Sec. 11.5.6.2 as

$$\begin{aligned} d_{bl} &= \frac{s}{24} \\ &= \frac{6 \text{ in}}{24} \\ &= 0.25 \text{ in} \\ &< 0.5 \text{ in} \quad [\text{no. 4 bars are satisfactory}] \end{aligned}$$

A check on the crushing of the concrete compression diagonals is provided by ACI Sec. 11.5.3.1. The left side of ACI Eq. (11-18) is

$$\begin{aligned} &\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \\ &= \sqrt{\left(\frac{110 \text{ in-kips}}{(12 \text{ in})(0.8)(30 \text{ in})}\right)^2 + \left(\frac{(150 \text{ in-kips})(74 \text{ in})}{(1.7)(261 \text{ in}^2)^2}\right)^2} \\ &= \sqrt{0.146 \frac{\text{kips}}{\text{in}^2} + 0.009 \frac{\text{kips}}{\text{in}^2}} \\ &= 0.394 \text{ kips/in}^2 \end{aligned}$$

The right side of ACI Eq. (11-18) is

$$\begin{aligned} &\phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \\ &= (0.75)\left(\frac{111,540 \text{ lbf}}{(12 \text{ in})(0.8)(30 \text{ in})} + 8\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}}\right) \\ &= \frac{1000 \frac{\text{lbf}}{\text{kip}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 0.755 \frac{\text{kips}}{\text{in}^2} \\ &> 0.394 \text{ kips/in}^2 \quad [\text{ACI Eq. (11-18) is satisfied}] \end{aligned}$$

### 3. PRESTRESS LOSSES

#### Nomenclature

|           |                                                                                                                          |                    |
|-----------|--------------------------------------------------------------------------------------------------------------------------|--------------------|
| $A_c$     | area of the gross concrete section                                                                                       | $\text{in}^2$      |
| $A_{ps}$  | area of prestressing tendon                                                                                              | $\text{in}^2$      |
| $b_w$     | web width, or diameter of circular section                                                                               | $\text{in}$        |
| $C$       | factor for relaxation losses                                                                                             | —                  |
| $E_c$     | modulus of elasticity of concrete at 28 days                                                                             | $\text{kips/in}^2$ |
| $E_{ci}$  | modulus of elasticity of concrete at time of initial prestress                                                           | $\text{kips/in}^2$ |
| $E_p$     | modulus of elasticity of prestressing tendon                                                                             | $\text{kips/in}^2$ |
| $f'_{ci}$ | concrete strength at transfer                                                                                            | $\text{lbf/in}^2$  |
| $f_{pd}$  | compressive stress at level of tendon centroid after elastic losses and including sustained dead load                    | $\text{lbf/in}^2$  |
| $f_{pi}$  | compressive stress at level of tendon centroid after elastic losses                                                      | $\text{lbf/in}^2$  |
| $f_{pp}$  | compressive stress at level of tendon centroid before elastic losses                                                     | $\text{lbf/in}^2$  |
| $g$       | sag of prestressing tendon                                                                                               | $\text{in}$        |
| $h$       | overall thickness or height of member                                                                                    | $\text{in}$        |
| $H$       | ambient relative humidity                                                                                                | —                  |
| $I_c$     | moment of inertia of the gross concrete section                                                                          | $\text{in}^4$      |
| $J$       | factor for relaxation losses                                                                                             | —                  |
| $K$       | wobble friction coefficient per foot of prestressing tendon                                                              | —                  |
| $K_{sh}$  | factor for shrinkage losses accounting for elapsed time between completion of casting and transfer of prestressing force | —                  |
| $K_{re}$  | factor for relaxation losses                                                                                             | —                  |
| $l_{px}$  | length of prestressing tendon from jacking end to any point $x$ measured along the curve                                 | $\text{ft}$        |
| $M_D$     | bending moment caused by superimposed dead load                                                                          | $\text{ft-kips}$   |
| $M_G$     | bending moment caused by member self-weight                                                                              | $\text{ft-kips}$   |
| $n$       | $E_p/E_c$                                                                                                                | —                  |
| $n_i$     | $E_p/E_{ci}$                                                                                                             | —                  |

|                 |                                                                     |                    |
|-----------------|---------------------------------------------------------------------|--------------------|
| $p_{cp}$        | outside perimeter of the concrete cross section                     | in                 |
| $P_i$           | prestressing tendon force after elastic losses                      | kips               |
| $P_p$           | prestressing tendon force before elastic losses                     | kips               |
| $P_{pj}$        | prestressing tendon force at jacking end                            | kips               |
| $P_{px}$        | prestressing tendon force at a distance of $x$ from the jacking end | kips               |
| $P_{\Delta cr}$ | loss of tendon force caused by creep                                | kips               |
| $P_{\Delta el}$ | loss of tendon force caused by elastic shortening                   | kips               |
| $P_{\Delta re}$ | loss of tendon force caused by relaxation                           | kips               |
| $P_{\Delta sh}$ | loss of tendon force caused by shrinkage                            | kips               |
| $R$             | radius of curvature of tendon profile                               | ft                 |
| $w_c$           | unit weight of concrete                                             | lb/ft <sup>3</sup> |

### Symbols

|                 |                                                                               |   |
|-----------------|-------------------------------------------------------------------------------|---|
| $\alpha$        | angular change in radians of tendon profile from jacking end to any point $x$ | — |
| $\epsilon_{sh}$ | basic shrinkage strain                                                        | — |
| $\mu$           | curvature friction coefficient                                                | — |

### Friction Losses

Friction losses, which occur in post-tensioned members because of friction and unintentional out-of-straightness of the ducts, are determined by ACI Sec. 18.6.2.1 as

$$P_{pj} = P_{px} e^{Kl_{px} + \mu_p \alpha_{px}} \quad [\text{ACI 18-1}]$$

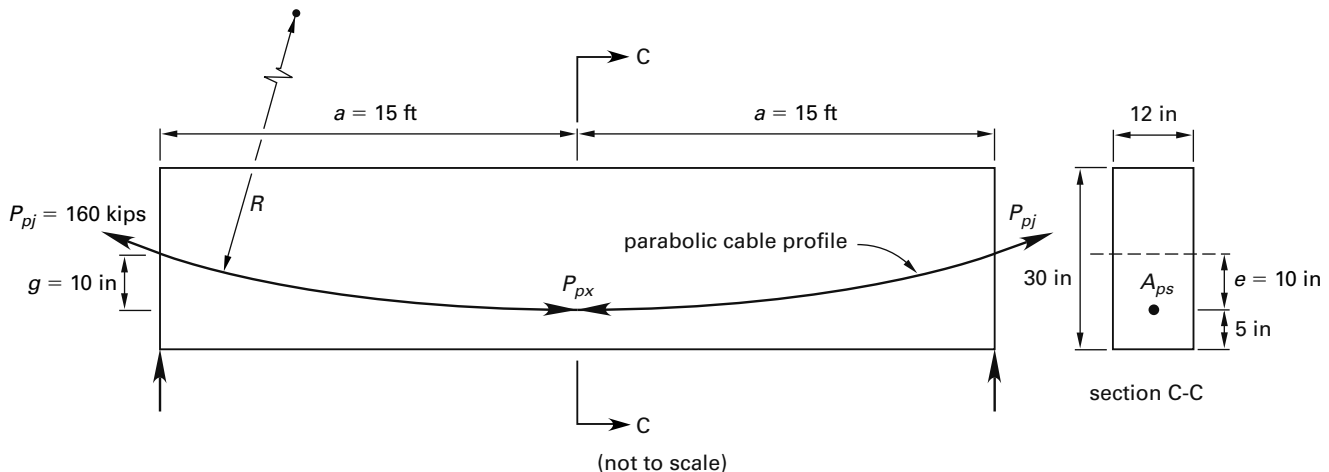
For a value of  $(Kl_{px} + \mu\alpha)$  not greater than 0.3, ACI Eq. (18-1) can be approximated by

$$P_{pj} = P_{px}(1 + Kl_{px} + \mu_p \alpha_{px}) \quad [\text{ACI 18-2}]$$

Rearranging the terms gives

$$P_{px} = P_{pj}(1 - Kl_{px} - \mu\alpha)$$

Illustration for Example 3.8



### Example 3.8

The post-tensioned beam shown in the illustration is simply supported over a span of 30 ft and has a 28 day concrete strength of 6000 lb/ft<sup>2</sup>. The area of the low-relaxation prestressing tendon provided is  $A_{ps} = 0.765 \text{ in}^2$ , with a specified tensile strength of  $f_{pu} = 270 \text{ kips/in}^2$ , and a yield strength of  $f_{py} = 243 \text{ kips/in}^2$ . The cable centroid, as shown, is parabolic in shape, and is stressed simultaneously from both ends with a jacking force of  $P_{pj} = 160 \text{ kips}$ . The value of the wobble friction coefficient is  $K = 0.0010/\text{ft}$ , and the curvature friction coefficient is  $\mu = 0.20$ . Determine the cable force at midspan of the member before elastic losses.

### Solution

The nominal radius of the cable profile is

$$\begin{aligned} R &= \frac{a^2}{2g} \\ &= \frac{(15 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{(2)(10.0 \text{ in})} \\ &= 135 \text{ ft} \end{aligned}$$

The cable length along the curve, from the jacking end to midspan is

$$\begin{aligned} l_{px} &= a + \frac{g^2}{3a} \\ &= 15 \text{ ft} + \frac{\left(\frac{10 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)^2}{(3)(15 \text{ ft})} \\ &= 15.02 \text{ ft} \end{aligned}$$

The angular change of the cable profile over length  $l_x$  is

$$\begin{aligned}\alpha &= \frac{l_{px}}{R} \\ &= \frac{15.02 \text{ ft}}{135 \text{ ft}} \\ &= 0.111 \text{ radians} \\ Kl_{px} + \mu\alpha &= (0.0010 \text{ ft}^{-1})(15.02 \text{ ft}) \\ &\quad + (0.20)(0.111 \text{ radians}) \\ &= 0.037 \\ &< 0.3 \quad [\text{ACI Eq. (18-2) is applicable}]\end{aligned}$$

The cable force at midspan is

$$\begin{aligned}P_{px} &= P_{pj}(1 - Kl_{px} - \mu\alpha) \\ &= (160 \text{ kips})(1 - 0.01502 - 0.0222) \\ &= 154 \text{ kips}\end{aligned}$$

### Anchor Seating Losses

Anchor seating losses result from slip or set that occur in the anchorage when prestressing force is transferred to the anchor device. Anchorage slip depends on the prestressing system used, and may be compensated by increasing the jacking force.

### Elastic Shortening Losses

Losses occur in a prestressed concrete beam at transfer due to the elastic shortening of the concrete. The concrete stress at the level of the centroid of the prestressing tendons after elastic shortening is

$$f_{pi} = P_i \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) - \frac{eM_G}{I_g}$$

Conservatively, this may be taken as the concrete stress at the level of the centroid of the prestressing tendons before elastic shortening, given by

$$f_{pi} \approx f_{pp} = P_p \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) - e \frac{M_G}{I_g}$$

If necessary, a process of iteration may now be used to refine the calculated value of the estimated losses.

In a pretensioned member with transfer occurring simultaneously in all tendons, the loss of prestressing force is

$$P_{\Delta el} = n_i A_{ps} f_{pi}$$

In a post-tensioned member with only one tendon, there is no loss from elastic shortening.

In a post-tensioned member with several tendons stressed sequentially, the maximum loss occurs in the first tendon stressed, and no loss occurs in the last

tendon stressed. The total loss is then half the value for a pretensioned member.

$$P_{\Delta el} = \frac{n_i A_{ps} f_{pi}}{2}$$

### Example 3.9

The post-tensioned beam described in Ex. 3.8 has a concrete strength at transfer of 4500 lbf/in<sup>2</sup>. The beam is simply supported over a span of 30 ft, and is prestressed with two cables having a combined area of  $A_{ps} = 0.765 \text{ in}^2$ . At the center of the span, the total initial force in the two tendons after friction losses is  $P_p = 154 \text{ kips}$ . The modulus of elasticity of the prestressing tendon is  $E_p = 28 \times 10^6 \text{ lbf/in}^2$ . Determine the loss of prestressing force caused by elastic shortening.

*Solution*

The self-weight of the beam is

$$\begin{aligned}w &= w_c b_w h \\ &= \left( 150 \frac{\text{lbf}}{\text{ft}^3} \right) (1.0 \text{ ft}) (2.5 \text{ ft}) \\ &= 375 \text{ lbf/ft}\end{aligned}$$

The self-weight bending moment at midspan is

$$\begin{aligned}M_G &= \frac{wl^2}{8} \\ &= \frac{\left( 375 \frac{\text{lbf}}{\text{ft}} \right) (30 \text{ ft})^2 \left( 12 \frac{\text{in}}{\text{ft}} \right)}{(8) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)} \\ &= 506 \text{ in-kips}\end{aligned}$$

From ACI Sec. 8.5, the modulus of elasticity of the concrete at transfer is

$$\begin{aligned}E_{ci} &= 57,000 \sqrt{f'_c} \\ &= 57,000 \sqrt{4500} \frac{\text{lbf}}{\text{in}^2} \\ &= 3,824,000 \frac{\text{lbf}}{\text{in}^2} \\ n_i &= \frac{E_p}{E_{ci}} \\ &= \frac{28 \times 10^6 \frac{\text{lbf}}{\text{in}^2}}{3,824,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 7.32\end{aligned}$$

The concrete stress at the level of the centroid of the prestressing tendons before elastic shortening is

$$\begin{aligned} f_{pp} &= P_p \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) - \frac{eM_G}{I_g} \\ &= (154 \text{ kips}) \left( \frac{1}{360 \text{ in}^2} + \frac{(10 \text{ in})^2}{27,000 \text{ in}^4} \right) \\ &\quad - \frac{(10 \text{ in})(506 \text{ in-kips})}{27,000 \text{ in}^4} \\ &= 0.811 \text{ kips/in}^2 \end{aligned}$$

The loss of prestressing force caused by elastic shortening may be estimated as

$$\begin{aligned} P_{\Delta el} &= \frac{n_i A_{ps} f_{pp}}{2} \\ &= \frac{(7.32)(0.765 \text{ in}^2) \left( 0.811 \frac{\text{kips}}{\text{in}^2} \right)}{2} \\ &= 2.3 \text{ kips} \end{aligned}$$

The force in the cables after friction and elastic losses is

$$\begin{aligned} P_i &= 154 \text{ kips} - 2.3 \text{ kips} \\ &= 151.7 \text{ kips} \end{aligned}$$

### Creep Losses

Creep is the increasing strain that occurs in a prestressed concrete member caused by the sustained compressive stress. The rate of creep is increased when the initial stress in the member is increased. The earlier the age at which the stress is applied to the member, the greater is the rate of creep. The concrete stress at the level of the centroid of the prestressing tendons after elastic shortening, allowing for sustained dead load, is

$$f_{pd} = P_i \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) - \frac{eM_G}{I_g} - \frac{eM_D}{I_g}$$

For post-tensioned members, with transfer at 28 days, the creep loss is given by<sup>1,2</sup>

$$P_{\Delta cr} = 1.6nA_{ps}f_{pd}$$

For pretensioned members, with transfer at 3 days, the creep loss is

$$P_{\Delta cr} = 2.0nA_{ps}f_{pd}$$

### Example 3.10

For the post-tensioned beam of Ex. 3.9, the cable force at midspan after elastic losses is  $P_i = 151.7$  kips. The

28 day concrete strength is  $f'_c = 6000$  lbf/in<sup>2</sup>. The superimposed dead load moment is  $M_D = 800$  in-kips. Determine the loss of prestressing force caused by creep.

*Solution*

From ACI Sec. 8.5, the modulus of elasticity of the concrete at 28 days is

$$\begin{aligned} E_c &= 57,000 \sqrt{f'_c} \\ &= 57,000 \sqrt{6000} \frac{\text{lbf}}{\text{in}^2} \\ &= 4,415,000 \text{ lbf/in}^2 \end{aligned}$$

The modular ratio at 28 days is

$$\begin{aligned} n &= \frac{E_p}{E_c} \\ &= \frac{28 \times 10^6 \frac{\text{lbf}}{\text{in}^2}}{4,415,000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 6.34 \end{aligned}$$

The concrete stress at the level of the centroid of the prestressing tendons, after elastic shortening and allowing for sustained dead load, is

$$\begin{aligned} f_{pd} &= P_i \left( \frac{1}{A_g} + \frac{e^2}{I_g} \right) - \frac{eM_G}{I_g} - \frac{eM_D}{I_g} \\ &= (151.7 \text{ kips}) \left( \frac{1}{360 \text{ in}^2} + \frac{(10 \text{ in})^2}{27,000 \text{ in}^4} \right) \\ &\quad - \frac{(10 \text{ in})(506 \text{ in-kips})}{27,000 \text{ in}^4} - \frac{(10 \text{ in})(800 \text{ in-kips})}{27,000 \text{ in}^4} \\ &= 0.499 \text{ kips/in}^2 \end{aligned}$$

The loss of prestressing force caused by creep is

$$\begin{aligned} P_{\Delta cr} &= 1.6nA_{ps}f_{pd} \\ &= (1.6)(6.34)(0.765 \text{ in}^2) \left( 0.499 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 3.9 \text{ kips} \end{aligned}$$

The force in the cable after friction, elastic and creep losses is

$$\begin{aligned} P &= 151.7 \text{ kips} - 3.9 \text{ kips} \\ &= 147.8 \text{ kips} \end{aligned}$$

<sup>1</sup>Zia, P., et al., 1979 (See References and Codes)

<sup>2</sup>Portland Cement Association, 2008 (See References and Codes)

### Shrinkage Losses

The shrinkage of a prestressed beam produces a shortening of the beam with a corresponding loss of prestress. The basic shrinkage strain is given by<sup>3,4</sup>

$$\epsilon_{sh} = 8.2 \times 10^{-6} \text{ in/in}$$

Allowing for the ambient relative humidity,  $H$ , and the ratio of the member's volume to surface area,  $A_g/p_{cp}$ , the shrinkage loss for a pretensioned member is

$$P_{\Delta sh} = A_{ps}\epsilon_{sh}E_p \left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H)$$

For a post-tensioned member, with transfer after some shrinkage has already occurred, the shrinkage loss is

$$P_{\Delta sh} = K_{sh}A_{ps}\epsilon_{sh}E_p \left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H)$$

### Example 3.11

The post-tensioned beam of Ex. 3.10 is located in an area with an ambient relative humidity of 60%, and transfer is effected 10 days after the completion of curing, giving a value<sup>5</sup> of 0.73 for  $K_{sh}$ . Determine the loss of prestressing force caused by shrinkage.

*Solution*

The shrinkage loss is

$$\begin{aligned} P_{\Delta sh} &= K_{sh}A_{ps}\epsilon_{sh}E_p \left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H) \\ &= (0.73)(0.765 \text{ in}^2) \left(8.2 \times 10^{-6} \frac{\text{in}}{\text{in}}\right) \\ &\quad \times \left(28 \times 10^3 \frac{\text{kips}}{\text{in}^2}\right) \\ &\quad \times \left(1 - \frac{(0.06 \text{ in}^{-1})(360 \text{ in}^2)}{84 \text{ in}}\right)(100 - 60) \\ &= 3.8 \text{ kips} \end{aligned}$$

The force in the cable after friction, elastic, creep, and shrinkage losses is

$$\begin{aligned} P &= 147.8 \text{ kips} - 3.8 \text{ kips} \\ &= 144 \text{ kips} \end{aligned}$$

### Relaxation Losses

Relaxation is the reduction in tensile force that occurs in a prestressing tendon under sustained tensile strain. The loss in prestress depends on the tendon properties, the initial force in the tendon, and the losses caused by

creep, shrinkage, and elastic shortening. The relaxation loss is given by<sup>5,3</sup>

$$P_{\Delta re} = (A_{ps}K_{re} - J(P_{\Delta cr} + P_{\Delta sh} + P_{\Delta el}))C$$

### Example 3.12

For the post-tensioned beam of Ex. 3.11, the values of the relevant parameters are<sup>4</sup>

$$K_{re} = 5 \text{ kips/in}^2$$

$$J = 0.04$$

$$C = 0.09$$

Determine the loss of prestressing force caused by relaxation.

*Solution*

The loss caused by relaxation is

$$\begin{aligned} P_{\Delta re} &= (A_{ps}K_{re} - J(P_{\Delta cr} + P_{\Delta sh} + P_{\Delta el}))C \\ &= \left( (0.765 \text{ in}^2) \left(5 \frac{\text{kips}}{\text{in}^2}\right) \right. \\ &\quad \left. - (0.04)(3.9 \text{ kips} + 3.8 \text{ kips} + 2.3 \text{ kips}) \right) \\ &\quad \times (0.9) \\ &= 3.1 \text{ kips} \end{aligned}$$

The force in the cable after friction, elastic, creep, shrinkage, and relaxation losses is

$$\begin{aligned} P_e &= P - P_{\Delta re} \\ &= 144 \text{ kips} - 3.1 \text{ kips} \\ &= 140.9 \text{ kips} \end{aligned}$$

## 4. COMPOSITE CONSTRUCTION

### Nomenclature

|          |                                                                                                          |                      |
|----------|----------------------------------------------------------------------------------------------------------|----------------------|
| $A_c$    | area of precast surface or area of contact surface for horizontal shear                                  | in <sup>2</sup>      |
| $A_g$    | gross area of precast girder                                                                             | in <sup>2</sup>      |
| $A_v$    | area of ties within a distance $s$                                                                       | in <sup>2</sup>      |
| $A_{vf}$ | area of friction reinforcement                                                                           | in                   |
| $b$      | effective flange width                                                                                   | in                   |
| $b_f$    | actual flange width                                                                                      | in                   |
| $b_v$    | width of girder at contact surface                                                                       | in                   |
| $b_w$    | width of girder web                                                                                      | in                   |
| $b/n$    | transformed flange width                                                                                 | in                   |
| $d$      | distance from extreme compression fiber to centroid of prestressed reinforcement (not less than $0.8h$ ) | in                   |
| $e$      | eccentricity of prestressing force                                                                       | in                   |
| $E_f$    | modulus of elasticity of flange concrete                                                                 | kips/in <sup>2</sup> |

<sup>3</sup>Zia, P., et al., 1979 (See References and Codes)

<sup>4</sup>Portland Cement Association, 2008 (See References and Codes)

<sup>5</sup>Ibid.

|             |                                                                                             |                      |
|-------------|---------------------------------------------------------------------------------------------|----------------------|
| $E_w$       | modulus of elasticity of precast girder concrete                                            | kips/in <sup>2</sup> |
| $f_b$       | stress in the bottom fiber of the composite section                                         | lb/in <sup>2</sup>   |
| $f_t$       | stress in the top fiber of the composite section                                            | lb/in <sup>2</sup>   |
| $h$         | depth of composite section                                                                  | in                   |
| $h_f$       | depth of flange of the composite section                                                    | in                   |
| $h_w$       | depth of precast girder                                                                     | in                   |
| $I_c$       | moment of inertia of composite section                                                      | in <sup>4</sup>      |
| $I_g$       | moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement | in <sup>4</sup>      |
| $l$         | span length                                                                                 | ft                   |
| $M$         | applied bending moment                                                                      | ft-kips              |
| $M_F$       | moment caused by the flange concrete                                                        | ft-kips              |
| $M_G$       | moment caused by the precast girder self-weight                                             | ft-kips              |
| $M_L$       | moment caused by the superimposed load                                                      | ft-kips              |
| $n$         | modular ratio, $E_w/E_f$                                                                    | —                    |
| $P_e$       | force in the prestressing tendons at service loads after allowance for all losses           | kips                 |
| $P_\Delta$  | total loss of prestress                                                                     | kips                 |
| $s$         | spacing of ties                                                                             | in                   |
| $s_b$       | spacing of precast girders                                                                  | ft                   |
| $S_{cb}$    | section modulus at bottom of composite section                                              | in <sup>3</sup>      |
| $S_{ct}$    | section modulus at top of composite section                                                 | in <sup>3</sup>      |
| $S_g$       | section modulus at bottom of precast girder                                                 | in <sup>3</sup>      |
| $V_{nh}$    | nominal horizontal shear strength                                                           | kips                 |
| $V_u$       | factored shear force at section                                                             | kips                 |
| $w_F$       | weight of concrete flange                                                                   | lb/ft                |
| $w_G$       | weight of precast girder                                                                    | lb/ft                |
| $w_L$       | superimposed load                                                                           | lb/ft                |
| $\bar{y}_c$ | distance from neutral axis of transformed composite section to bottom of precast girder     | in                   |

### Symbols

|           |                                                                         |   |
|-----------|-------------------------------------------------------------------------|---|
| $\lambda$ | correlation factor related to unit weight of concrete                   | — |
| $\mu$     | coefficient of friction                                                 | — |
| $\rho_v$  | ratio of tie reinforcement area to area of contact surface, $A_v/b_v s$ | — |
| $\phi$    | strength reduction factor                                               | — |

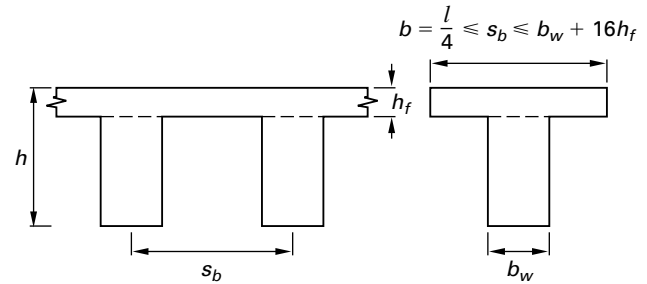
### General Considerations

As shown in Fig. 3.7, a composite beam consists of a concrete flange acting integrally with a precast prestressed concrete girder to resist the applied loads. Transfer of horizontal shear at the interface of the flange and the girder is required to provide composite action between the two components. The compressive stress in the concrete flange is not uniform over the width of the flange, and an effective width is assumed in calculations.

As shown in Fig. 3.7, the effective width of the flange is defined by ACI Sec. 8.12 as the least of

- $l/4$
- $b_w + 16h_f$
- $s_b$

Figure 3.7 Composite Section

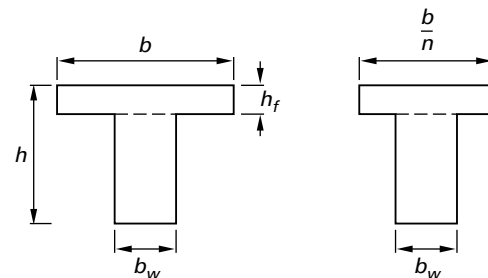


For service load design, composite section properties are calculated by assuming that plane sections remain plane, and that strain varies linearly with distance from the neutral axis of the composite section. When the 28 day compressive strengths of the flange and the precast member are different, the flange area is transformed into an equivalent area corresponding to the properties of the precast member. This equivalent area is obtained by dividing the effective slab width by the modular ratio  $n$ .

As shown in Fig. 3.8, the flange transformed area is

$$A_f = \frac{b h_f}{n}$$

Figure 3.8 Transformed Section



The bending moment applied to the composite section produces a stress in the bottom fiber of the precast girder, given by

$$f_b = \frac{M}{S_{cb}}$$

$$S_{cb} = \frac{I_c}{\bar{y}_c}$$

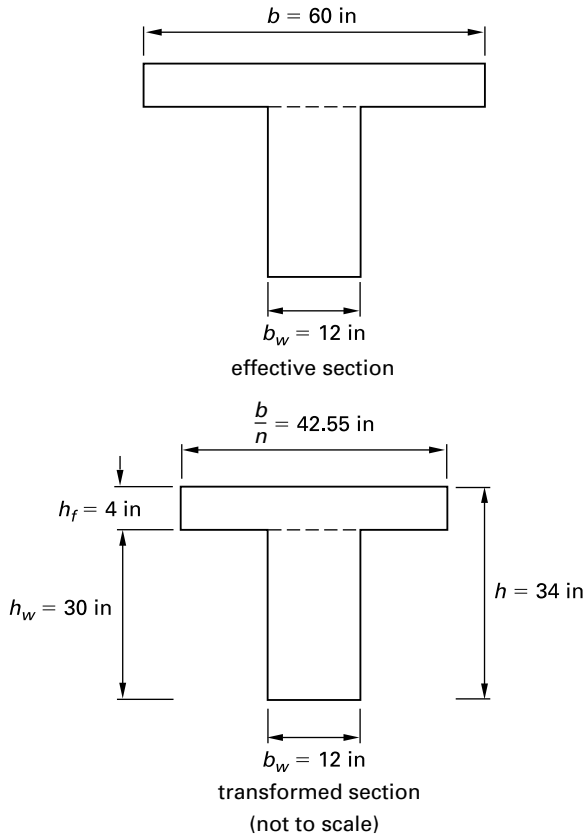
The stress produced in the top of the flange by the bending moment applied to the composite section is

$$f_t = \frac{M}{nS_{ct}}$$

$$S_{ct} = \frac{I_c}{h - \bar{y}_c}$$

### Example 3.13

A composite beam has an effective span of  $l = 30$  ft.



The precast, prestressed girders have a 28 day concrete strength of  $6000 \text{ lbf/in}^2$  and are spaced at  $s_b = 5$  ft centers. The flange has a 28 day concrete strength of  $3000 \text{ lbf/in}^2$ . Determine the transformed section properties of the composite section.

#### Solution

The modulus of elasticity of the girder and flange concrete is obtained from ACI Sec. 8.5 as

$$E_w = 57,000 \sqrt{f'_c} = 57,000 \sqrt{6000} \frac{\text{lbf}}{\text{in}^2}$$

$$= 4,415,000 \text{ lbf/in}^2$$

$$E_f = 57,000 \sqrt{f'_c} = 57,000 \sqrt{3000} \frac{\text{lbf}}{\text{in}^2}$$

$$= 3,122,000 \text{ lbf/in}^2$$

The modular ratio is

$$n = \frac{E_w}{E_f}$$

$$= \frac{4,415,000 \frac{\text{lbf}}{\text{in}^2}}{3,122,000 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 1.41$$

The effective flange width is limited to the lesser of

$$b = \frac{l}{4}$$

$$= \frac{(30 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{4}$$

$$= 90 \text{ in}$$

$$b = b_w + 16h_f$$

$$= 12 \text{ in} + (16)(4 \text{ in})$$

$$= 76 \text{ in}$$

$$b = s_b$$

$$= 60 \text{ in} \quad [\text{governs}]$$

The transformed flange width is

$$\frac{b}{n} = \frac{60 \text{ in}}{1.41}$$

$$= 42.55 \text{ in}$$

The relevant properties of the precast girder are

$$A_g = 360 \text{ in}^2$$

$$I_g = 27,000 \text{ in}^4$$

$$S_g = 1800 \text{ in}^3$$

The relevant properties of the flange are

$$A_f = 170 \text{ in}^2$$

$$I_f = 227 \text{ in}^4$$

The properties of the transformed section are shown in Table 3.1.

**Table 3.1** Transformed Section Moments and Centroids

| part   | $A \text{ (in}^2\text{)}$ | $y \text{ (in)}$ | $I \text{ (in}^4\text{)}$ | $Ay \text{ (in}^3\text{)}$ | $Ay^2 \text{ (in}^4\text{)}$ |
|--------|---------------------------|------------------|---------------------------|----------------------------|------------------------------|
| girder | 360                       | 15               | 27,000                    | 5400                       | 81,000                       |
| flange | 170                       | 32               | 227                       | 5440                       | 174,080                      |
| total  | 530                       | —                | 27,227                    | 10,840                     | 255,080                      |



The height of the centroid of the composite section is

$$\begin{aligned}\bar{y}_c &= \frac{\sum Ay}{\sum A} \\ &= \frac{10,840 \text{ in}^3}{530 \text{ in}^2} \\ &= 20.45 \text{ in}\end{aligned}$$

The moment of inertia of the composite section is

$$\begin{aligned}I_c &= \sum I + \sum Ay^2 + \bar{y}_c^2 \sum A - 2\bar{y} \sum Ay \\ &= 27,227 \text{ in}^4 + 255,080 \text{ in}^4 + (20.45 \text{ in})^2(530 \text{ in}^2) \\ &\quad - (2)(20.45 \text{ in})(10,840 \text{ in}^3) \\ &= 60,598 \text{ in}^4\end{aligned}$$

The section modulus of the composite section referred to the top of the flange is

$$\begin{aligned}S_{ct} &= \frac{I_c}{h - y_c} \\ &= \frac{60,598 \text{ in}^4}{34 \text{ in} - 20.45 \text{ in}} \\ &= 4472 \text{ in}^3\end{aligned}$$

The section modulus of the composite section referred to the bottom of the girder is

$$\begin{aligned}S_{cb} &= \frac{I_c}{\bar{y}_c} \\ &= \frac{60,598 \text{ in}^4}{20.45 \text{ in}} \\ &= 2963 \text{ in}^3\end{aligned}$$

### Shear Connection

Transfer of the horizontal shear force across the interface is necessary to provide full composite action. This may be effected by intentionally roughening the top surface of the precast girder and providing reinforcement to tie the two components together. ACI Sec. 17.5.3 specifies that the factored shear force at a section must not exceed

$$V_u = \phi V_{nh} \quad [\text{ACI 17-1}]$$

When the interface is intentionally roughened, ACI Sec. 17.5.3.1 specifies that the nominal horizontal shear strength must not be greater than

$$V_{nh} = 80b_v d$$

When the interface is smooth, with the minimum ties required by ACI Sec. 11.4.6.3 provided across the

interface, ACI Sec. 17.5.3.2 specifies that the nominal horizontal shear strength must not be greater than

$$V_{nh} = 80b_v d$$

When the interface is roughened to 0.25 in amplitude, with the minimum ties required by ACI Sec. 11.4.6.3 provided across the interface, ACI Sec. 17.5.3.3 specifies that the nominal horizontal shear strength is

$$\begin{aligned}V_{nh} &= (260 + 0.6\rho_v f_y) \lambda b_v d \\ &\leq 500b_v d\end{aligned}$$

The correction factor related to the unit weight of concrete is given by ACI Sec. 11.6.4.3 as

$$\begin{aligned}\lambda &= 1.0 \quad [\text{normal-weight concrete}] \\ &= 0.85 \quad [\text{sand-lightweight concrete}] \\ &= 0.75 \quad [\text{all lightweight concrete}]\end{aligned}$$

When the factored shear force exceeds  $\phi(500b_v d)$ , ACI Sec. 17.5.3.4 requires that the design shall be based on the shear-friction method given in ACI Sec. 11.6.4, with the nominal horizontal shear strength given by

$$\begin{aligned}V_{nh} &= A_{vf} f_y \mu \quad [\text{ACI 11-25}] \\ &\leq 0.2 f'_c A_c \\ &\leq 800 A_c\end{aligned}$$

The coefficient of friction is given by ACI Sec. 11.6.4.3 as

$$\begin{aligned}\mu &= 1.0 \lambda \quad [\text{interface roughened to an amplitude of 0.25 in}] \\ &= 0.6 \lambda \quad [\text{interface not roughened}]\end{aligned}$$

In accordance with ACI Sec. 17.6.1, the tie spacing must not exceed four times the least dimension of the supported element, or 24 in.

### Example 3.14

The composite section of Ex. 3.13 has a factored shear force of  $V_u = 100$  kips at the critical section. Normal-weight concrete is used for both the prestressed girder and the flange. Determine the required tie reinforcement at the interface, which is intentionally roughened to an amplitude of 0.25 in. If ties are needed, use 12 in spacing.

#### Solution

From ACI Sec. 17.5.2, the effective depth of the prestressing tendons may be taken as

$$\begin{aligned}d &= 0.8h \\ &= (0.8)(34 \text{ in}) \\ &= 27.2 \text{ in}\end{aligned}$$

The limiting value of the factored shear force for the application of the shear-friction method is given by ACI Sec. 17.5.3.4 as

$$\begin{aligned} 500\phi b_v d &= \frac{\left(500 \frac{\text{lbf}}{\text{in}^2}\right)(0.75)(12 \text{ in})(27.2 \text{ in})}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 122 \text{ kips} \\ &> V_u \end{aligned}$$

ACI Sec. 17.5.3.3 applies, and

$$\begin{aligned} V_u &= (100 \text{ kips}) \left(1000 \frac{\text{lbf}}{\text{kip}}\right) \\ &= 100,000 \text{ lbf} \\ &= \phi(260 + 0.6\rho_v)\lambda b_v d \\ &= (0.75) \left(260 \frac{\text{lbf}}{\text{in}^2} + 0.6\rho_v \left(60,000 \frac{\text{lbf}}{\text{in}^2}\right)\right) \\ &\quad \times (1.0)(12 \text{ in})(27.2 \text{ in}) \\ \rho_v &= 0.0041 \end{aligned}$$

The required area of vertical ties at a spacing of 12 in is

$$\begin{aligned} A_v &= \rho_v b_v s \\ &= (0.0041)(12 \text{ in})(12 \text{ in}) \\ &= 0.59 \text{ in}^2 \end{aligned}$$

Provide no. 5 grade 60 ties (two legs) at a spacing of 12 in to give an area of

$$\begin{aligned} A_v &= 0.62 \text{ in}^2 \\ &> 0.59 \text{ in}^2 \quad [\text{satisfactory}] \end{aligned}$$

### Design for Flexure

Composite beams may be constructed using either of two methods, shored construction or unshored construction. In shored construction, the precast girder is propped before casting the flange, in order to eliminate stresses in the precast girder caused by the deposited concrete. On removal of the props, the weight of the flange, additional superimposed dead load, and live loads are supported by the composite section. In unshored construction, the flange is cast without propping the precast girder. The precast girder alone must be adequate to support the weight of the flange and all other loads that are applied before the flange concrete has attained its design strength. The composite section supports additional superimposed dead load and live loads.

For both shored and unshored construction, in accordance with ACI Sec. 17.2.4, the design of the composite section for ultimate loads is identical.

### Example 3.15

The precast prestressed girder of the composite section described in Ex. 3.13 has a final prestressing force of  $P_e = 140$  kips, with an eccentricity of  $e = 10$  in at midspan. The loss of prestress may be assumed to occur before the flange is cast. The precast, prestressed girders have a span of  $l = 30$  ft, and are spaced at  $s_b = 5$  ft centers. The composite section has a span of  $l = 30$  ft. The superimposed applied load is  $w_L = 1000$  lbf/ft, and the precast section is not propped. Determine the stresses at midspan in the composite section.

#### Solution

The self-weight bending moment at the midspan of the precast girder is

$$\begin{aligned} M_G &= \frac{w_G l^2}{8} \\ &= \frac{\left(375 \frac{\text{lbf}}{\text{ft}}\right)(30 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{(8) \left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 506 \text{ in-kips} \end{aligned}$$

The bending moment produced at midspan by the weight of the flange concrete is

$$\begin{aligned} M_F &= \frac{w_F l^2}{8} \\ &= \frac{\left(250 \frac{\text{lbf}}{\text{ft}}\right)(30 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{(8) \left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 338 \text{ in-kips} \end{aligned}$$

The bending moment produced at midspan by the applied superimposed load is

$$\begin{aligned} M_L &= \frac{w_L l^2}{8} \\ &= \frac{\left(1000 \frac{\text{lbf}}{\text{ft}}\right)(30 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{(8) \left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 1350 \text{ in-kips} \end{aligned}$$

The stress in the bottom fiber of the prestressed girder caused by the final prestressing force is

$$\begin{aligned} f_b &= P_e \left( \frac{1}{A_g} + \frac{e}{S_g} \right) \\ &= (140 \text{ kips}) \left( \frac{1}{360 \text{ in}^2} + \frac{10 \text{ in}}{1800 \text{ in}^3} \right) \\ &= 1.167 \text{ kips/in}^2 \quad [\text{compression}] \end{aligned}$$

The stress in the bottom fiber of the prestressed girder caused by its self-weight and the weight of the flange concrete is

$$\begin{aligned} f_b &= \frac{M_G + M_F}{S_g} \\ &= \frac{506 \text{ in-kips} + 338 \text{ in-kips}}{1800 \text{ in}^3} \\ &= 0.469 \text{ kips/in}^2 \quad [\text{tension}] \end{aligned}$$

The stress in the bottom fiber of the prestressed girder caused by the applied superimposed load on the composite section is

$$\begin{aligned} f_b &= \frac{M_L}{S_{cb}} \\ &= \frac{1350 \text{ in-kips}}{2963 \text{ in}^3} \\ &= 0.456 \text{ kips/in}^2 \quad [\text{tension}] \end{aligned}$$

The final stress in the bottom fiber of the prestressed girder caused by all loads is

$$\begin{aligned} f_b &= 1.167 \frac{\text{kips}}{\text{in}^2} - 0.469 \frac{\text{kips}}{\text{in}^2} - 0.456 \frac{\text{kips}}{\text{in}^2} \\ &= 0.242 \text{ kips/in}^2 \quad [\text{compression}] \end{aligned}$$

The final stress in the top fiber of the flange caused by all loads is

$$\begin{aligned} f_t &= \frac{M_L}{nS_{ct}} \\ &= \frac{1350 \text{ in-kips}}{(1.41)(4472 \text{ in}^3)} \\ &= 0.214 \text{ kips/in}^2 \end{aligned}$$

## 5. LOAD BALANCING PROCEDURE

### Nomenclature

|       |                                                        |                          |
|-------|--------------------------------------------------------|--------------------------|
| $A_g$ | area of concrete section                               | $\text{in}^2$            |
| $f_c$ | concrete stress                                        | $\text{lbf/in}^2$        |
| $g$   | sag of prestressing tendon                             | $\text{in}$              |
| $l$   | span length                                            | $\text{ft}$              |
| $M_B$ | balancing load moment caused by $w_B$                  | $\text{in-lbf}$          |
| $M_o$ | out-of-balance moment caused by $w_o$ ,<br>$M_W - M_B$ | $\text{in-lbf}$          |
| $M_W$ | applied load moment caused by $w_W$                    | $\text{in-lbf}$          |
| $P$   | prestressing force                                     | $\text{kips}$            |
| $S$   | section modulus                                        | $\text{in}^3$            |
| $w_B$ | balancing-load produced by<br>prestressing tendon      | $\text{kips/ft or kips}$ |
| $w_o$ | out-of-balance load, $w_W - w_B$                       | $\text{kips/ft or kips}$ |
| $w_W$ | superimposed applied load                              | $\text{kips/ft or kips}$ |

The load balancing technique uses the upward pressure of the prestressing tendons on a prestressed beam to balance the downward pressure of the applied loads. As shown in Fig. 3.9, a tendon with a parabolic profile produces a uniformly distributed upward pressure on a beam of  $w_B$ . Equating moments in the free-body diagram gives

$$\frac{w_B l^2}{8} = M_B = Pg$$

The balancing load is

$$w_B = \frac{8Pg}{l^2}$$

The applied load  $w_W$  on the beam produces a moment

$$M_W = \frac{w_W l^2}{8}$$

When the applied load  $w_W$  equals the balancing load  $w_B$ , the moment produced by the applied load is

$$M_W = \frac{w_B l^2}{8} = Pg$$

Figure 3.9 Load Balancing Concepts

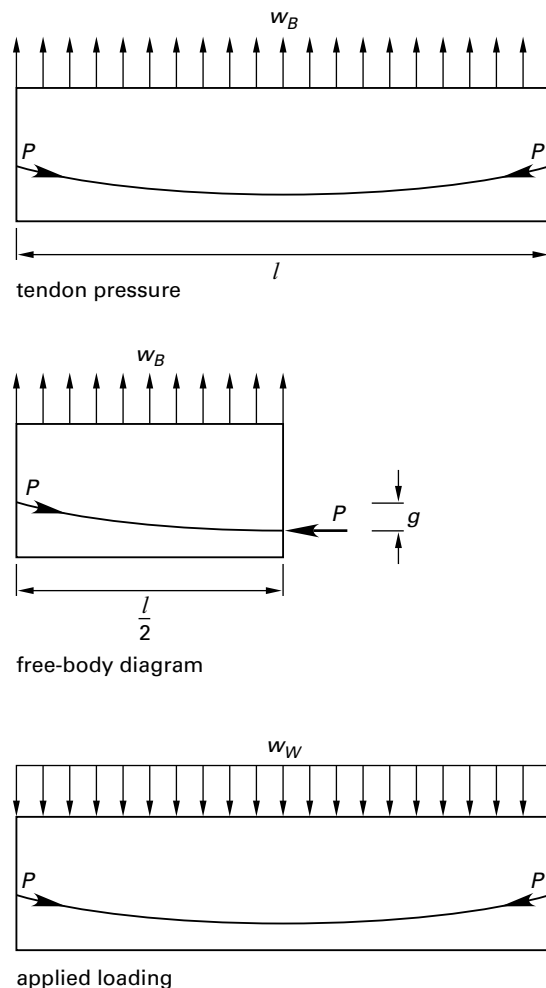
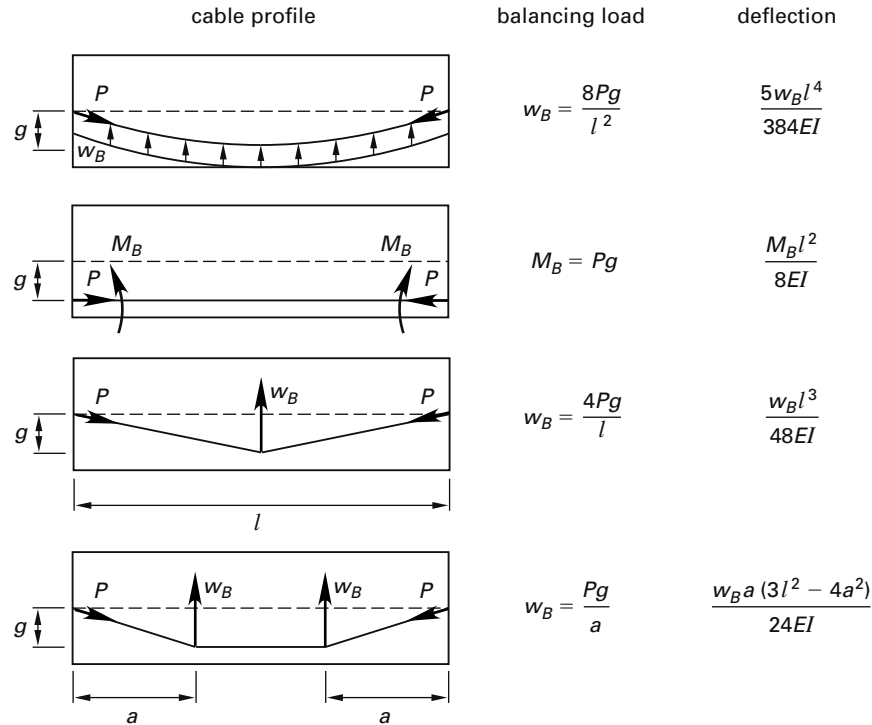


Figure 3.10 Alternative Tendon Profiles



When the applied load  $w_W$  equals the balancing load  $w_B$ , there is no moment on the beam. At any section, there is a constant compressive stress of

$$f_c = \frac{P}{A_g}$$

If the downward load is not fully balanced by the upward force, the out-of-balance load on the beam is

$$w_o = w_W - w_B$$

This produces an out-of-balance moment of

$$M_o = M_W - M_B$$

The stress in the concrete is then

$$f_c = \frac{P}{A_g} \pm \frac{M_o}{S}$$

The balancing loads produced by other tendon profiles may be derived,<sup>6,7,8</sup> and are shown in Fig. 3.10. Deflections also may be readily calculated using this method.

### Example 3.16

The beam shown in the illustration is prestressed with a tendon having a parabolic profile and a final prestressing force of  $P_e = 140$  kips. Determine the uniformly

distributed load, including the weight of the beam, which will exactly balance the prestressing force, and determine the resulting stress in the beam. Calculate the stresses in the beam if the uniformly distributed load is increased by 20%.

### Solution

The sag of the tendon is

$$g = 10 \text{ in}$$

The uniformly distributed load required to exactly balance the prestressing force is

$$\begin{aligned} w_W &= \frac{8P_e g}{l^2} \\ &= \frac{(8)(140 \text{ kips})(10 \text{ in})}{(30 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)} \\ &= 1.037 \text{ kips/ft} \end{aligned}$$

The uniform compressive stress throughout the beam is then

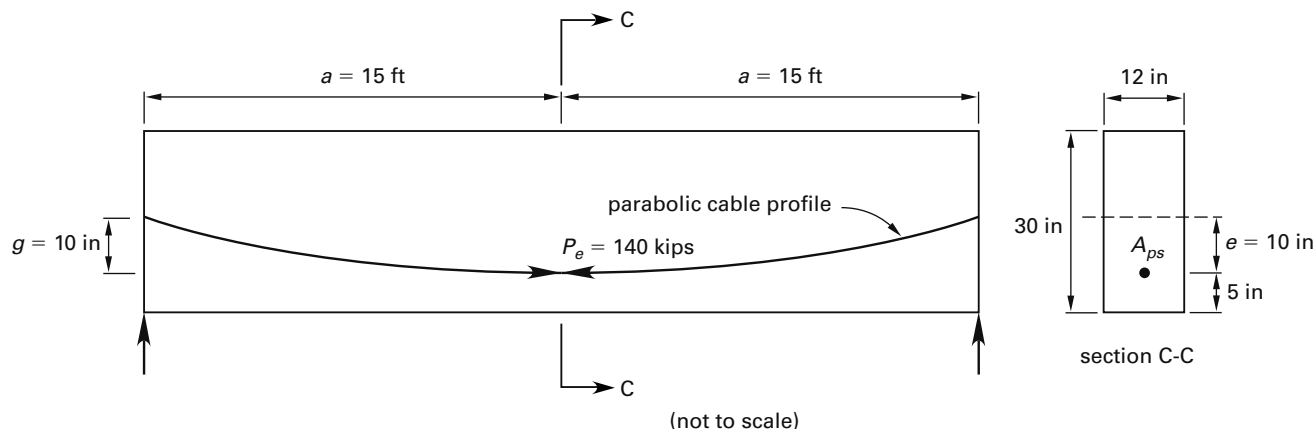
$$\begin{aligned} f_c &= \frac{P}{A_g} \\ &= \frac{140 \text{ kips}}{360 \text{ in}^2} \\ &= 0.389 \text{ kips/in}^2 \quad [\text{compression}] \end{aligned}$$

<sup>6</sup>Lin, T.Y., 1963 (See References and Codes)

<sup>7</sup>Prestressed Concrete Institute, 2004 (See References and Codes)

<sup>8</sup>Freyermuth, C.L., and Schoolbred, R.A., 1967 (See References and Codes)

Illustration for Example 3.16



An increase of 20% in  $w_W$  produces an out-of-balance load of

$$w_o = (0.2) \left( 1.037 \frac{\text{kips}}{\text{ft}} \right) = 0.207 \text{ kips/ft}$$

The out-of-balance moment produced at midspan by  $w_o$  is

$$M_o = \frac{w_o l^2}{8} = \frac{\left( 0.207 \frac{\text{kips}}{\text{ft}} \right) (30 \text{ ft})^2 \left( 12 \frac{\text{in}}{\text{ft}} \right)}{8} = 279 \text{ in-kips}$$

The resultant stresses at mid-span are

$$f_b = f_c - \frac{M_o}{S} = 0.389 \frac{\text{kips}}{\text{in}^2} - \frac{279 \text{ in-kips}}{1800 \text{ in}^3} = 0.234 \text{ kips/in}^2 \quad [\text{compression}]$$

$$f_b = f_c + \frac{M_o}{S} = 0.389 \frac{\text{kips}}{\text{in}^2} + \frac{279 \text{ in-kips}}{1800 \text{ in}^3} = 0.544 \text{ kips/in}^2 \quad [\text{compression}]$$

## 6. CONCORDANT CABLE PROFILE

### Nomenclature

|       |                                    |               |
|-------|------------------------------------|---------------|
| $A_g$ | area of concrete section transfer  | $\text{in}^2$ |
| $e$   | eccentricity of prestressing force | in            |
| $e'$  | resultant cable eccentricity       | in            |

|       |                                                                                   |                          |
|-------|-----------------------------------------------------------------------------------|--------------------------|
| $EI$  | flexural stiffness of compression member                                          | $\text{in}^2\text{-lbf}$ |
| $m$   | moment produced by unit value of the redundant                                    | in-kips                  |
| $M_p$ | primary moment produced by prestressing force                                     | in-kips                  |
| $M_r$ | resultant moment produced by prestressing force and secondary effects, $Pe + M_s$ | in-kips                  |
| $M_s$ | secondary moment produced by secondary effects                                    | in-kips                  |
| $P$   | prestressing force                                                                | kips                     |
| $R$   | reaction, support restraint                                                       | kips                     |
| $S$   | section modulus                                                                   | $\text{in}^3$            |

### Symbols

|            |                       |    |
|------------|-----------------------|----|
| $\delta_p$ | deflection at midspan | in |
|------------|-----------------------|----|

As shown in Fig. 3.11, prestressing a simply supported beam with a tendon having a constant eccentricity produces a uniform bending moment in the beam that has no effect on the support reactions. This bending moment, known as the *primary bending moment*, is

$$M_p = Pe$$

The primary bending moment is shown in Fig. 3.11, using the sign convention that moment causing compressive stress in the bottom fiber of the beam is positive. The primary bending moment produces an upward deflection in the beam. The deflection at midspan is

$$\delta_p = \frac{M_p(2l)^2}{8EI} = \frac{Pe(2l)^2}{8EI}$$

Introducing an additional support 3 to the beam at midspan produces an indeterminate two-span continuous beam. Upward deflection of the beam at the central

support is now restrained by the support. The support reaction is obtained from Fig. 3.10 as

$$R_3 = \frac{48\delta_p EI}{(2l)^3} \\ = \frac{6Pe}{2l}$$

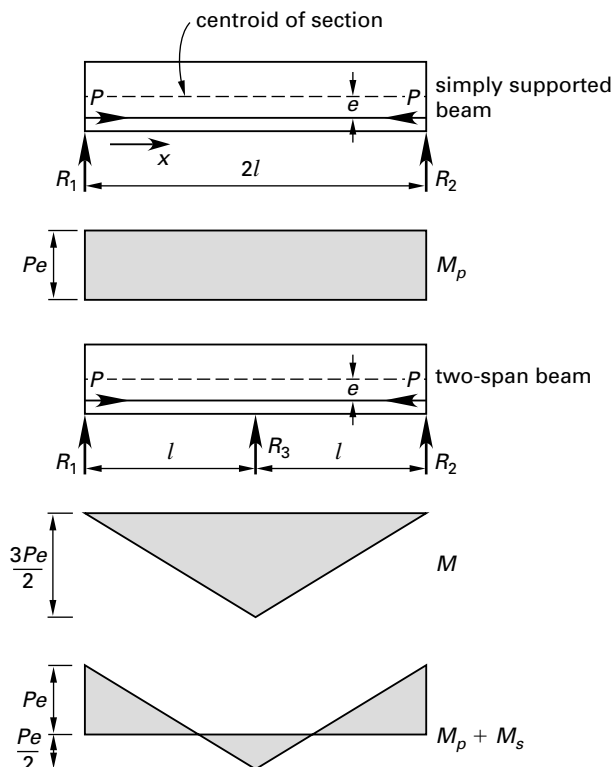
This reaction produces a secondary bending moment in the beam, with a maximum value at the central support of

$$M_s = \frac{R_3(2l)}{4} \\ = \frac{3Pe}{2}$$

Superimposing the primary and secondary moment diagrams gives the combined moment diagram shown in Fig. 3.11. The value of the resultant moment at the central support is

$$M_r = M_p - M_s \\ = Pe - \frac{3Pe}{2} \\ = -\frac{Pe}{2}$$

Figure 3.11 Secondary Effects



In general, the equation for the resultant moment is

$$M_r = Pe - \frac{3Pex}{2l}$$

The effective tendon eccentricity is

$$e' = \frac{M_r}{P} \\ = e - \frac{3ex}{2l}$$

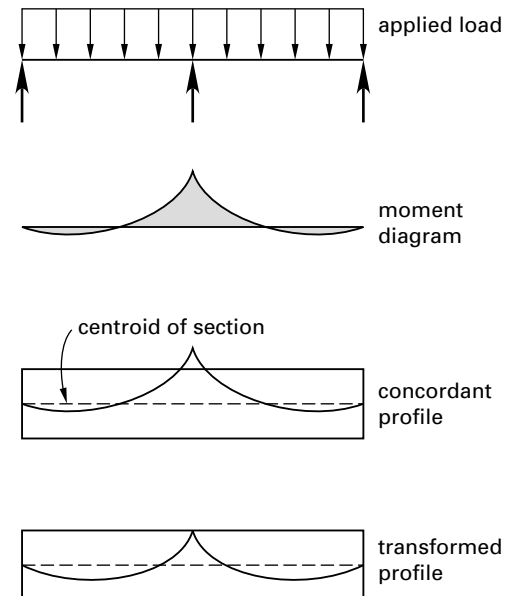
Hence, a tendon with an initial eccentricity of  $e'$  produces no secondary effects in the member and no support reactions. It is termed the *concordant cable*.

External loads applied to a continuous beam on unyielding supports produce a bending moment diagram with magnitude  $M_x$  at any section  $x$ . To some convenient scale, this is the concordant profile for this system of loads because the effective eccentricity is

$$e' = \frac{M_x}{P}$$

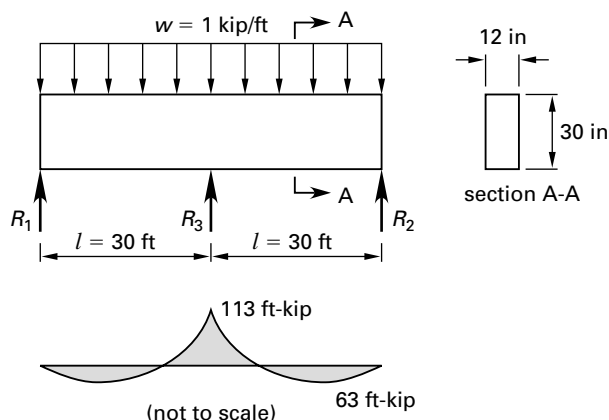
In addition, a concordant profile may be modified as shown in Fig. 3.12, using a linear transformation by varying the location of the tendon at interior supports, without changing the resultant moment.

Figure 3.12 Concordant Cable Profile



### Example 3.17

The beam shown in the illustration is prestressed with a tendon having a final prestressing force of  $Pe = 140$  kips. Determine the eccentricity of the concordant cable at support 3 to balance a uniformly distributed load of 1.0 kip/ft applied to the beam, and determine the resulting stress in the beam.



### Solution

The bending moment produced at support 3 by the distributed load is

$$\begin{aligned} M_3 &= 0.125wl^2 \\ &= (0.125) \left( 1 \frac{\text{kip}}{\text{ft}} \right) (30 \text{ ft})^2 \left( 12 \frac{\text{in}}{\text{ft}} \right) \\ &= 1350 \text{ in-kips} \end{aligned}$$

The eccentricity of the concordant cable at support 3 is

$$\begin{aligned} e' &= \frac{M_3}{Pe} \\ &= \frac{1350 \text{ in-kips}}{140 \text{ kips}} \\ &= 9.6 \text{ in} \quad [\text{above the centroid of the section}] \end{aligned}$$

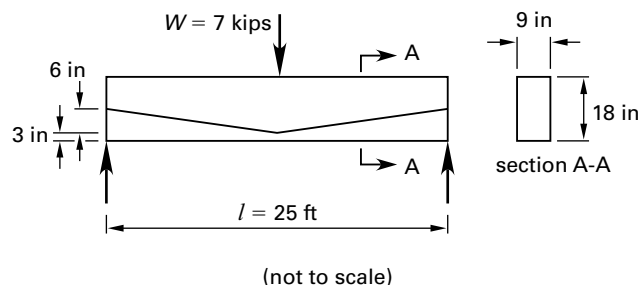
The resultant stresses at support 3 are

$$\begin{aligned} f_b &= \frac{Pe}{A_g} - \frac{M_3}{S} \\ &= \frac{140 \text{ kips}}{360 \text{ in}^2} - \frac{1350 \text{ in-kips}}{1800 \text{ in}^3} \\ &= -0.361 \text{ kips/in}^2 \quad [\text{tension}] \end{aligned}$$

$$\begin{aligned} f_t &= \frac{Pe}{A_g} + \frac{M_3}{S} \\ &= \frac{140 \text{ kips}}{360 \text{ in}^2} + \frac{1350 \text{ in-kips}}{1800 \text{ in}^3} \\ &= 1.139 \text{ kips/in}^2 \quad [\text{compression}] \end{aligned}$$

### PRACTICE PROBLEMS

Problems 1–5 refer to the normal weight concrete prestressed beam shown in the illustration. The beam is simply supported over a span of 25 ft, and has a 28 day concrete strength of 6000 lbf/in<sup>2</sup>. The area of the low-relaxation prestressing tendons provided is  $A_{ps} = 0.612 \text{ in}^2$ , with a specified tensile strength of  $f_{pu} = 270 \text{ kips/in}^2$ , a yield strength of  $f_{py} = 243 \text{ kips/in}^2$ , and an effective stress of  $f_{se} = 150 \text{ kips/in}^2$  after all losses.



1. What is most nearly the cracking moment of the beam at midspan?

- (A) 650.0 in-kips
- (B) 863.0 in-kips
- (C) 1108 in-kips
- (D) 1324 in-kips

2. The post-tensioned unbonded beam has an area of 0.6 in<sup>2</sup> of grade 60 auxiliary reinforcement provided at a height of 3 in above the beam soffit. Which of the given statements is/are true?

- I. The design flexural strength of the beam is 1718 in-kips.
- II. The design flexural strength of the beam is 1431 in-kips.
- III. The beam complies with ACI Sec. 18.8.2.
- IV. The beam does not comply with ACI Sec. 18.8.2.

- (A) I and III only
- (B) I and IV only
- (C) II and III only
- (D) II and IV only

3. The beam supports a concentrated load of 7 kips at midspan. What is most nearly the prestressing force required in the tendons to exactly balance the applied load?

- (A) 44 kips
- (B) 88 kips
- (C) 130 kips
- (D) 180 kips

4. What is most nearly the resulting stress in the beam at midspan?

- (A) 270 lbf/in<sup>2</sup>
- (B) 460 lbf/in<sup>2</sup>
- (C) 540 lbf/in<sup>2</sup>
- (D) 810 lbf/in<sup>2</sup>

5. The prestressed beam of Prob. 3 supports an additional uniformly distributed load of 200 lbf/ft. What are most nearly the resulting stresses in the beam at midspan?

- (A)  $f_b = -115 \text{ lbf/in}^2$ ,  $f_t = 655 \text{ lbf/in}^2$
- (B)  $f_b = 154 \text{ lbf/in}^2$ ,  $f_t = 926 \text{ lbf/in}^2$
- (C)  $f_b = 425 \text{ lbf/in}^2$ ,  $f_t = 1190 \text{ lbf/in}^2$
- (D)  $f_b = 695 \text{ lbf/in}^2$ ,  $f_t = 1470 \text{ lbf/in}^2$

## SOLUTIONS

1. The relevant properties of the beam are

$$\begin{aligned}
 A_g &= bh \\
 &= (9 \text{ in})(18 \text{ in}) \\
 &= 162 \text{ in}^2 \\
 S &= \frac{A_g h}{6} \\
 &= \frac{(162 \text{ in}^2)(18 \text{ in})}{6} \\
 &= 486 \text{ in}^3 \\
 e &= \frac{h}{2} - 3 \text{ in} \\
 &= \frac{18 \text{ in}}{2} - 3 \text{ in} \\
 &= 6 \text{ in} \\
 R_b &= \frac{1}{A_g} + \frac{e}{S} \\
 &= \frac{1}{162 \text{ in}^2} + \frac{6 \text{ in}}{486 \text{ in}^3} \\
 &= 0.0185 \text{ in}^{-2}
 \end{aligned}$$

After allowance for all losses, the force in the prestressing tendons at service loads is

$$\begin{aligned}
 Pe &= A_{ps} f_{se} \\
 &= (0.612 \text{ in}^2) \left( 150 \frac{\text{kips}}{\text{in}^2} \right) \\
 &= 91.80 \text{ kips}
 \end{aligned}$$

The modulus of rupture of the concrete is given by ACI Eq. (9-10) as

$$\begin{aligned}
 f_r &= 7.5 \lambda \sqrt{f'_c} \\
 &= (7.5)(1.0) \sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} \\
 &= 581 \text{ lbf/in}^2
 \end{aligned}$$

The cracking moment is

$$\begin{aligned}
 M_{cr} &= S(PeR_b + f_r) \\
 &= (486 \text{ in}^3) \left( (91.80 \text{ kips})(0.0185 \text{ in}^{-2}) + (0.581 \frac{\text{kips}}{\text{in}^2}) \right) \\
 &= 1108 \text{ in-kips}
 \end{aligned}$$

The answer is (C).



2. Because the ratio of the effective stress to the tensile strength  $f_{se}/f_{pu}$  is greater than 0.5, the method of ACI Sec. 18.7.2 may be used. The ratio of prestressed reinforcement is

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{0.612 \text{ in}^2}{(9 \text{ in})(15 \text{ in})} = 0.00453$$

For unbonded tendons and a span-to-depth ratio of 35 or less, the stress in the unbonded tendons at nominal strength is given by ACI Sec. 18.7.2 as

$$\begin{aligned} f_{ps} &= f_{se} + 10 \frac{\text{kips}}{\text{in}^2} + \frac{f'_c}{100\rho_p} \quad [\text{ACI 18-4}] \\ &= 150 \frac{\text{kips}}{\text{in}^2} + 10 \frac{\text{kips}}{\text{in}^2} + \frac{6 \frac{\text{kips}}{\text{in}^2}}{(100)(0.00453)} \\ &= 173 \text{ kips/in}^2 \\ &< f_{py} \quad [\text{satisfactory}] \\ &< f_{se} + 60 \text{ kips/in}^2 \quad [\text{satisfactory}] \end{aligned}$$

The minimum required area of auxiliary reinforcement is specified by ACI Sec. 18.9.2 as

$$\begin{aligned} A_s &= 0.004A_{ct} \quad [\text{ACI 18-6}] \\ &= (0.004)(9 \text{ in})(9 \text{ in}) \\ &= 0.324 \text{ in}^2 \\ &< 0.60 \text{ in}^2 \text{ provided} \quad [\text{satisfactory}] \end{aligned}$$

Assuming full utilization of the auxiliary reinforcement, the depth of the stress block is

$$\begin{aligned} a &= \frac{A_{ps}f_{ps} + A_sf_y}{0.85f'_cb} \\ &= \frac{(0.612 \text{ in}^2)\left(173 \frac{\text{kips}}{\text{in}^2}\right) + (0.6 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(6 \frac{\text{kips}}{\text{in}^2}\right)(9 \text{ in})} \\ &= 3.09 \text{ in} \end{aligned}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$\begin{aligned} a_t &= 0.375\beta_1 d_p \\ &= (0.375)(0.75)(15 \text{ in}) \\ &= 4.22 \text{ in} \\ &> a \end{aligned}$$

The section is tension-controlled and

$$\phi = 0.9$$

The nominal flexural strength of the section is

$$\begin{aligned} M_n &= A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_sf_y\left(d - \frac{a}{2}\right) \\ &= (0.612 \text{ in}^2)\left(173 \frac{\text{kips}}{\text{in}^2}\right)\left(15 \text{ in} - \frac{3.09 \text{ in}}{2}\right) \\ &\quad + (0.60 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(15 \text{ in} - \frac{3.09 \text{ in}}{2}\right) \\ &= 1909 \text{ in-kips} \end{aligned}$$

The design flexural strength of the beam is

$$\begin{aligned} \phi M_n &= (0.9)(1909 \text{ in-kips}) \\ &= 1718 \text{ in-kips} \end{aligned}$$

The cracking moment of the beam is determined in Prob. 1 as

$$\begin{aligned} M_{cr} &= 1108 \text{ in-kips} \\ \frac{\phi M_n}{M_{cr}} &= \frac{1718 \text{ in-kips}}{1108 \text{ in-kips}} \\ &= 1.55 \\ &> 1.2 \end{aligned}$$

The beam complies with ACI Sec. 18.8.2.

**The answer is (A).**

3. The sag of the tendon is

$$g = 6 \text{ in}$$

The prestressing force required to exactly balance the applied load is obtained from Fig. 3.10 as

$$\begin{aligned} P &= \frac{Wl}{4g} \\ &= \frac{(7 \text{ kips})(300 \text{ in})}{(4)(6 \text{ in})} \\ &= 87.5 \text{ kips} \quad (88 \text{ kips}) \end{aligned}$$

**The answer is (B).**

4. The uniform compressive stress throughout the beam is

$$\begin{aligned} f_c &= \frac{P}{A_g} = \frac{87,500 \text{ lbf}}{162 \text{ in}^2} \\ &= 540 \text{ lbf/in}^2 \end{aligned}$$

**The answer is (C).**

5. The moment produced at midspan by the additional distributed load is

$$\begin{aligned} M &= \frac{wl^2}{8} = \frac{\left(200 \frac{\text{lbf}}{\text{ft}}\right)(25 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{8} \\ &= 187,500 \text{ in-lbf} \end{aligned}$$

The resultant stresses at midspan are

$$\begin{aligned} f_b &= f_c - \frac{M}{S} \\ &= 540 \frac{\text{lbf}}{\text{in}^2} - \frac{187,500 \text{ in-lbf}}{486 \text{ in}^3} \\ &= 154 \text{ lbf/in}^2 \\ f_t &= f_c + \frac{M}{S} \\ &= 540 \frac{\text{lbf}}{\text{in}^2} + \frac{187,500 \text{ in-lbf}}{486 \text{ in}^3} \\ &= 926 \text{ lbf/in}^2 \end{aligned}$$

**The answer is (B).**



# 4

# Structural Steel Design

|                                               |      |
|-----------------------------------------------|------|
| 1. Plastic Design . . . . .                   | 4-1  |
| 2. Eccentrically Loaded Bolt Groups . . . . . | 4-11 |
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|           |                                                                                                     |                      |
|-----------|-----------------------------------------------------------------------------------------------------|----------------------|
| $L_p$     | limiting laterally unbraced length for full plastic bending capacity                                | ft or in             |
| $L_{pd}$  | limiting laterally unbraced length for plastic analysis                                             | ft or in             |
| $L_r$     | limiting laterally unbraced length for inelastic lateral torsional buckling                         | ft or in             |
| $L_r$     | roof live load                                                                                      | kips or lbf          |
| $L_u$     | maximum unbraced segment length at which the allowable bending stress may be taken to be $0.60F_y$  | ft                   |
| $m_i$     | number of independent collapse mechanisms in a structure                                            | —                    |
| $M/M_p$   | end moment ratio, positive when the segment is bent in reverse curvature                            | —                    |
| $M_m$     | maximum moment that can be resisted by the member in the absence of axial load                      | kips/in <sup>2</sup> |
| $M_{nx}$  | nominal flexural strength about the strong axis in the absence of axial load                        | kips                 |
| $M_{ny}$  | nominal flexural strength about the weak axis in the absence of axial load                          | kips                 |
| $M_p$     | plastic moment of resistance of a member, larger of the moments at the ends of the unbraced segment | ft-kips              |
| $M'_p$    | plastic moment of resistance modified for axial compression                                         | ft-kips              |
| $M_s$     | bending moment produced by factored loads acting on the cut-back structure                          | ft-kips              |
| $M_{ux}$  | required flexural strength about the strong axis, including second-order effects                    | kips                 |
| $M_{uy}$  | required flexural strength about the weak axis, including second-order effects                      | kips                 |
| $M_y$     | moment at which the extreme fibers of a member yield in flexure                                     | ft-kips              |
| $M_1$     | smaller moment at end of unbraced length of beam                                                    | ft-kips              |
| $M_2$     | larger moment at end of unbraced length of beam                                                     | ft-kips              |
| $M_1/M_2$ | positive when moments cause reverse curvature and negative for single curvature                     | —                    |
| $p$       | number of possible hinge locations in a structure                                                   | —                    |
| $P$       | axial force                                                                                         | kips or lbf          |

## 1. PLASTIC DESIGN

### Nomenclature

|          |                                                                                                              |                      |
|----------|--------------------------------------------------------------------------------------------------------------|----------------------|
| $A$      | gross area of member [ASD]                                                                                   | in <sup>2</sup>      |
| $A_f$    | area of flange                                                                                               | in <sup>2</sup>      |
| $A_g$    | gross area of member [LRFD]                                                                                  | in <sup>2</sup>      |
| $BF$     | tabulated factor used to calculate the design flexural strength for unbraced lengths between $L_p$ and $L_r$ | kips                 |
| $C_b$    | bending coefficient                                                                                          | —                    |
| $C_c$    | column slenderness ratio separating elastic and inelastic buckling                                           | —                    |
| $C_m$    | factor relating actual moment diagram to an equivalent uniform moment diagram                                | —                    |
| $D$      | dead load                                                                                                    | kips or lbf          |
| $D$      | degree of indeterminacy of a structure                                                                       | —                    |
| $E$      | earthquake load                                                                                              | kips or lbf          |
| $E$      | modulus of elasticity of steel, 29,000 kips/in <sup>2</sup>                                                  | —                    |
| $F_a$    | allowable axial stress                                                                                       | kips/in <sup>2</sup> |
| $F_{cr}$ | critical stress                                                                                              | kips/in <sup>2</sup> |
| $F'_e$   | factored Euler critical stress tabulated in AISC Table 8                                                     | kips/in <sup>2</sup> |
| $F_y$    | yield stress                                                                                                 | kips/in <sup>2</sup> |
| $g$      | stiffness ratio                                                                                              | —                    |
| $H$      | horizontal force                                                                                             | kips or lbf          |
| $I$      | moment of inertia of section about centroidal axis                                                           | in <sup>4</sup>      |
| $I_c$    | moment of inertia of column                                                                                  | in <sup>4</sup>      |
| $I_g$    | moment of inertia of beam                                                                                    | in <sup>4</sup>      |
| $K$      | effective-length factor                                                                                      | —                    |
| $l$      | span length                                                                                                  | ft                   |
| $l_{cr}$ | critical unbraced segment length adjacent to a plastic hinge                                                 | in                   |
| $L$      | live load due to occupancy                                                                                   | kips or lbf          |
| $L_b$    | unbraced segment length                                                                                      | in                   |
| $L_c$    | maximum unbraced segment length at which the allowable bending stress may be taken to be $0.66F_y$           | ft                   |

|           |                                                                                               |             |
|-----------|-----------------------------------------------------------------------------------------------|-------------|
| $P$       | factored axial load                                                                           | kips        |
| $P_{cr}$  | maximum strength of an axially loaded compression member                                      | kips        |
| $P_e$     | Euler buckling load                                                                           | kips        |
| $P_{max}$ | maximum load on a column                                                                      | kips        |
| $P_n$     | nominal axial strength in the absence of bending moment                                       | kips        |
| $P_u$     | required axial strength in compression                                                        | kips        |
| $P_y$     | plastic axial load                                                                            | kips        |
| $r$       | radius of gyration of cross section                                                           | in          |
| $r_b$     | corresponding radius of gyration                                                              | in          |
| $r_T$     | radius of gyration of compression flange plus one-third of the compression portion of the web | in          |
| $r_x$     | radius of gyration of the member about its strong axis                                        | in          |
| $r_y$     | radius of gyration of the member about its weak axis                                          | in          |
| $R$       | rainwater or ice load                                                                         | kips or lbf |
| $S$       | elastic section modulus, $M_y/F_y$                                                            | ft-kips     |
| $S$       | snow load                                                                                     | kips or lbf |
| $V$       | shear force                                                                                   | kips or lbf |
| $W$       | applied force                                                                                 | kips or lbf |
| $W$       | wind load                                                                                     | kips or lbf |
| $Z$       | plastic section modulus, $M_p/F_y$                                                            | ft-kips     |

#### Symbols

|             |                                         |     |
|-------------|-----------------------------------------|-----|
| $\theta$    | plastic hinge rotation                  | deg |
| $\lambda$   | load factor                             | —   |
| $\lambda_c$ | column slenderness parameter            | —   |
| $\phi$      | resistance factor                       | —   |
| $\phi_b$    | resistance factor for flexure, 0.90     | —   |
| $\phi_c$    | resistance factor for compression, 0.85 | —   |
| $v$         | shape factor, $Z/S$                     | —   |

### General Considerations

The plastic method, or inelastic method, of structural analysis<sup>1,2</sup> is used to determine the maximum loads that a structure can support prior to collapse. The plastic analysis method has several advantages over the ASD and LRFD design techniques. The principal advantages of the plastic analysis technique are that it

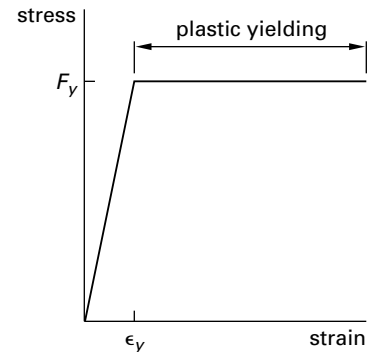
- produces a more economical structure
- provides a simple and direct design technique
- accurately models the structure at ultimate loads
- realistically predicts the ultimate strength

### Plastic Hinges

The plastic method of structural analysis is applicable to structures constructed with a ductile material possessing ideal elastic-plastic characteristics. As shown in Fig. 4.1, such a material initially exhibits a linear relationship between stress and strain, until the yield point

is reached. After this point, the stress remains constant at the yield stress  $F_y$ , while the strain continues to increase indefinitely as plastic yielding of the material occurs. The plastic hinge formed at the location where yielding occurs has a plastic moment of resistance,  $M_p$ . Rotation continues at the hinge without any increase in resisting moment.

Figure 4.1 Elastic-Plastic Material

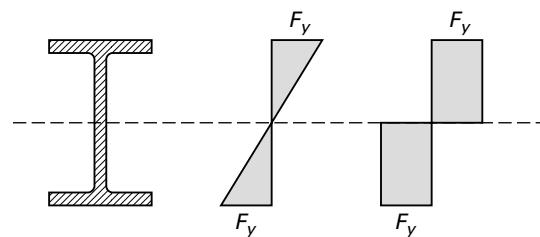


### Shape Factor

As shown in Fig. 4.2, an increasing applied bending moment on a steel beam eventually causes the extreme fibers to reach the yield stress. The resisting moment developed in the beam is the yield moment given by

$$M_y = F_y S$$

Figure 4.2 Shape Factor Determination



As the moment on the section continues to increase, the yielding at the extreme fibers progresses toward the equal area axis until finally the whole of the section has yielded. The resisting moment developed in the section is the plastic moment given by

$$M_p = F_y Z \quad [\text{AISC F2-1}]$$

The plastic section modulus  $Z$  is calculated as the arithmetic sum of the first moments of the area about the neutral axis. A plastic hinge has now formed in the section, and rotation of the hinge continues without

<sup>1</sup>Horne, M.R., and L.J. Morris, 1982 (See References and Codes)

<sup>2</sup>Neal, B.G., 1970 (See References and Codes)

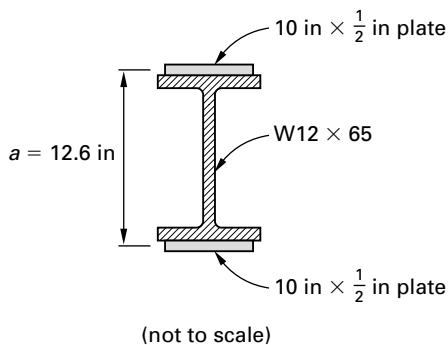
increase in the resisting moment. The shape factor is defined as

$$\begin{aligned} v &= \frac{Z}{S} = \frac{M_p}{M_s} \\ &\approx 1.14 \quad [\text{for an I section}] \\ &= 1.5 \quad [\text{for a solid rectangular section}] \end{aligned}$$

Values of the plastic modulus are tabulated in the AISC Manual. To ensure that adequate plastic rotations can occur, AISC App. 1.2 and 1.3 specify that members must be adequately braced compact sections with a yield stress not exceeding 65 kips/in<sup>2</sup>.

#### Example 4.1

A W12 × 65 section has 10 in × 1/2 in plates welded to both flanges, as shown in the illustration. The yield stress of the W section and the plates is 50 kips/in<sup>2</sup>. Determine the plastic moment of resistance and the shape factor for the combination section.



#### Solution

From AISC Table 1-1, the properties of the W12 × 65 section are

$$\begin{aligned} d &= 12.1 \text{ in} \\ A_W &= 19.1 \text{ in}^2 \\ S_W &= 87.9 \text{ in}^3 \\ I_W &= 533 \text{ in}^4 \\ Z_W &= 96.8 \text{ in}^3 \end{aligned}$$

The area of each plate is

$$A_P = 5 \text{ in}^2$$

The distance between the centroids of the two plates is

$$a = 12.6 \text{ in}$$

Neglecting the moment of inertia of the plates about their individual centroidal axes, the moment of inertia of the combination section is

$$\begin{aligned} I_C &= I_W + \frac{A_P a^2}{2} = 533 \text{ in}^4 + \frac{(5 \text{ in}^2)(12.6 \text{ in})^2}{2} \\ &= 930 \text{ in}^4 \end{aligned}$$

The elastic section modulus of the combination section is

$$\begin{aligned} S_C &= \frac{I_C}{\frac{d}{2} + 0.5 \text{ in}} = \frac{930 \text{ in}^4}{\frac{12.1 \text{ in}}{2} + 0.5 \text{ in}} \\ &= 142 \text{ in}^3 \end{aligned}$$

The plastic section modulus of the combination section is

$$\begin{aligned} Z_C &= Z_W + A_P a \\ &= 96.8 \text{ in}^3 + (5 \text{ in}^2)(12.6 \text{ in}) \\ &= 160 \text{ in}^3 \end{aligned}$$

The plastic moment of resistance of the combination section is

$$\begin{aligned} M_p &= F_y Z_C \\ &= \left( 50 \frac{\text{kips}}{\text{in}^2} \right) (160 \text{ in}^3) \\ &= \frac{8000 \text{ in-kips}}{12 \frac{\text{in}}{\text{ft}}} \\ &= 667 \text{ ft-kips} \end{aligned}$$

The shape factor is

$$\begin{aligned} v &= \frac{Z_C}{S_C} = \frac{160 \text{ in}^3}{142 \text{ in}^3} \\ &= 1.13 \end{aligned}$$

#### Plastic Hinge Formation and Load Factors

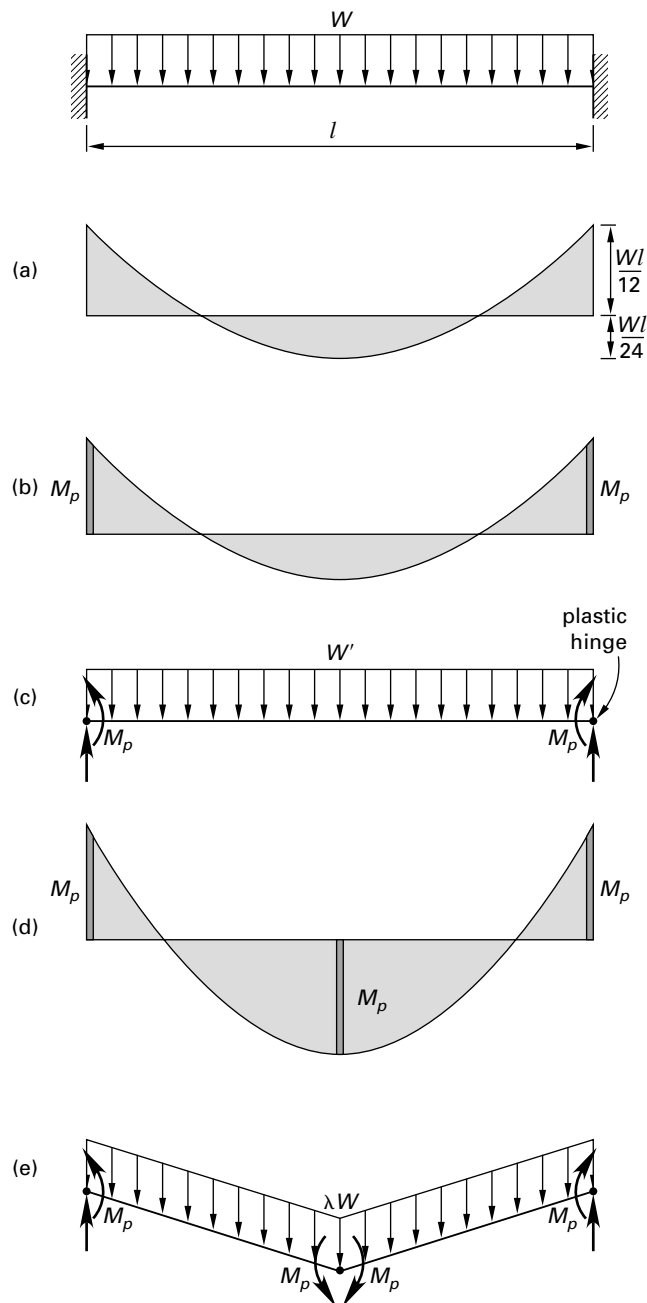
A plastic hinge is formed in a structure as the bending moment at a specific location reaches the plastic moment of resistance at that location. As shown in Fig. 4.3, a fixed-ended beam supports a uniformly distributed service load,  $W$ .

The bending moments produced in the beam are shown in Fig. 4.3(a). The moments at the ends of the beam are twice that at the center.

As the load  $W$  is progressively increased to a value  $W'$ , plastic hinges are formed simultaneously at both ends of the beam. The bending moments in the beam are as shown in Fig. 4.3(b).

As shown in Fig. 4.3(c), the system is now equivalent to a simply supported beam with an applied load  $W'$  and moments  $M_p$  at both ends. Progressively increasing the applied load causes the two plastic hinges to rotate, while the moments at both ends remain constant at the value  $M_p$ .

Figure 4.3 Formation of Plastic Hinges



Finally, as the applied load is increased to the value  $\lambda W$ , a third plastic hinge forms in the center of the span, giving the distribution of bending moment shown in Fig. 4.3(d).

The system is now an unstable mechanism shown in Fig. 4.3(e), and collapse occurs under the ultimate, or plastic-limit, load  $\lambda W$ . Immediately before collapse, the system is statically determinate and the ultimate load

may be calculated. Taking moments about the center of the span for the left half of the beam gives

$$2M_p = \left(\frac{\lambda W}{2}\right)\left(\frac{l}{2}\right) - \left(\frac{\lambda W}{2}\right)\left(\frac{l}{4}\right)$$

$$\lambda W = \frac{16M_p}{l}$$

The ratio of the collapse load to the service load is

$$\frac{\lambda W}{W} = \lambda$$

AISC App. 1.1 specifies that plastic design is not permitted using ASD design methods.

AISC Sec. B2 specifies that load factors and load combinations shall be as stipulated in the appropriate code or in ASCE<sup>3</sup>. The required ultimate loads are given by IBC<sup>4</sup> Sec. 1605.2 as

$$\lambda W = 1.4D \quad [\text{IBC 16-1}]$$

$$\lambda W = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad [\text{IBC 16-2}]$$

$$\lambda W = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.8W) \quad [\text{IBC 16-3}]$$

$$\lambda W = 1.2D + 1.6W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R) \quad [\text{IBC 16-4}]$$

$$\lambda W = 1.2D \pm 1.0E + 0.5L + 0.2S \quad [\text{IBC 16-5}]$$

$$\lambda W = 0.9D \pm 1.6W \quad [\text{IBC 16-6}]$$

$$\lambda W = 0.9D \pm 1.0E \quad [\text{IBC 16-7}]$$

For garages, places of public assembly, and areas where  $L > 100 \text{ lbf/ft}^2$ , replace  $0.5L$  with  $1.0L$  in IBC Eqs. (16-4) and (16-5). For roof configurations that do not shed snow, replace  $0.2S$  with  $0.7S$  in IBC Eq. (16-5).

The plastic modulus of a structure that is required to support a given load combination may be determined by either the statical design method or the mechanism design method.

### Statical Design Method

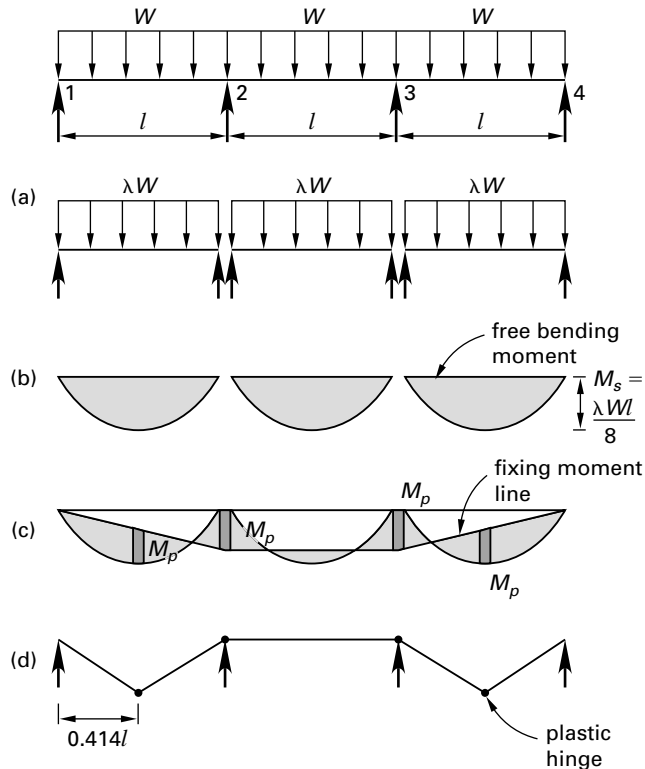
The statical method is a convenient technique to use for continuous beams, as illustrated in Fig. 4.4. The three-span continuous beam of uniform section is first cut back to a statically determinate condition and the factored loads applied, as shown in Fig. 4.4(a). The free bending moment diagram for this condition is shown in Fig. 4.4(b). It consists of three moment envelopes, each with a maximum value of

$$M_s = \frac{\lambda W l}{8}$$

<sup>3</sup>American Society of Civil Engineers, 2005 (See References and Codes)

<sup>4</sup>International Code Council, 2009 (See References and Codes)

Figure 4.4 Statical Design Method



The fixing moment line that represents the support moments in the original structure is now superimposed on the free moment diagram, as shown in Fig. 4.4(c). The fixing moment line is positioned to make the moments at supports 1 and 2 and in spans 12 and 34 equal to  $M_p$ , the required plastic moment of resistance. Hence, the collapse mechanism shown in Fig. 4.4(d) is formed, and collapse occurs simultaneously in the two end spans, with the plastic hinges occurring at a distance of  $0.414l$  from the end supports. The required plastic moment of resistance is determined from the geometry of the figure as

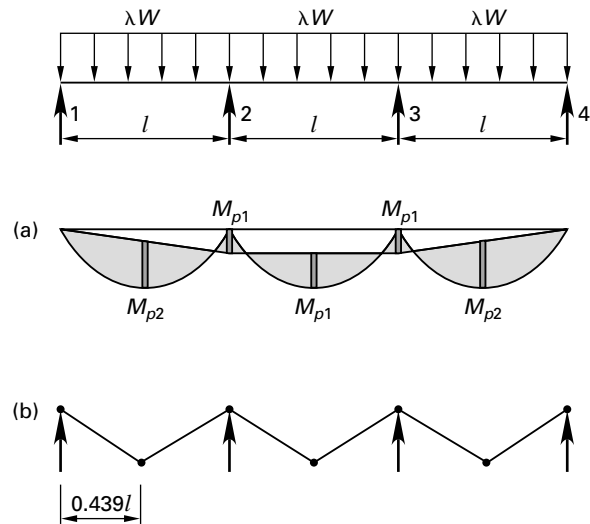
$$\begin{aligned} M_p &= 0.686M_s \\ &= 0.0858\lambda Wl \end{aligned}$$

When the continuous beam shown in Fig. 4.5 is of nonuniform section, a complete collapse mechanism is possible, with plastic hinges forming in all spans and at both interior supports. The location of the fixing moment line that produces this condition is shown in Fig. 4.5(a). The required plastic moments of resistance are

$$\begin{aligned} M_{p1} &= 0.5M_s \\ &= \frac{\lambda Wl}{16} \\ M_{p2} &= 0.766M_s \\ &= 0.0958\lambda Wl \end{aligned}$$

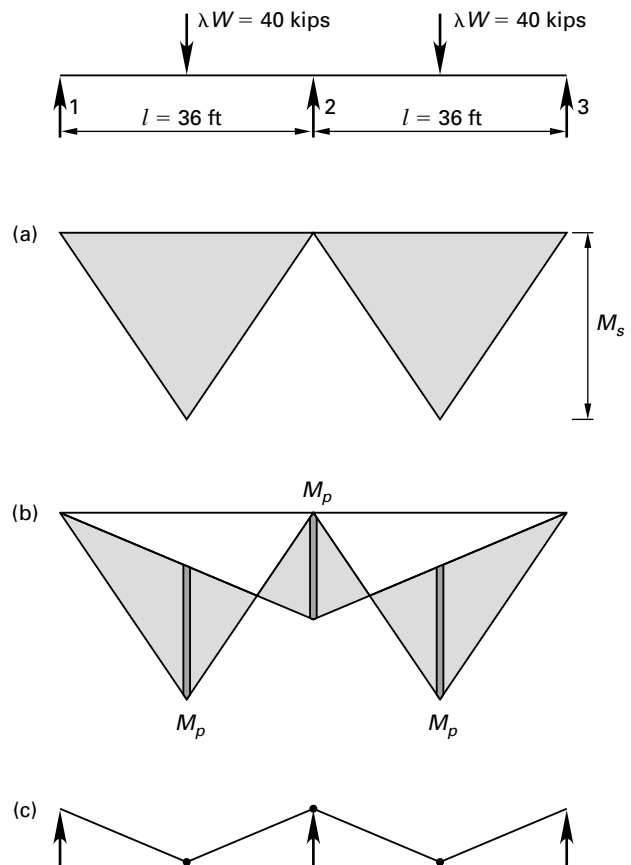
The collapse mechanism is shown in Fig. 4.5(b). The plastic hinges occur at a distance of  $0.439l$  from the end supports.

Figure 4.5 Nonuniform Beam



### Example 4.2

The two-span continuous beam shown in the illustration supports the factored load indicated, including an allowance for the self-weight of the beam. Assuming that adequate lateral support is provided to the beam, determine the lightest W12 section required, using grade 50 steel.





### Solution

As indicated in illustration (a), the free bending moment in each span is

$$\begin{aligned} M_s &= \frac{\lambda W l}{4} \\ &= \frac{(40 \text{ kips})(36 \text{ ft})}{4} \\ &= 360 \text{ ft-kips} \end{aligned}$$

The first plastic hinge forms at the central support. To produce collapse, three plastic hinges are necessary, as shown in illustration (c). The fixing moment line is located as shown in illustration (b). The required plastic moment of resistance is derived from the geometry of the illustration as

$$\begin{aligned} M_p &= \frac{2}{3} M_s \\ &= \left(\frac{2}{3}\right)(360 \text{ ft-kips}) \\ &= 240 \text{ ft-kips} \end{aligned}$$

From AISC Table 3-6, a W12 × 45 has

$$\begin{aligned} \phi_b M_p &= 241 \text{ ft-kips} \\ &> 240 \text{ ft-kips} \quad [\text{satisfactory}] \end{aligned}$$

### Beam Bracing Requirements

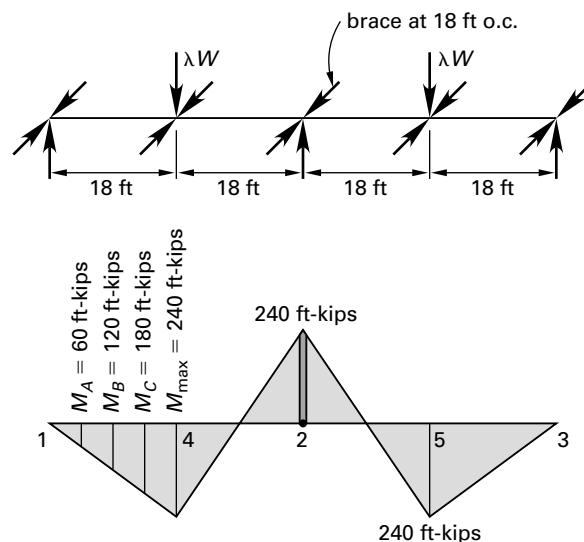
To ensure that the necessary hinge rotations can occur before collapse, the maximum unbraced length of a member adjacent to a plastic hinge is restricted by AISC Eq. (A-1-7) to a value of

$$L_{pd} = \left( 0.12 + (0.076) \left( \frac{M_1}{M_2} \right) \right) \frac{E r_y}{F_y}$$

Rotation does not occur in the last hinge to form, and the bracing requirements of AISC Sec. F2.2 are applicable. Similarly, AISC Sec. F2.2 applies to segments remote from a plastic hinge.

### Example 4.3

The two-span continuous beam of Ex. 4.2 is laterally braced at the supports and at the center of the spans, as shown in the illustration. Determine whether the beam is adequately braced.



### Solution

From AISC Tables 1-1 and 3-6, the relevant properties for a W12 × 45 are

$$\begin{aligned} r_y &= 1.95 \text{ in} \\ \phi_b M_p &= 241 \text{ ft-kips} \\ \phi_b M_r &= 151 \text{ ft-kips} \\ L_p &= 6.89 \text{ ft} \\ L_r &= 22.4 \text{ ft} \\ BF &= 5.75 \text{ kips} \\ F_y &= 50 \frac{\text{kips}}{\text{in}^2} \end{aligned}$$

The unbraced length in the end span is

$$\begin{aligned} L_b &= L_{14} = L_{24} \\ &= 0.5 L_{12} = (0.5)(36 \text{ ft}) \\ &= 18 \text{ ft} \\ &> L_p \\ &< L_r \end{aligned}$$

For section 14, the plastic hinge at 4 is the last hinge to form, and the bracing requirements of AISC Sec. F2.2 are applicable. The bending moments acting at the quarter points of section 14 are shown in the moment diagram. The relative values of these moments are

$$\begin{aligned} M_{\max} &= 4 \\ M_A &= 1 \\ M_B &= 2 \\ M_C &= 3 \end{aligned}$$

The bending coefficient is given by AISC Eq. (F1-1) where  $R_M$  is 1.0, as

$$\begin{aligned} C_b &= \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C} R_M \\ &= \left( \frac{(12.5)(4)}{(2.5)(4) + (3)(1) + (4)(2) + (3)(3)} \right) (1.0) \\ &= 1.67 \end{aligned}$$

The nominal flexural strength for an unbraced length between the full plastic bending capacity length  $L_p$  and the inelastic lateral torsional buckling length  $L_r$  is given by AISC Sec. F2.2b as

$$M_n = C_b \left( M_p - (M_p - 0.7 F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) \quad [\text{AISC F2-2}]$$

The flexural design strength is derived as

$$\begin{aligned} \phi_b M_n &= C_b (\phi_b M_p - (BF)(L_b - L_p)) \\ &= (1.67) \left( \begin{array}{l} (241 \text{ ft-kips}) \\ - (5.75 \text{ kips})(18 \text{ ft} - 6.89 \text{ ft}) \end{array} \right) \\ &\leq \phi_b M_p \\ &= 241 \text{ ft-kips} \quad [\text{maximum}] \\ &> M_{\max} \quad [\text{satisfactory}] \end{aligned}$$

For section 24, the bracing requirements of AISC App. 1.7 are applicable.

$$\begin{aligned} \frac{M_{24}}{M_{42}} &= \frac{+M_p}{M_p} \quad [\text{moments cause reverse curvature}] \\ &= 1.0 \end{aligned}$$

The maximum allowable unbraced length is given by AISC Eq. (A-1-7) as

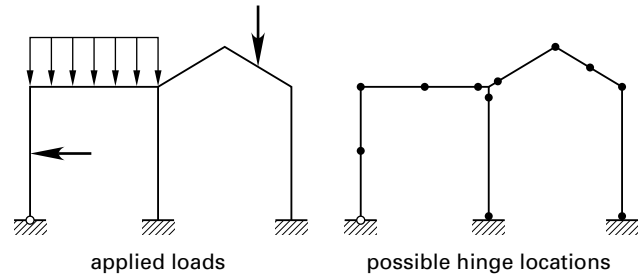
$$\begin{aligned} L_{pd} &= \left( 0.12 + (0.076) \left( \frac{M_{24}}{M_{42}} \right) \right) \frac{E r_y}{F_y} \\ &= (0.12 + (0.076)(1.0)) \\ &\quad \times \left( \frac{\left( 29,000 \frac{\text{kips}}{\text{in}^2} \right) (1.95 \text{ in})}{\left( 50 \frac{\text{kips}}{\text{in}^2} \right) \left( 12 \frac{\text{in}}{\text{ft}} \right)} \right) \\ &= 18.5 \text{ ft} \\ &> L_{25} \quad [\text{satisfactory}] \end{aligned}$$

The beam is adequately braced.

## Mechanism Design Method

The mechanism design method may be used to determine the required plastic moment of resistance in rigid frames and beams. As shown in Fig. 4.6, plastic hinges may form at the point of application of a concentrated load, at the ends of members, and at the location of zero shear in a prismatic beam. In the case of two members meeting at a joint, a plastic hinge forms in the weaker member. In the case of three or more members meeting at a joint, a plastic hinge may form at the ends of each of the members.

Figure 4.6 Hinge Locations



An independent collapse mechanism corresponds to a condition of unstable equilibrium in a structure. A structure that is indeterminate to the degree  $D$  becomes determinate when  $D$  plastic hinges have formed. The formation of one more plastic hinges produces a collapse mechanism. Hence, in a structure that has  $p$  possible hinge locations, the number of possible independent mechanisms is

$$m_i = p - D$$

In addition, the independent mechanisms may be combined to form combined mechanisms.

The structure shown in Fig. 4.6 is five degrees indeterminate, with 11 possible hinge locations. The number of possible independent mechanisms is

$$\begin{aligned} m_i &= p - D = 11 - 5 \\ &= 6 \end{aligned}$$

As shown in Fig. 4.7, the independent mechanisms consist of the beam mechanisms  $B_1$ ,  $B_2$ , and  $B_3$ ; the sway mechanism  $S$ ; the gable mechanism  $G$ ; and the joint mechanism  $J$ . Any of these independent mechanisms may be combined to form a combined mechanism. Two such combinations,  $(B_2 + S + J)$  and  $(B_2 + G + J)$ , are shown in Fig. 4.8. In combining mechanisms, the objective is to eliminate hinges in order to produce a more critical loading condition. If the correct mechanism has been selected, the bending moment diagram of the structure at collapse must show that the plastic moment of resistance is not anywhere exceeded.

Figure 4.7 Independent Mechanisms

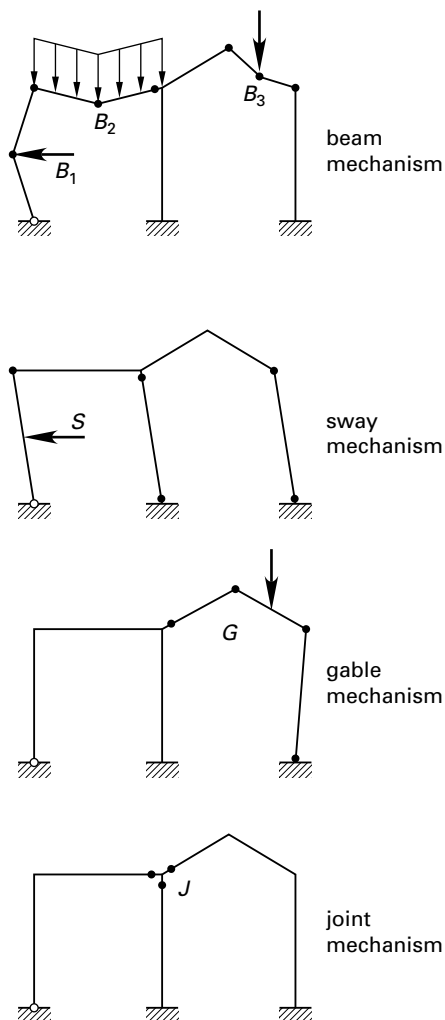
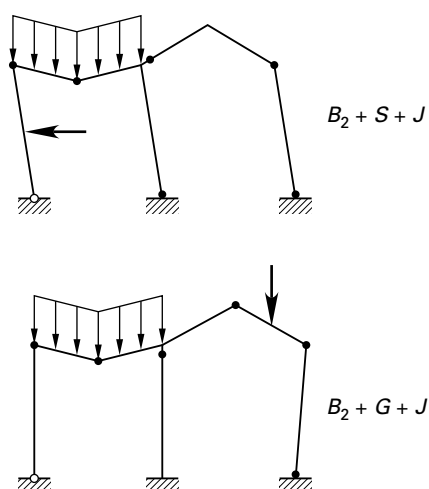


Figure 4.8 Combined Mechanisms

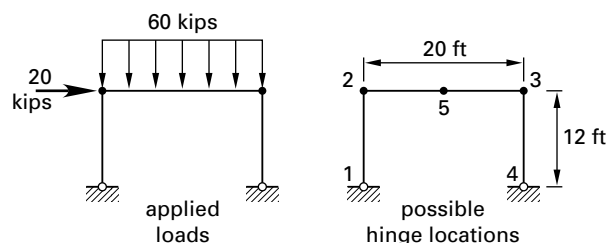


The virtual work principle, which is used in the mechanism design method, entails establishing equilibrium relationships resulting from real forces undergoing virtual displacements. The principle states that if a structure in equilibrium under a system of applied forces is subjected to a system of virtual displacements compatible with the external restraints and the geometry of the structure, the total external work done by the applied forces equals the work done by the internal forces (caused by the internal deformations corresponding to the external displacements). The expression *virtual displacement* implies an extremely small imaginary displacement, and the external work done is the product of a real loading system and imaginary displacements. The internal work, or internal strain energy, is the product of the internal forces in the system and the deformations produced by the imaginary displacements. The equilibrium relationship is: Internal work equals external work.

The application of the method is illustrated in Ex. 4.4.

#### Example 4.4

The rigid frame shown in the example illustration is fabricated from members of a uniform section in grade 50 steel. For the factored loading indicated, ignoring the member self-weight and assuming adequate lateral support is provided, determine the plastic moment of resistance required.



#### Solution

The three possible collapse mechanisms are the independent beam mechanism, the independent sway mechanism, and the combined mechanism. These are shown in the solution illustration. Applying a virtual displacement to each of these mechanisms in turn, and equating internal and external work provides three equations. A value of  $M_p$  may be obtained from each of the three equations. The largest value of  $M_p$  governs.

For the beam mechanism, applying a virtual vertical displacement at midspan of the beam, and setting internal work equal to external work,

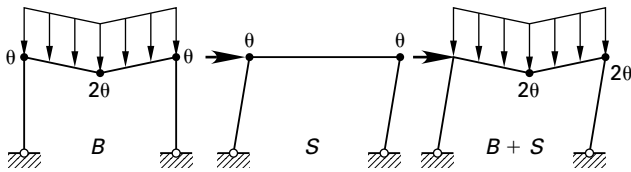
$$\begin{aligned} 4M_p(\theta) &= \frac{(2) \left( 3 \frac{\text{kips}}{\text{ft}} \right) (10 \text{ ft})(10 \text{ ft})(\theta)}{2} \\ &= (300 \text{ ft-kips})(\theta) \\ M_p &= 75 \text{ ft-kips} \end{aligned}$$

For the sway mechanism, applying a virtual horizontal displacement at the top of the columns, and setting internal work equal to external work,

$$\begin{aligned} 2M_p(\theta) &= (20 \text{ kips})(12 \text{ ft})(\theta) \\ &= (240 \text{ ft-kips})(\theta) \\ M_p &= 120 \text{ ft-kips} \end{aligned}$$

For the combined mechanism, adding internal and external work of both independent mechanisms, and setting internal work equal to external work gives

$$\begin{aligned} 4M_p(\theta) &= (300 \text{ ft-kips})(\theta) + (240 \text{ ft-kips})(\theta) \\ &= (540 \text{ ft-kips})(\theta) \\ M_p &= 135 \text{ ft-kips} \quad [\text{governs}] \end{aligned}$$



#### Example 4.5

For the rigid frame described in Ex. 4.4, determine the lightest W10 section required using grade 50 steel.

*Solution*

From LRFD Table 3-6, a W10 × 30 has

$$\begin{aligned} \phi_b M_p &= 137 \text{ ft-kips} \\ &> 135 \text{ ft-kips} \quad [\text{satisfactory}] \end{aligned}$$

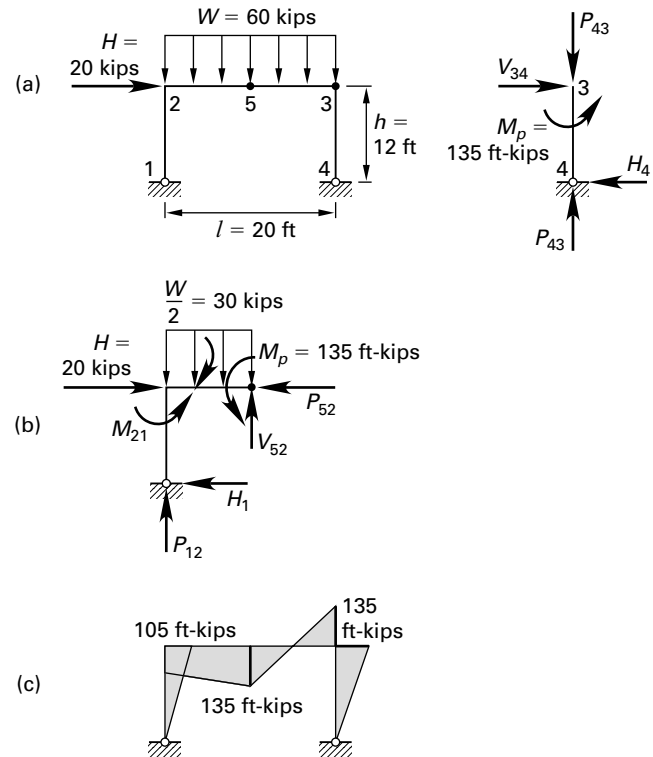
#### Static Equilibrium Check

The mechanism method provides an upper bound on the collapse load. Using static equilibrium methods, a bending moment diagram of the assumed collapse mode may be constructed. If the correct collapse mode has been selected, the value of  $M_p$  should nowhere be exceeded.

#### Example 4.6

Calculate the member forces and draw the bending moment diagram for the assumed collapse mechanism of Ex. 4.4.

*Solution*



For the free-body diagram of member 34, as shown in illustration (a), taking moments about point 3 gives the horizontal reaction at support 4 as

$$\begin{aligned} H_4 &= \frac{M_p}{h} = \frac{135 \text{ ft-kips}}{12 \text{ ft}} \\ &= 11.25 \text{ kips} \quad [\text{acting to the left as shown}] \\ &= V_{34} \\ &= P_{52} \end{aligned}$$

For the free-body diagram of member 125, as shown in illustration (b), resolving forces gives the horizontal reaction at support 1 as

$$\begin{aligned} H_1 &= H - P_{52} = 20 \text{ kips} - 11.25 \text{ kips} \\ &= 8.75 \text{ kips} \quad [\text{acting to the left as shown}] \end{aligned}$$

Taking moments about point 5 gives the axial force in member 12 as

$$\begin{aligned} P_{12} &= \frac{M_p + \left(\frac{W}{2}\right)\left(\frac{l}{4}\right) - H_1 h}{\frac{l}{2}} \\ &= \frac{135 \text{ ft-kips} + (30 \text{ kips})\left(\frac{20 \text{ ft}}{4}\right) - (8.75 \text{ kips})(12 \text{ ft})}{\frac{20 \text{ ft}}{2}} \\ &= 18 \text{ kips} \end{aligned}$$

Taking moments about joint 2 for member 12, gives the moment at joint 2 as

$$\begin{aligned} M_{21} &= H_1 h = (8.75 \text{ kips})(12 \text{ ft}) \\ &= 105 \text{ ft-kips} \end{aligned}$$

Resolving forces gives the shear force at point 5 as

$$\begin{aligned} V_{52} &= \frac{W}{2} - P_{12} = 30 \text{ kips} - 18 \text{ kips} \\ &= 12 \text{ kips} \end{aligned}$$

For the whole structure, resolving forces gives the axial force in member 43 as

$$\begin{aligned} P_{43} &= W - P_{12} = 60 \text{ kips} - 18 \text{ kips} \\ &= 42 \text{ kips} \end{aligned}$$

The bending moment diagram is shown in illustration (c). Because  $M_p = 135$  ft-kips is not exceeded at any point in the frame, the combined mechanism is the correct failure mode.

### Column Design Requirements

The column maximum unbraced length,  $L$ , for plastic design is limited by AISC App 1.6 to

$$\begin{aligned} \frac{L}{r} &= 4.71 \sqrt{\frac{E}{F_y}} \\ &= 113 \quad [\text{for } F_y = 50 \text{ kips/in}^2] \end{aligned}$$

The actual unbraced length in the plane of bending is  $L$ . The corresponding radius of gyration is  $r$ .

In accordance with AISC App. 1.5, the axial load on a column shall not exceed the value

$$\begin{aligned} P_{\max} &= 0.85 \phi_c A_g F_y \quad [\text{braced frame}] \\ P_{\max} &= 0.75 \phi_c A_g F_y \quad [\text{sway frame}] \end{aligned}$$

AISC Comm. Sec. 1.5 permits low rise frames to be designed without consideration of second-order ( $P$ -delta) effects.

As for beams, the maximum unbraced length of a column segment must not exceed the value

$$L_{pd} = \left( 0.12 + (0.076) \left( \frac{M_1}{M_2} \right) \right) \frac{E r_y}{F_y} \quad [\text{AISC A-1-7}]$$

For combined axial force and flexure, the interaction expressions from LRFD Sec. H1 apply, and

$$\begin{aligned} \frac{P_u}{\phi_c P_n} + \left( \frac{8}{9} \right) \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) &\leq 1.0 \quad \left[ \frac{P_u}{\phi_c P_n} \geq 0.2 \right] \\ \frac{P_u}{2 \phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) &\leq 1.0 \quad \left[ \frac{P_u}{\phi_c P_n} < 0.2 \right] \end{aligned}$$

### Example 4.7

Determine whether column 34 of the sway frame in Ex. 4.4 is satisfactory. The column and girder consist of a grade 50, W10  $\times$  30 section, and the column is laterally braced at 4 ft centers. It may be assumed that the plastic hinge located at point 5 is the last to form.

#### Solution

The relevant properties of a W10  $\times$  30 section are obtained from AISC Tables 1-1 and 3-6 as

$$A_g = 8.84 \text{ in}^2$$

$$I = 170 \text{ in}^4$$

$$r_x = 4.38 \text{ in}$$

$$r_y = 1.37 \text{ in}$$

$$\phi_b M_p = 137 \text{ ft-kips}$$

$$L_p = 4.84 \text{ ft}$$

The maximum unbraced length of a column segment shall not exceed the value given by AISC Eq. (A-1-7) as

$$\begin{aligned} L_{pd} &= \left( 0.12 - (0.076) \left( \frac{M_1}{M_2} \right) \right) \frac{E r_y}{F_y} \\ &= \left( 0.12 + (0.076) \left( \frac{2 M_p}{3 M_p} \right) \right) \\ &\quad \times \left( \frac{\left( \left( 29,000 \frac{\text{kips}}{\text{in}^2} \right) (1.37 \text{ in}) \right)}{50 \frac{\text{kips}}{\text{in}^2}} \right) \\ &\quad \times \left( \frac{12 \frac{\text{in}}{\text{ft}}}{12 \frac{\text{in}}{\text{ft}}} \right) \\ &= 4.6 \text{ ft} \\ &> 4 \text{ ft} \quad [\text{satisfactory}] \end{aligned}$$

$$\frac{L_x}{r_x} = \frac{144 \text{ in}}{4.38 \text{ in}} = 32.9$$

$$< 113 \quad [\text{satisfies AISC App. 1.6}]$$

$$\frac{L_y}{r_y} = \frac{48 \text{ in}}{1.37 \text{ in}} = 35.0$$

$$< 113 \quad [\text{satisfies AISC App. 1.6}]$$

For the pinned connection at joint 4, AISC Comm. Sec. C2 specifies a stiffness ratio of  $G_A = 10$ . At joint 3,

$$G_B = \frac{\sum \left( \frac{I_c}{l_c} \right)}{\sum \left( \frac{I_g}{l_g} \right)} = \frac{\frac{I}{12}}{\frac{I}{20}} = 1.7$$

From the alignment chart for sway frames in AISC Comm. Fig. C-C2.4, the effective length factor is

$$K_{34} = 2.0$$

The slenderness ratio about the  $x$ -axis is

$$\begin{aligned} \frac{K_{34}L_x}{r_x} &= \frac{(2.0)(12 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{4.38 \text{ in}} \\ &= 66 \\ &< 200 \quad [\text{satisfies AISC Sec. E2}] \end{aligned}$$

The frame is braced in the lateral direction and the effective length factor about the  $y$ -axis is given by AISC Table C-C2.2 as

$$K_{34} = 1.0$$

The slenderness ratio about the  $y$ -axis is

$$\begin{aligned} \frac{K_{34}L_y}{r_y} &= \frac{(1.0)(4 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{1.37 \text{ in}} \\ &= 35.0 \end{aligned}$$

The slenderness ratio about the  $x$ -axis governs.

In accordance with AISC App. 1.5, the maximum, axial load in the column of a sway frame is restricted to

$$\begin{aligned} P_{\max} &= 0.75\phi_c A_g F_y \\ &= (0.75)(0.90)(8.84 \text{ in}^2) \left( 50 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 298 \text{ kips} \\ &> P_{43} \quad [\text{satisfactory}] \end{aligned}$$

From AISC Table 3-6, for a W10  $\times$  30,

$$\begin{aligned} L_p &= 4.84 \text{ ft} \\ &> 4.0 \text{ ft} \quad [\text{full plastic bending capacity available}] \end{aligned}$$

$$\begin{aligned} \phi_b M_{nx} &= \phi_b M_p \\ &= 137 \text{ ft-kips} \end{aligned}$$

From AISC Table 4-22, for a  $K_{34}L_x/r_x$  value of 66, the design stress for axial load is

$$\phi_c F_{cr} = 32.7 \text{ kips/in}^2$$

The design axial strength is

$$\begin{aligned} \phi_c P_n &= \phi_c F_{cr} A_g \quad [\text{AISC E3-1}] \\ &= \left( 32.7 \frac{\text{kips}}{\text{in}^2} \right) (8.84 \text{ in}^2) \\ &= 289 \text{ kips} \\ \frac{P_{34}}{\phi_c P_n} &= \frac{42 \text{ kips}}{289 \text{ kips}} \\ &= 0.15 \\ &< 0.20 \quad [\text{AISC Eq. (H1-1b) governs}] \end{aligned}$$

Secondary effects may be neglected and AISC Eq. (H1-1b) reduces to

$$\begin{aligned} \frac{P_{34}}{2\phi_c P_n} + \frac{M_p}{\phi_b M_{nx}} &\leq 1.0 \\ \frac{42 \text{ kips}}{(2)(289 \text{ kips})} + \frac{135 \text{ ft-kips}}{137 \text{ ft-kips}} &= 1.06 \\ &\approx 1.0 \quad [\text{satisfactory}] \end{aligned}$$

The column is satisfactory.

## 2. ECCENTRICALLY LOADED BOLT GROUPS

### Nomenclature

|          |                                                                                          |                    |
|----------|------------------------------------------------------------------------------------------|--------------------|
| $A_b$    | nominal unthreaded body area of bolt                                                     | $\text{in}^2$      |
| $C$      | coefficient for eccentrically loaded bolt and weld groups                                | —                  |
| $d_b$    | nominal bolt diameter                                                                    | in                 |
| $d_m$    | moment arm between resultant tensile and compressive forces caused by an eccentric force | in                 |
| $D$      | lateral distance between bolts                                                           | in                 |
| $e$      | eccentricity of applied load                                                             | in                 |
| $f_v$    | computed shear stress                                                                    | $\text{kips/in}^2$ |
| $F_{nt}$ | nominal tensile stress of bolt                                                           | $\text{kips/in}^2$ |
| $F_{nv}$ | nominal shear stress of bolt                                                             | $\text{kips/in}^2$ |
| $F_u$    | specified minimum tensile strength                                                       | $\text{kips/in}^2$ |
| $n$      | number of bolts in one vertical row                                                      | —                  |
| $n'$     | number of bolts above the neutral axis (in tension)                                      | —                  |
| $N$      | total number of bolts in a group                                                         | —                  |
| $P$      | externally applied eccentric load                                                        | kips               |
| $P_u$    | factored load on connection                                                              | kips               |
| $P_v$    | allowable shear capacity of bolt                                                         | kips               |
| $R$      | resultant force on bolt                                                                  | kips               |
| $R_n$    | nominal strength                                                                         | kips               |
| $s$      | center-to-center pitch of two consecutive bolts                                          | in                 |
| $T$      | unfactored tensile force                                                                 | kips               |
| $T_b$    | minimum pre-tension force                                                                | kips               |
| $T_u$    | factored tensile force                                                                   | kips               |
| $V_P$    | shear force on bolt                                                                      | kips               |

### Symbols

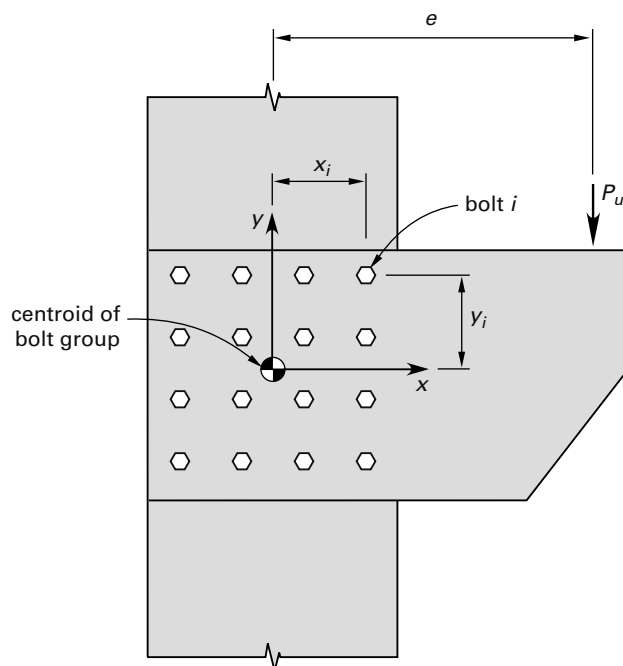
|        |                   |   |
|--------|-------------------|---|
| $\phi$ | resistance factor | — |
|--------|-------------------|---|

### Bolt Group Eccentrically Loaded in the Plane of the Faying Surface, LRFD Method

Eccentrically loaded bolt groups of the type shown in Fig. 4.9 may be conservatively designed by means of the elastic unit area method. The moment of inertia of the bolt group about the  $x$ -axis is

$$I_x = \sum y_i^2$$

Figure 4.9 Eccentrically Loaded Bolt Group, LRFD Method



The moment of inertia of the bolt group about the  $y$ -axis is

$$I_y = \sum x_i^2$$

The polar moment of inertia of the bolt group about the centroid is

$$I_o = I_x + I_y$$

The vertical force on bolt  $i$ , caused by the applied load  $P_u$ , is

$$V_P = \frac{P_u}{N}$$

The vertical force on bolt  $i$ , caused by the eccentricity  $e$ , is

$$V_e = \frac{P_u e x_i}{I_o}$$

The horizontal force on bolt  $i$ , caused by the eccentricity  $e$ , is

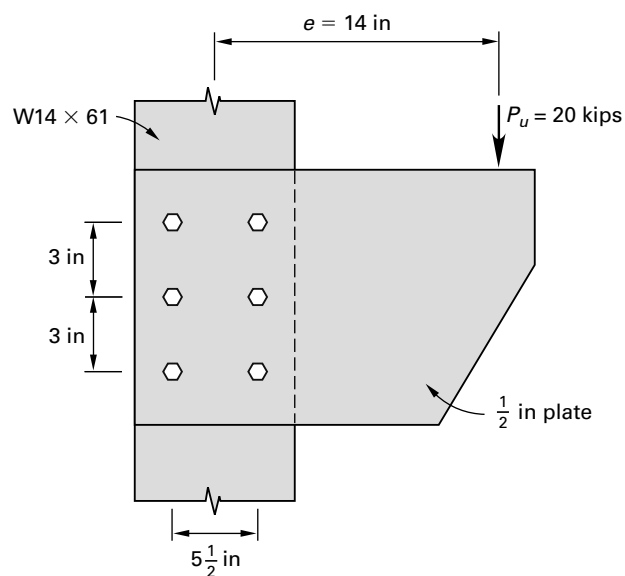
$$H_e = \frac{P_u e y_i}{I_o}$$

The resultant force on bolt  $i$  is

$$R = \sqrt{(V_P + V_e)^2 + H_e^2}$$

### Example 4.8

Using the LRFD method, determine the diameter of the A325 bearing-type bolts required in the bolted bracket shown in the illustration. Threads are included in the shear plane.



(not to scale)

### Solution

The geometrical properties of the bolt group are obtained by applying the unit area method. The moment of inertia about the  $x$ -axis is

$$\begin{aligned} I_x &= \sum y^2 = (4)(3 \text{ in})^2 \\ &= 36 \text{ in}^4/\text{in}^2 \end{aligned}$$

The moment of inertia about the  $y$ -axis is

$$\begin{aligned} I_y &= \sum x^2 = (6)(2.75 \text{ in})^2 \\ &= 45.38 \text{ in}^4/\text{in}^2 \end{aligned}$$

The polar moment of inertia about the centroid is

$$\begin{aligned} I_o &= I_x + I_y = 36 \frac{\text{in}^4}{\text{in}^2} + 45.38 \frac{\text{in}^4}{\text{in}^2} \\ &= 81.38 \text{ in}^4/\text{in}^2 \end{aligned}$$

The top right bolt is the most heavily loaded, and the coexistent forces on this bolt are the following. The vertical force caused by the applied load is

$$V_P = \frac{P_u}{N} = \frac{20 \text{ kips}}{6} = 3.33 \text{ kips}$$

The vertical force caused by the eccentricity is

$$\begin{aligned} V_e &= \frac{P_u e x_i}{I_o} \\ &= \frac{(20 \text{ kips})(14 \text{ in})(2.75 \text{ in})}{81.38 \frac{\text{in}^4}{\text{in}^2}} \\ &= 9.46 \text{ kips} \end{aligned}$$

The horizontal force caused by the eccentricity is

$$\begin{aligned} H_e &= \frac{P_u e y_i}{I_o} \\ &= \frac{(20 \text{ kips})(14 \text{ in})(3 \text{ in})}{81.38 \frac{\text{in}^4}{\text{in}^2}} \\ &= 10.32 \text{ kips} \end{aligned}$$

The resultant force is

$$\begin{aligned} R &= \sqrt{(V_P + V_e)^2 + H_e^2} \\ &= \sqrt{(3.33 \text{ kips} + 9.46 \text{ kips})^2 + (10.32 \text{ kips})^2} \\ &= 16.4 \text{ kips} \end{aligned}$$

Shear controls. From AISC Table 7-1, the design shear strength of a  $\frac{7}{8}$  in diameter A325N bolt in a standard hole, in single shear with threads included in the shear plane, is

$$\begin{aligned} \phi R_n &= 21.6 \text{ kips} \\ &> 16.4 \text{ kips} \quad [\text{satisfactory}] \end{aligned}$$

### Bolt Group Eccentrically Loaded in the Plane of the Faying Surface, ASD Method

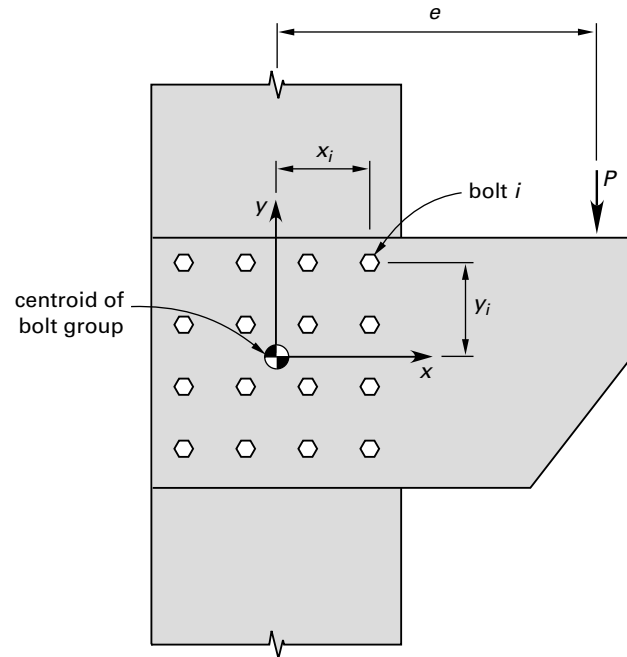
Eccentrically loaded bolt groups of the type shown in Fig. 4.10 may be conservatively designed by means of the elastic unit area method. The moment of inertia of the bolt group about the  $x$ -axis is

$$I_x = \sum y^2$$

The moment of inertia of the bolt group about the  $y$ -axis is

$$I_y = \sum x^2$$

Figure 4.10 Eccentrically Loaded Bolt Group, ASD Method



The polar moment of inertia of the bolt group about the centroid is

$$I_o = I_x + I_y$$

The vertical force on bolt  $i$ , caused by the applied load  $P$ , is

$$V_P = \frac{P}{N}$$

The vertical force on bolt  $i$ , caused by the eccentricity  $e$ , is

$$V_e = \frac{P e x_i}{I_o}$$

The horizontal force on bolt  $i$ , caused by the eccentricity  $e$ , is

$$H_e = \frac{P e y_i}{I_o}$$

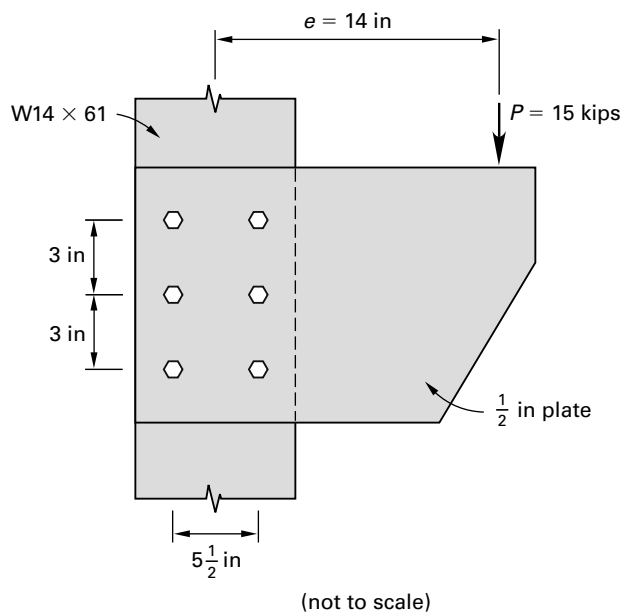
The resultant force on bolt  $i$  is

$$R = \sqrt{(V_P + V_e)^2 + H_e^2}$$

### Example 4.9

Using the ASD method, determine the diameter of the A325 bearing-type bolts required in the bolted bracket shown in the illustration. Threads are included in the shear plane.





### Solution

The geometrical properties of the bolt group are obtained by applying the unit area method. The moment of inertia about the  $x$ -axis is

$$I_x = \sum y^2 = (4)(3 \text{ in})^2 = 36 \text{ in}^4/\text{in}^2$$

The moment of inertia about the  $y$ -axis is

$$I_y = \sum x^2 = (6)(2.75 \text{ in})^2 = 45.38 \text{ in}^4/\text{in}^2$$

The polar moment of inertia about the centroid is

$$I_o = I_x + I_y = 36 \frac{\text{in}^4}{\text{in}^2} + 45.38 \frac{\text{in}^4}{\text{in}^2} = 81.38 \text{ in}^4/\text{in}^2$$

The top right bolt is the most heavily loaded, and the coexistent forces on this bolt are the following. The vertical force caused by the applied load is

$$V_P = \frac{P}{N} = \frac{15 \text{ kips}}{6} = 2.50 \text{ kips}$$

The vertical force caused by the eccentricity is

$$V_e = \frac{P e x_i}{I_o} = \frac{(15 \text{ kips})(14 \text{ in})(2.75 \text{ in})}{81.38 \frac{\text{in}^4}{\text{in}^2}} = 7.10 \text{ kips}$$

The horizontal force caused by the eccentricity is

$$H_e = \frac{P e y_i}{I_o} = \frac{(15 \text{ kips})(14 \text{ in})(3 \text{ in})}{81.38 \frac{\text{in}^4}{\text{in}^2}} = 7.74 \text{ kips}$$

The resultant force is

$$R = \sqrt{(V_P + V_e)^2 + H_e^2} = \sqrt{(2.50 \text{ kips} + 7.10 \text{ kips})^2 + (7.74 \text{ kips})^2} = 12.3 \text{ kips}$$

Shear controls. From AISC Table 7-1, the allowable shear strength of a  $7/8$  in diameter A325N bolt in a standard hole, in single shear with threads included in the shear plane, is

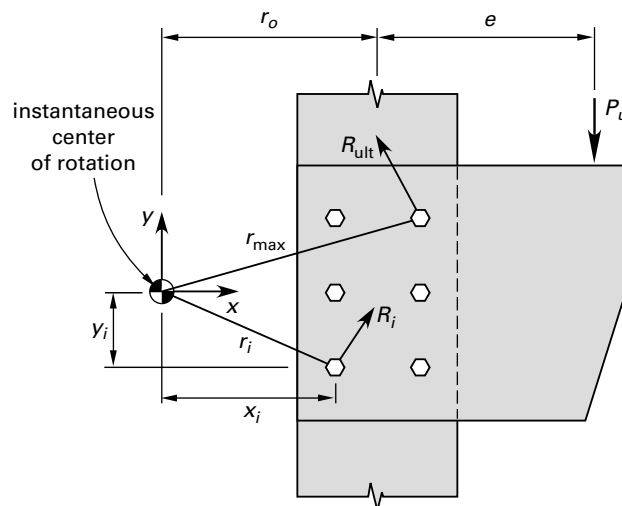
$$P_v = 14.4 \text{ kips} > 12.3 \text{ kips} \quad [\text{satisfactory}]$$

### Instantaneous Center of Rotation, LRFD Method

The instantaneous center of rotation method for analyzing eccentrically loaded bolt groups affords a more realistic estimate of a bolt group's capacity. The eccentrically loaded member is assumed to rotate about a point designated the instantaneous center of rotation.

As shown in Fig. 4.11, the bolt that is farthest from the instantaneous center will experience the greatest deformation, and will be the first bolt to fail as the applied load is increased. The deformations in the remaining bolts in the group are proportional to their distances from the instantaneous center.

Figure 4.11 Instantaneous Center of Rotation, LRFD Method



For common bolt group patterns, AISC Tables 7-7-7-14 tabulate the coefficient  $C$ , given by

$$C = \frac{P_u}{\phi R_n}$$

The method may be applied to bearing-type and slip-critical bolts, in single or double shear connections.

#### Example 4.10

Using the instantaneous center of rotation method, determine the diameter of the A325 bearing-type bolts required in the bolted bracket design in Ex. 4.8. Threads are included in the shear plane.

##### Solution

The relevant details obtained from Ex. 4.8 are the following. The number of bolts in one vertical row is

$$n = 3$$

The bolt pitch is

$$s = 3 \text{ in}$$

The lateral distance between bolts is

$$D = 5\frac{1}{2} \text{ in}$$

The eccentricity is

$$e = 14 \text{ in}$$

From AISC Table 7-9, the coefficient  $C$  is

$$C = 1.36$$

The required design strength of an individual bolt, based on the instantaneous center of rotation method, is

$$\begin{aligned} \phi R_n &= \frac{P_u}{C} = \frac{20 \text{ kips}}{1.36} \\ &= 14.7 \text{ kips} \end{aligned}$$

Shear controls. From AISC Table 7-1, the design shear strength of a  $\frac{3}{4}$  in diameter A325N bolt in a standard hole, in single shear with threads included in the shear plane, is

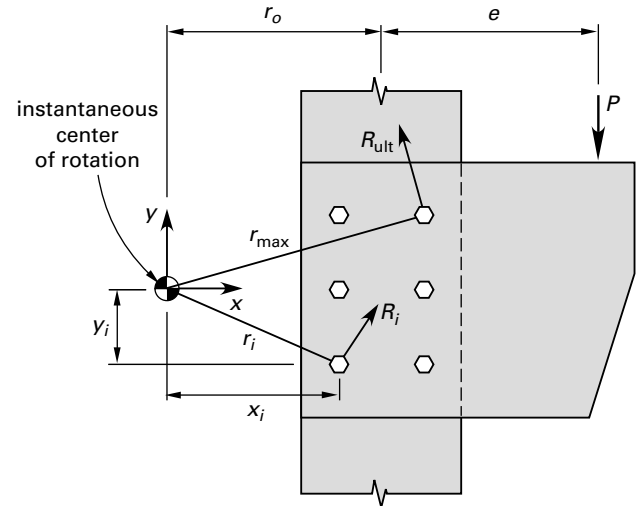
$$\begin{aligned} \phi R_n &= 15.9 \text{ kips} \\ &> 14.7 \text{ kips} \quad [\text{satisfactory}] \end{aligned}$$

#### Instantaneous Center of Rotation, ASD Method

The instantaneous center of rotation method for analyzing eccentrically loaded bolt groups affords a more realistic estimate of a bolt group's capacity. The eccentrically loaded member is assumed to rotate about a point designated the instantaneous center of rotation.

As shown in Fig. 4.12, the bolt that is farthest from the instantaneous center will experience the greatest deformation, and will be the first bolt to fail as the applied load is increased. The deformations in the remaining bolts in the group are proportional to their distances from the instantaneous center.

Figure 4.12 Instantaneous Center of Rotation, ASD Method



AISC Part 7, Tables 7-7-7-14 tabulate the coefficient  $C$  for common bolt group patterns. Here, the coefficient  $C$  is

$$C = \frac{P}{P_v}$$

The method may be applied to bearing-type and slip-critical bolts, in single or double shear connections.

#### Example 4.11

Using the instantaneous center of rotation method, determine the diameter of the A325 bearing-type bolts required in the bolted bracket designed in Ex. 4.9. Threads are included in the shear plane.

##### Solution

The relevant details from Ex. 4.9 are the following. The number of bolts in one vertical row is

$$n = 3$$

The bolt pitch is

$$b = 3 \text{ in}$$

The lateral distance between bolts is

$$D = 5\frac{1}{2} \text{ in}$$

The eccentricity is

$$e = 14 \text{ in}$$

From AISC Table 7-9, the coefficient  $C$  is

$$C = 1.36$$

The required strength of an individual bolt, based on the instantaneous center of rotation method, is

$$P_v = \frac{P}{C} = \frac{15 \text{ kips}}{1.36} = 11.0 \text{ kips}$$

Shear controls. From AISC Table 7-1, the allowable shear strength of a  $\frac{7}{8}$  in diameter A325N bolt in a standard hole, in single shear with threads included in the shear plane, is

$$P_v = 14.4 \text{ kips} > 11.0 \text{ kips} \quad [\text{satisfactory}]$$

### Bolt Group Eccentrically Loaded Normal to the Faying Surface, LRFD Method

Eccentrically loaded bolt groups of the type shown in Fig. 4.13 may be conservatively designed by assuming that the neutral axis is located at the centroid of the bolt group, and that a plastic stress distribution is produced in the bolts. The tensile force in each bolt above the neutral axis caused by the eccentricity is

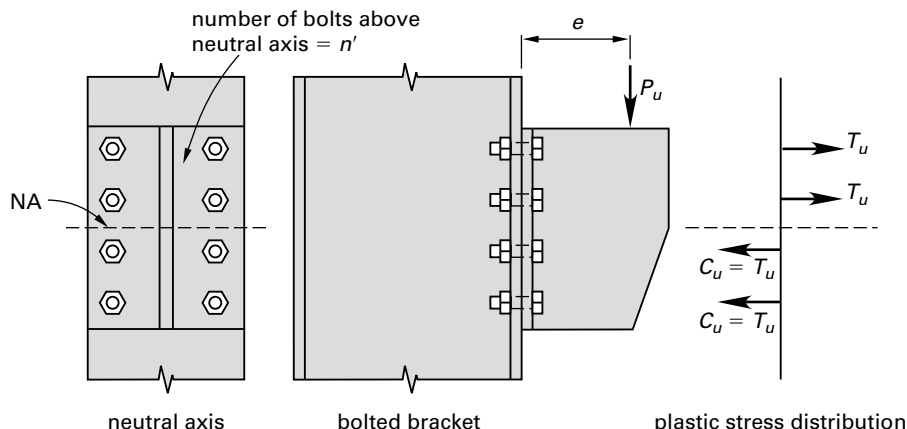
$$T_u = \frac{P_u e}{n' d_m}$$

The shear force in each bolt caused by the factored applied load is

$$V_P = \frac{P_u}{N}$$

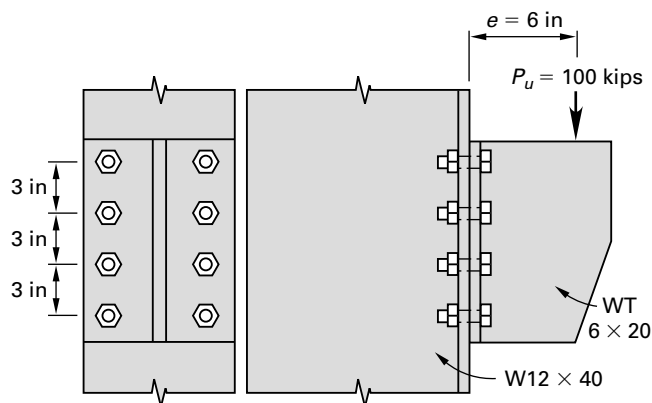
Bearing-type connectors may now be selected for combined shear and tensile forces based on the requirements of AISC Sec. J3.7.

**Figure 4.13** Bolt Group Eccentrically Loaded Normal to the Faying Surface, LRFD Method



### Example 4.12

Determine whether the  $\frac{7}{8}$  in diameter A325N bearing-type bolts in the bolted bracket shown in the illustration are adequate. Prying action may be neglected.



### Solution

The tensile force in each bolt above the neutral axis caused by the eccentricity is

$$T_u = \frac{P_u e}{n' d_m} = \frac{(100 \text{ kips})(6 \text{ in})}{(4)(6 \text{ in})} = 25 \text{ kips}$$

The shear force in each bolt caused by the applied load is

$$V_P = \frac{P_u}{N} = \frac{100 \text{ kips}}{8} = 12.5 \text{ kips}$$

The calculated shear stress on each bolt is

$$f_v = \frac{V_P}{A_b} = \frac{12.5 \text{ kips}}{0.601 \text{ in}^2} = 20.8 \text{ kips/in}^2$$

The design shear stress for grade A325 bolts, with threads included in the shear plane, is obtained from AISC Table J3.2 as

$$\begin{aligned}\phi F_{nv} &= (0.75) \left( 48 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 36 \frac{\text{kips}}{\text{in}^2} \\ &> f_v \quad [\text{satisfactory}] \\ f_v &> 0.2\phi F_{nv}\end{aligned}$$

The design tensile stress for grade A325 bolts is obtained from AISC Table J3.2 as

$$\begin{aligned}\phi F_{nt} &= (0.75) \left( 90 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 67.5 \frac{\text{kips}}{\text{in}^2}\end{aligned}$$

The factored tensile stress in each  $\frac{7}{8}$  in diameter bolt is

$$\begin{aligned}f_t &= \frac{T_u}{A_b} = \frac{25 \text{ kips}}{0.601 \text{ in}^2} \\ &= 41.60 \frac{\text{kips}}{\text{in}^2} \\ &< \phi F_{nt} \quad [\text{satisfactory}] \\ &> 0.2\phi F_{nt}\end{aligned}$$

It is necessary to investigate the effects of the combined shear and tensile stress. The nominal tensile stress  $F'_{nt}$  of a bolt that is subjected to combined shear and tension is given by AISC Eq. (J3-3a) as

$$\begin{aligned}F'_{nt} &= 1.3F_{nt} - \frac{f_v F_{nt}}{\phi F_{nv}} \\ &= (1.3) \left( 90 \frac{\text{kips}}{\text{in}^2} \right) - \frac{\left( 20.8 \frac{\text{kips}}{\text{in}^2} \right) \left( 90 \frac{\text{kips}}{\text{in}^2} \right)}{36 \frac{\text{kips}}{\text{in}^2}} \\ &= 65 \text{ kips/in}^2\end{aligned}$$

The design tensile stress  $\phi F'_{nt}$  of a bolt that is subjected to combined shear and tension is given by AISC Eq. (J3-2) as

$$\begin{aligned}\phi F'_{nt} &= (0.75) \left( 65 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 48.75 \frac{\text{kips}}{\text{in}^2} \\ &> f_t \quad [\text{satisfactory}]\end{aligned}$$

### Bolt Group Eccentrically Loaded Normal to the Faying Surface, ASD Method

Eccentrically loaded bolt groups of the type shown in Fig. 4.14 may be conservatively designed by assuming that the neutral axis is located at the centroid of the bolt group, and that an elastic stress distribution is produced in the bolts. The moment of inertia of the bolt group about the  $x$ -axis is

$$I_x = \sum y^2$$

The total tensile force on the bolts a distance  $y_i$  from the neutral axis, caused by the eccentricity  $e$ , is

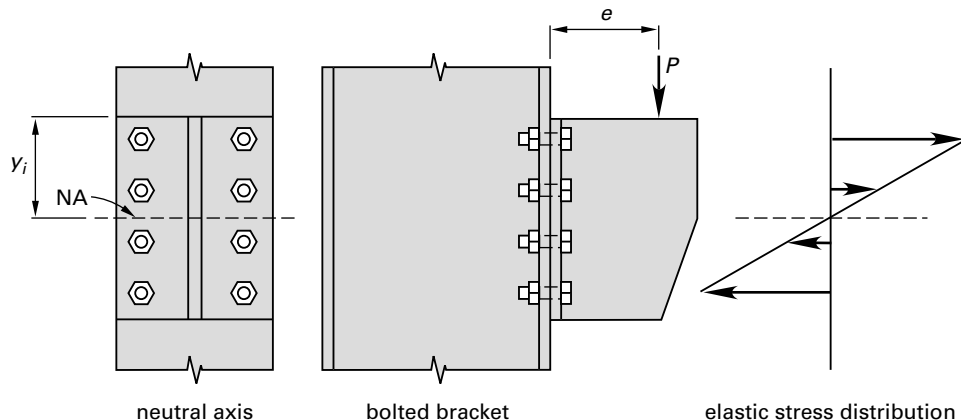
$$T_i = \frac{Pey_i}{I_x}$$

The shear force on each bolt caused by the applied load  $P$  is

$$V = \frac{P}{n}$$

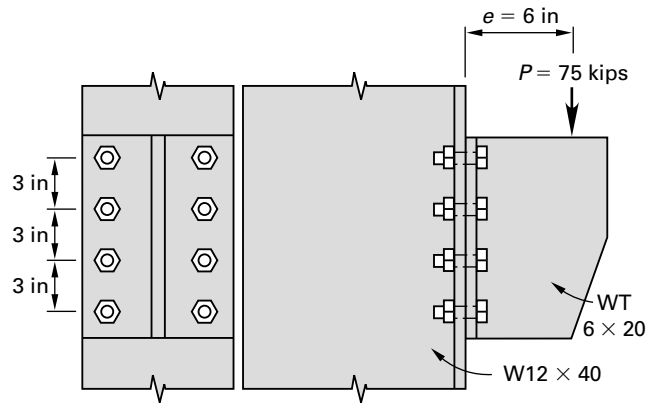
Bearing-type connectors may now be selected for combined shear and tensile forces, based on the requirements of AISC Sec. J3.7.

Figure 4.14 Bolt Group Eccentrically Loaded Normal to Faying Surface, ASD Method



### Example 4.13

Determine whether the  $\frac{7}{8}$  in diameter A325N bearing-type bolts in the bolted bracket shown in the illustration are adequate. Prying action may be neglected.



#### Solution

Assuming the neutral axis occurs at the centroid of the bolt group, and applying the unit area method, the inertia about the  $x$ -axis is

$$I_x = \sum y^2 = (4)(1.5 \text{ in})^2 + (4)(4.5 \text{ in})^2 = 90 \text{ in}^2$$

The applied moment on the bolt group is

$$M = Pe = (75 \text{ kips})(6 \text{ in}) = 450 \text{ in-kips}$$

The tensile force on each of the top bolts caused by the moment is

$$T = \frac{My}{2I_x} = \frac{(450 \text{ in-kips})(4.5 \text{ in})}{(2)(90 \text{ in}^2)} = 11.25 \text{ kips}$$

The shear force on each bolt caused by the vertical load is

$$V = \frac{P}{8} = \frac{75 \text{ kips}}{8} = 9.38 \text{ kips}$$

Bearing is not critical. From AISC Table J3.2, the allowable shear stress on a  $\frac{7}{8}$  in A325N bolt in a standard hole in single shear is

$$\frac{F_{nv}}{\Omega} = \frac{48 \frac{\text{kips}}{\text{in}^2}}{2} = 24 \text{ kips/in}^2$$

The shear stress on a top bolt is

$$\begin{aligned} f_v &= \frac{V}{A_b} = \frac{9.38 \text{ kips}}{0.601 \text{ in}^2} \\ &= 15.61 \text{ kips/in}^2 \\ &< \frac{F_{nv}}{\Omega} \quad [\text{satisfactory}] \\ &> \frac{0.2F_{nv}}{\Omega} \end{aligned}$$

The allowable tensile stress for grade A325N bolts is obtained from AISC Table J3.2 as

$$\begin{aligned} \frac{F_{nt}}{\Omega} &= \frac{90 \frac{\text{kips}}{\text{in}^2}}{2} \\ &= 45 \text{ kips/in}^2 \end{aligned}$$

The tensile stress in each of the top  $\frac{7}{8}$  in diameter bolt is

$$\begin{aligned} f_t &= \frac{T}{A_b} = \frac{11.25 \text{ kips}}{0.601 \text{ in}^2} \\ &= 18.72 \frac{\text{kips}}{\text{in}^2} \\ &< \frac{F_{nt}}{\Omega} \quad [\text{satisfactory}] \\ &> \frac{0.2F_{nt}}{\Omega} \end{aligned}$$

It is necessary to investigate the effects of the combined shear and tensile stress. The nominal tensile stress  $F'_{nt}$  of a bolt subjected to combined shear and tension is given by AISC Eq. (J3-3b) as

$$\begin{aligned} F'_{nt} &= 1.3F_{nt} - \frac{\Omega f_v F_{nt}}{F_{nv}} \\ &= (1.3) \left( 90 \frac{\text{kips}}{\text{in}^2} \right) - \frac{(2) \left( 15.61 \frac{\text{kips}}{\text{in}^2} \right) \left( 90 \frac{\text{kips}}{\text{in}^2} \right)}{48 \frac{\text{kips}}{\text{in}^2}} \\ &= 58.46 \text{ kips/in}^2 \end{aligned}$$

The allowable tensile stress  $F'_{nt}/\Omega$  of a bolt subjected to combined shear and tension, is given by AISC Eq. (J3-2) as

$$\begin{aligned} \frac{F'_{nt}}{\Omega_t} &= \frac{58.46 \frac{\text{kips}}{\text{in}^2}}{2} \\ &= 29.23 \text{ kips/in}^2 \\ &> f_t \quad [\text{satisfactory}] \end{aligned}$$

### 3. ECCENTRICALLY LOADED WELD GROUPS

#### Nomenclature

|           |                                                                                             |                      |
|-----------|---------------------------------------------------------------------------------------------|----------------------|
| $a$       | coefficient for eccentrically loaded weld group                                             | —                    |
| $A_w$     | effective area of the weld                                                                  | in <sup>2</sup>      |
| $C$       | coefficient for eccentrically loaded weld group                                             | —                    |
| $C_1$     | correction factor for electrode strength given in Tables 4.1 and 4.2                        | —                    |
| $D$       | number of sixteenths of an inch in the weld size                                            | —                    |
| $F_{EXX}$ | classification of weld metal                                                                | —                    |
| $F_w$     | nominal strength of weld electrode                                                          | kips/in <sup>2</sup> |
| $k$       | coefficient for eccentrically loaded weld group                                             | —                    |
| $l$       | characteristic length of weld group used in tabulated values of instantaneous center method | in                   |
| $\bar{l}$ | total length of weld                                                                        | in                   |
| $M$       | maximum unfactored moment caused by service loads                                           | in-lbf               |
| $P$       | externally applied load                                                                     | kips                 |
| $P_u$     | externally applied factored load                                                            | kips                 |
| $q$       | allowable strength of a 1/16 in fillet weld per in run                                      | kips/in per 1/16 in  |
| $q_u$     | design strength of a 1/16 in fillet weld per in run                                         | kips/in per 1/16 in  |
| $R_n$     | nominal strength                                                                            | kips                 |
| $t_e$     | effective throat thickness of fillet weld                                                   | in                   |
| $w$       | fillet weld size                                                                            | in                   |

#### Weld Group Eccentrically Loaded in the Plane of the Faying Surface, LRFD Method

Eccentrically loaded weld groups of the type shown in Fig. 4.15 may be conservatively designed by means of the elastic vector analysis technique, assuming unit size of weld. The polar moment of inertia of the bolt group about the centroid is

$$I_o = I_x + I_y$$

For a total length of weld  $\bar{l}$ , the vertical force per linear in of weld caused by the applied load  $P_u$ , is

$$V_P = \frac{P_u}{\bar{l}}$$

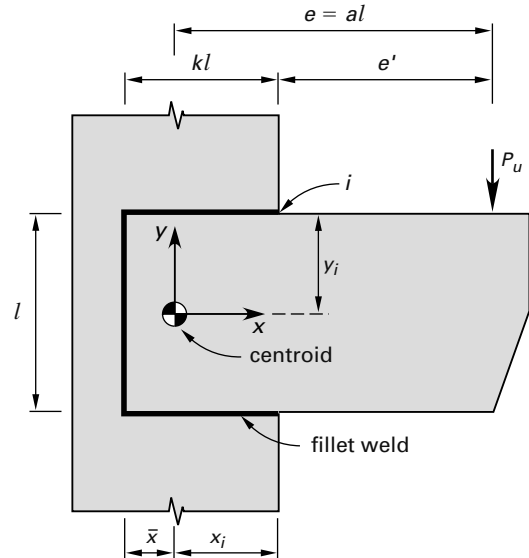
The vertical force at point  $i$ , caused by the eccentricity  $e$ , is

$$V_e = \frac{P_u e x_i}{I_o}$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$H_e = \frac{P_u e y_i}{I_o}$$

Figure 4.15 Eccentrically Loaded Weld Group, LRFD Method



The resultant force at point  $i$  is

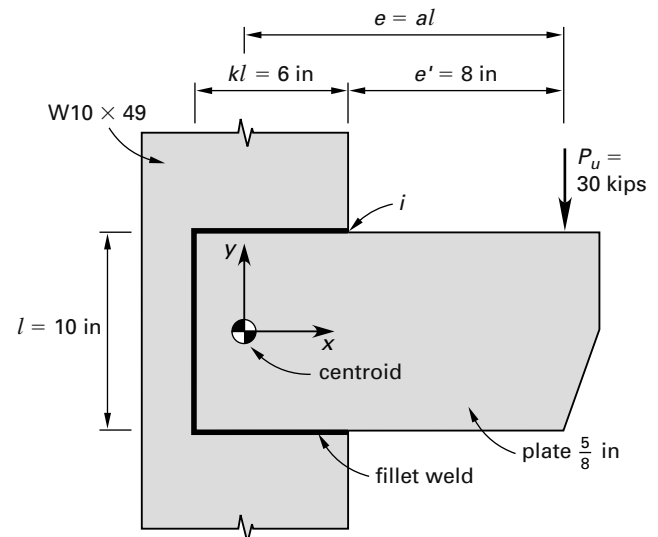
$$R = \sqrt{(V_P + V_e)^2 + H_e^2}$$

For design purposes, it is convenient to determine the design strength of a 1/16 in fillet weld per in run of E70XX grade electrodes, which is

$$\begin{aligned} q_u &= \phi F_w t_e \\ &= (0.75)(0.6) \left( 70 \frac{\text{kips}}{\text{in}^2} \right) (0.707) \left( \frac{1}{16} \text{ in} \right) \\ &= 1.39 \text{ kips/in per } 1/16 \text{ in} \end{aligned}$$

#### Example 4.14

Determine the size of E70XX fillet weld required in the welded bracket shown in the illustration. Use the elastic unit area method.



### Solution

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\begin{aligned}\bar{l} &= l + 2kl = 10 \text{ in} + (2)(6 \text{ in}) \\ &= 22 \text{ in}\end{aligned}$$

The centroid location, given by AISC Table 8-8 for a value of the coefficient  $k=0.60$ , for an eccentrically loaded weld group, is

$$\begin{aligned}\bar{x} &= xl = (0.164)(10 \text{ in}) \\ &= 1.64 \text{ in}\end{aligned}$$

The moment of inertia about the  $x$ -axis is

$$\begin{aligned}I_x &= \frac{l^3}{12} + 2kl\left(\frac{l}{2}\right)^2 \\ &= \frac{(10 \text{ in})^3}{12} + (2)(6 \text{ in})\left(\frac{10 \text{ in}}{2}\right)^2 \\ &= 383 \text{ in}^4/\text{in}\end{aligned}$$

The moment of inertia about the  $y$ -axis is

$$\begin{aligned}I_y &= \frac{2(kl)^3}{12} + 2kl\left(\frac{kl}{2} - \bar{x}\right)^2 + l\bar{x}^2 \\ &= \frac{(2)(6 \text{ in})^3}{12} + (2)(6 \text{ in})\left(\frac{6 \text{ in}}{2} - 1.64 \text{ in}\right)^2 \\ &\quad + (10 \text{ in})(1.64 \text{ in})^2 \\ &= 85 \text{ in}^4/\text{in}\end{aligned}$$

The polar moment of inertia is

$$\begin{aligned}I_o &= I_x + I_y \\ &= 383 \frac{\text{in}^4}{\text{in}} + 85 \frac{\text{in}^4}{\text{in}} \\ &= 468 \text{ in}^4/\text{in}\end{aligned}$$

The eccentricity of the applied load about the centroid of the weld profile is

$$\begin{aligned}e &= e' + kl - \bar{x} \\ &= 8 \text{ in} + 6 \text{ in} - 1.64 \text{ in} \\ &= 12.36 \text{ in}\end{aligned}$$

The top right-hand corner of the weld profile is the most highly stressed, and the coexistent forces acting at this point in the  $x$ -direction and  $y$ -direction are the following.

The vertical force caused by the applied load is

$$\begin{aligned}V_P &= \frac{P_u}{\bar{l}} = \frac{30 \text{ kips}}{22 \text{ in}} \\ &= 1.36 \text{ kips/in}\end{aligned}$$

The vertical force caused by the eccentricity is

$$\begin{aligned}V_e &= \frac{P_u ex_i}{I_o} \\ &= \frac{(30 \text{ kips})(12.36 \text{ in})(4.36 \text{ in})}{468 \frac{\text{in}^4}{\text{in}}} \\ &= 3.45 \text{ kips/in}\end{aligned}$$

The horizontal force caused by the eccentricity is

$$\begin{aligned}H_e &= \frac{P_u ey_i}{I_o} \\ &= \frac{(30 \text{ kips})(12.36 \text{ in})(5 \text{ in})}{468 \frac{\text{in}^4}{\text{in}}} \\ &= 3.96 \text{ kips/in}\end{aligned}$$

The resultant force is

$$\begin{aligned}R &= \sqrt{(V_P + V_e)^2 + H_e^2} \\ &= \sqrt{\left(1.36 \frac{\text{kips}}{\text{in}} + 3.45 \frac{\text{kips}}{\text{in}}\right)^2 + \left(3.96 \frac{\text{kips}}{\text{in}}\right)^2} \\ &= 6.23 \text{ kips/in}\end{aligned}$$

The required fillet weld size per  $1/16$  in is

$$\begin{aligned}D &= \frac{R}{q_u} = \frac{6.23 \frac{\text{kips}}{\text{in}}}{1.39 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}} \\ &= 4.5 \text{ sixteenths}\end{aligned}$$

Use a weld size of

$$w = \frac{5}{16} \text{ in}$$

The flange thickness of the W10  $\times$  49 is

$$\begin{aligned}t_f &= 0.560 \text{ in} \\ &\approx \frac{9}{16} \text{ in}\end{aligned}$$

From AISC Table J2.4, the minimum size of fillet weld for  $9/16$  in thick material is

$$\begin{aligned}w_{\min} &= \frac{1}{4} \text{ in} \\ &< \frac{5}{16} \text{ in} \quad [\text{satisfactory}]\end{aligned}$$

From AISC Sec. J2.2b, the maximum size of fillet weld for the  $\frac{5}{8}$  in plate is

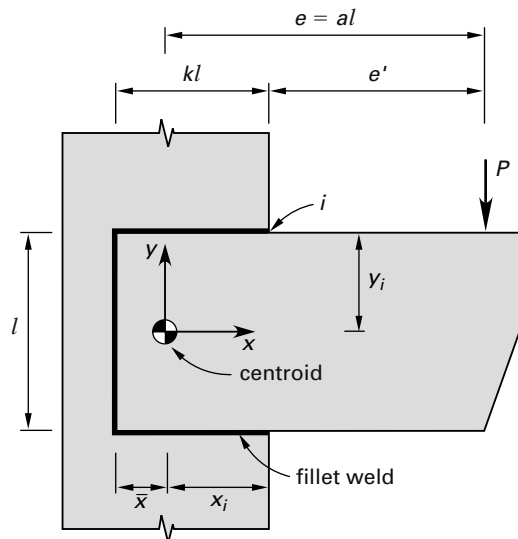
$$\begin{aligned} w_{\max} &= \frac{5}{8} \text{ in} - \frac{1}{16} \text{ in} \\ &= \frac{9}{16} \text{ in} \\ &> \frac{5}{16} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

### Weld Group Eccentrically Loaded in the Plane of the Faying Surface, ASD Method

Eccentrically loaded weld groups of the type shown in Fig. 4.16 may be conservatively designed by means of the elastic vector analysis technique, assuming unit size of weld. The polar moment of inertia of the bolt group about the centroid is

$$I_o = I_x + I_y$$

Figure 4.16 Eccentrically Loaded Weld Group, ASD Method



For a total length of weld  $\bar{l}$ , the vertical force per linear inch of weld, caused by the applied load  $P$ , is

$$V_P = \frac{P}{\bar{l}}$$

The vertical force at point  $i$ , caused by the eccentricity  $e$ , is

$$V_e = \frac{Pe x_i}{I_o}$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$H_e = \frac{Pe y_i}{I_o}$$

The resultant force at point  $i$  is

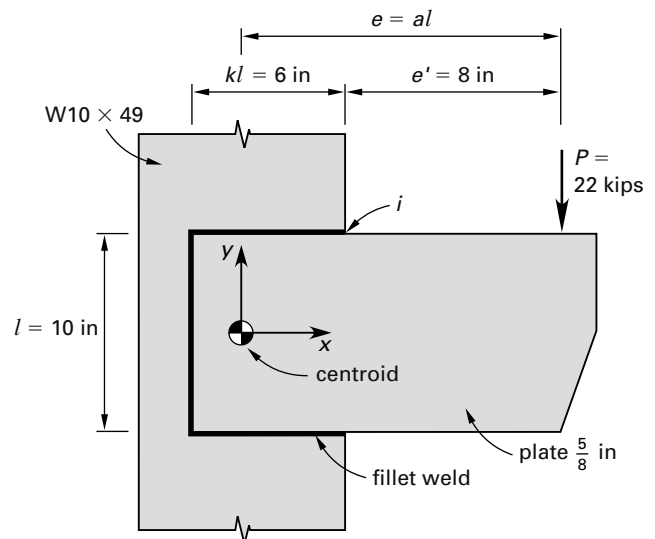
$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$

For design purposes, it is convenient to determine the strength of a  $\frac{1}{16}$  in fillet weld per in run of E70XX grade electrodes, which is

$$\begin{aligned} q &= 0.3 F_{u, \text{weld}} t_e \\ &= (0.3) \left( 70 \frac{\text{kips}}{\text{in}^2} \right) (0.707) \left( \frac{1}{16} \text{ in} \right) \\ &= 0.928 \text{ kips/in per } 1/16 \text{ in} \end{aligned}$$

### Example 4.15

Determine the size of E70XX fillet weld required in the welded bracket shown in the illustration. Use the elastic unit area method.



### Solution

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\begin{aligned} \bar{l} &= l + 2kl = 10 \text{ in} + (2)(6 \text{ in}) \\ &= 22 \text{ in} \end{aligned}$$

The centroid location is given by AISC Table 8-8, for a value of  $k = 0.60$ , as

$$\begin{aligned} \bar{x} &= xl = (0.163)(10 \text{ in}) \\ &= 1.63 \text{ in} \end{aligned}$$



The moment of inertia about the  $x$ -axis is

$$\begin{aligned} I_x &= \frac{l^3}{12} + 2kl\left(\frac{l}{2}\right)^2 \\ &= \frac{(10 \text{ in})^3}{12} + (2)(6 \text{ in})\left(\frac{10 \text{ in}}{2}\right)^2 \\ &= 383 \text{ in}^4/\text{in} \end{aligned}$$

The moment of inertia about the  $y$ -axis is

$$\begin{aligned} I_y &= \frac{(2kl)^3}{12} + 2kl\left(\frac{kl}{2} - \bar{x}\right)^2 + l\bar{x}^2 \\ &= \frac{(2)(6 \text{ in})^3}{12} + (2)(6 \text{ in})\left(\frac{6 \text{ in}}{2} - 1.63 \text{ in}\right)^2 \\ &\quad + (10 \text{ in})(1.63 \text{ in})^2 \\ &= 85 \text{ in}^4/\text{in} \end{aligned}$$

The polar moment of inertia is

$$\begin{aligned} I_o &= I_x + I_y \\ &= 383 \frac{\text{in}^4}{\text{in}} + 85 \frac{\text{in}^4}{\text{in}} \\ &= 468 \text{ in}^4/\text{in} \end{aligned}$$

The eccentricity of the applied load about the centroid of the weld profile is

$$\begin{aligned} e &= e' + kl - \bar{x} \\ &= 8 \text{ in} + 6 \text{ in} - 1.63 \text{ in} \\ &= 12.37 \text{ in} \end{aligned}$$

The top right-hand corner of the weld profile is the most highly stressed, and the coexistent forces acting at this point in the  $x$ -direction and  $y$ -direction are the following.

The vertical force caused by the applied load is

$$\begin{aligned} V_P &= \frac{P}{l} = \frac{22 \text{ kips}}{22 \text{ in}} \\ &= 1.00 \text{ kips/in} \end{aligned}$$

The vertical force caused by the eccentricity is

$$\begin{aligned} V_e &= \frac{Pex_i}{I_o} \\ &= \frac{(22 \text{ kips})(12.37 \text{ in})(4.37 \text{ in})}{468 \frac{\text{in}^4}{\text{in}}} \\ &= 2.54 \text{ kips/in} \end{aligned}$$

The horizontal force caused by the eccentricity is

$$\begin{aligned} H_e &= \frac{Pey_i}{I_o} \\ &= \frac{(22 \text{ kips})(12.37 \text{ in})(5 \text{ in})}{468 \frac{\text{in}^4}{\text{in}}} \\ &= 2.91 \text{ kips/in} \end{aligned}$$

The resultant force is

$$\begin{aligned} R &= \sqrt{(V_P + V_e)^2 + H_e^2} \\ &= \sqrt{\left(1.00 \frac{\text{kips}}{\text{in}} + 2.54 \frac{\text{kips}}{\text{in}}\right)^2 + \left(2.91 \frac{\text{kips}}{\text{in}}\right)^2} \\ &= 4.58 \text{ kips/in} \end{aligned}$$

The required fillet weld size per  $1/16$  in is

$$\begin{aligned} D &= \frac{R}{q} \\ &= \frac{4.58 \frac{\text{kips}}{\text{in}}}{0.928 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}} \\ &= 4.94 \text{ sixteenths} \end{aligned}$$

Use a weld size of

$$w = \frac{5}{16} \text{ in}$$

The flange thickness of the W10  $\times$  49 is

$$\begin{aligned} t_f &= 0.560 \text{ in} \\ &\approx \frac{9}{16} \text{ in} \end{aligned}$$

From AISC Table J2.4, the minimum size of fillet weld for  $9/16$  in thick material is

$$\begin{aligned} w_{\min} &= \frac{1}{4} \text{ in} \\ &< \frac{5}{16} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

From AISC Sec. J2.2b, the maximum size of fillet weld for the  $5/8$  in plate is

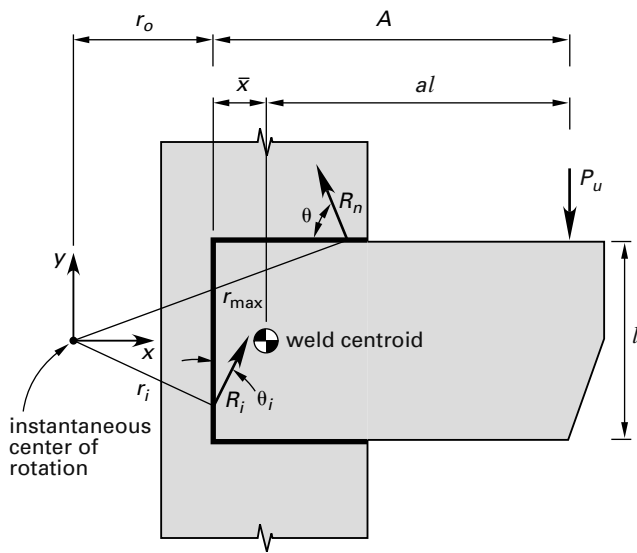
$$\begin{aligned} w_{\max} &= \frac{5}{8} \text{ in} - \frac{1}{16} \text{ in} \\ &= \frac{9}{16} \text{ in} \\ &> \frac{5}{16} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

### Instantaneous Center of Rotation, LRFD Method

The instantaneous center of rotation method for analyzing eccentrically loaded weld groups affords a more realistic estimate of a weld group's capacity. The eccentrically loaded member is assumed to rotate about a point designated the instantaneous center of rotation.

As shown in Fig. 4.17, the weld that is farthest from the instantaneous center will experience the greatest deformation. It will reach the nominal strength first as the applied load is increased. The deformations of other weld elements are proportional to their distances from the instantaneous center.

**Figure 4.17** Instantaneous Center of Rotation, LRFD Method



AISC Tables 8-4–8-11 tabulate the coefficient  $C$  for common weld group patterns given by

$$C = \frac{R_n}{C_1 D l}$$

$$R_n = \frac{P_u}{\phi}$$

Correction factors  $C_1$  are given in Table 4.1.

**Table 4.1** Correction Factor for Electrode Strength, LRFD

| electrode | E60   | E70 | E80  | E90  | E100 |
|-----------|-------|-----|------|------|------|
| $C_1$     | 0.857 | 1.0 | 1.03 | 1.16 | 1.21 |

### Example 4.16

Using the instantaneous center of rotation method, determine the size of E70XX fillet weld required in the welded bracket designed in Ex. 4.14.

#### Solution

The relevant details obtained from Ex. 4.14 are

$$l = 10 \text{ in}$$

$$e' = 8 \text{ in}$$

$$kl = 6 \text{ in}$$

$$k = 0.6$$

From AISC Table 8-8,

$$\begin{aligned} \bar{x} &= 0.164l \\ &= (0.164)(10 \text{ in}) \\ &= 1.64 \text{ in} \\ a &= \frac{e' + kl - \bar{x}}{l} \\ &= \frac{8 \text{ in} + 6 \text{ in} - 1.64 \text{ in}}{10 \text{ in}} \\ &= 1.24 \text{ in} \end{aligned}$$

From AISC Table 8-8, for values of  $a = 1.24$  and  $k = 0.6$ , the coefficient  $C$  is given as

$$C = 1.59$$

For E70XX electrodes, the correction factor for electrode strength is obtained from Table 4.1 as

$$C_1 = 1.0$$

The required weld size is

$$\begin{aligned} D &= \frac{P_u}{\phi C C_1 l} \\ &= \frac{30 \text{ kips}}{(0.75)(1.59)(1.0)(10 \text{ in})} \\ &= 2.5 \text{ sixteenths} \end{aligned}$$

The flange thickness of the W10 × 49 is

$$\begin{aligned} t_f &= 0.560 \text{ in} \\ &\approx \frac{9}{16} \text{ in} \end{aligned}$$

From AISC Table J2.4, the minimum size of fillet weld for  $\frac{9}{16}$  in thick material is

$$w_{\min} = \frac{1}{4} \text{ in}$$

$$w = \frac{1}{4} \text{ in}$$

The instantaneous center of rotation method for analyzing eccentrically loaded weld groups affords a more realistic estimate of a weld group's capacity. The eccentrically loaded member is assumed to rotate about a point designated the instantaneous center of rotation.

As shown in Fig. 4.18, the weld that is farthest from the instantaneous center will experience the greatest deformation. It will reach maximum strength first as the applied load is increased. The deformations of other weld elements are proportional to their distances from the instantaneous center.

$$C = \frac{R_n}{C_1 D l}$$

$$R_n = \Omega P$$

Correction factors  $C_1$  are given in Table 4.2.

| electrode | E60   | E70 | E80  | E90  | E100 |
|-----------|-------|-----|------|------|------|
| $C_1$     | 0.857 | 1.0 | 1.03 | 1.16 | 1.21 |

Using the instantaneous center of rotation method, determine the size of E70XX fillet weld required in the welded bracket designed in Ex. 4.15.

The relevant details from Ex. 4.15 are

$l = 10$  in

$$e' = 8 \text{ in}$$

$kl = 6$  in

$k = 0.6$

From AISC Table 8-8,

$$\begin{aligned}\bar{x} &= 0.164l \\ &= (0.164)(10 \text{ in}) \\ &= 1.64 \text{ in} \\ a &= \frac{e' + kl - \bar{x}}{l} \\ &= \frac{8 \text{ in} + 6 \text{ in} - 1.64 \text{ in}}{10 \text{ in}} \\ &= 1.24 \text{ in}\end{aligned}$$

From AISC Table 8-8 for values of  $a = 1.24$  and  $k = 0.6$ , the coefficient  $C$  is given as

$$C = 1.59$$

For E70XX electrodes, the correction factor for electrode strength is obtained from Table 4.2 as

$$C_1 = 1.0$$

The required weld size is

$$\begin{aligned} D &= \frac{\Omega P}{C C_1 l} \\ &= \frac{(2)(22 \text{ kips})}{(1.59)(1.0)(10 \text{ in})} \\ &= 2.8 \text{ sixteenths} \end{aligned}$$

Use a weld size of

$$w = \frac{1}{4} \text{ in}$$

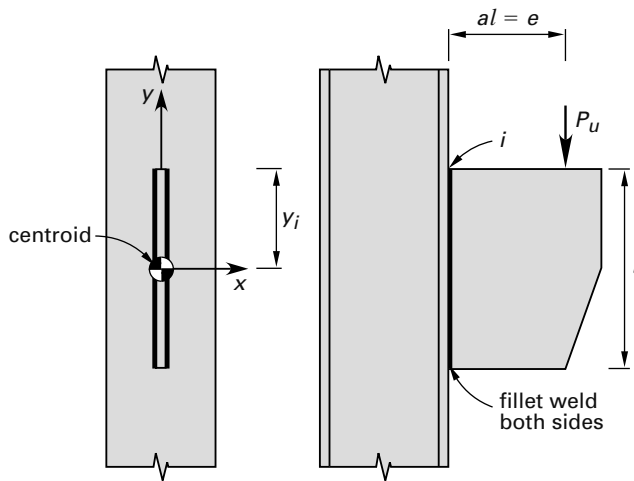
### Weld Group Eccentrically Loaded Normal to the Faying Surface, LRFD Method

Eccentrically loaded weld groups of the type shown in Fig. 4.19 may be conservatively designed by means of the elastic vector analysis technique, assuming unit size

of weld. For a total length of weld  $\bar{l}$ , the vertical force per linear inch of weld caused by the applied load  $P_u$  is

$$V_P = \frac{P_u}{\bar{l}} \\ = P_u/2l$$

**Figure 4.19** Weld Group Eccentrically Loaded Normal to the Faying Surface, LRFD Method



The moment of inertia about the  $x$ -axis is

$$I_x = \frac{2l^3}{12} \\ = l^3/6$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$H_e = \frac{P_u e y_i}{I_x} \\ = 3P_u e/l^2$$

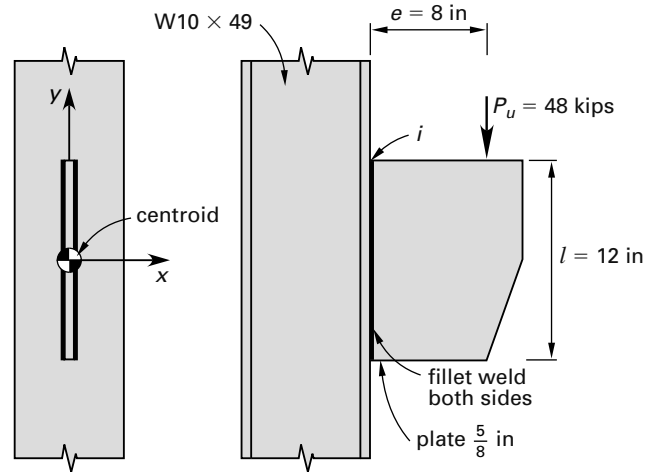
The resultant force at point  $i$  is

$$R = \sqrt{V_P^2 + H_e^2}$$

The instantaneous center of rotation method may also be used to analyze weld groups eccentrically loaded normal to the faying surface. AISC Table 8-4 provides a means of designing weld groups by this method.

#### Example 4.18

Determine the required size of E70XX fillet weld in the welded gusset plate shown in the illustration. Use the elastic unit area method, and compare with the instantaneous center of rotation method.



**Solution**

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\bar{l} = 2l = (2)(12 \text{ in}) \\ = 24 \text{ in}$$

The vertical force per linear inch of weld, caused by the applied load  $P_u$ , is

$$V_P = \frac{P_u}{\bar{l}} = \frac{48 \text{ kips}}{(2)(12 \text{ in})} \\ = 2 \text{ kips/in}$$

The moment of inertia about the  $x$ -axis is

$$I_x = \frac{2l^3}{12} = \frac{(2)(12 \text{ in})^3}{12} \\ = 288 \text{ in}^4/\text{in}$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$H_e = \frac{P_u e y_i}{I_x} = \frac{(48 \text{ kips})(8 \text{ in})(6 \text{ in})}{288 \frac{\text{in}^4}{\text{in}}} \\ = 8 \text{ kips/in}$$

The resultant force at point  $i$  is

$$R = \sqrt{V_P^2 + H_e^2} \\ = \sqrt{\left(2.0 \frac{\text{kips}}{\text{in}}\right)^2 + \left(8.0 \frac{\text{kips}}{\text{in}}\right)^2} \\ = 8.25 \text{ kips/in}$$

The required fillet weld size per  $\frac{1}{16}$  inch is

$$\begin{aligned} D &= \frac{R}{q_u} \\ &= \frac{5.46 \frac{\text{kips}}{\text{in}}}{1.39 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}} \\ &= 3.9 \text{ sixteenths} \end{aligned}$$

Use a weld size of

$$w = \frac{3}{8} \text{ in}$$

From AISC Table 8-4, for values of  $a = 0.67$  and  $k = 0$ , the coefficient  $C$  is given as 1.83. The required fillet weld size per  $\frac{1}{16}$  in, based on the instantaneous center of rotation method, is

$$\begin{aligned} D &= \frac{P_u}{\phi C l} \\ &= \frac{48 \text{ kips}}{(0.75) \left( 1.83 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in} \right) (12 \text{ in})} \\ &= 2.9 \text{ sixteenths} \end{aligned}$$

The flange thickness of the W10  $\times$  49 is

$$\begin{aligned} t_f &= 0.560 \text{ in} \\ &\approx \frac{9}{16} \text{ in} \end{aligned}$$

From AISC Table J2.4, the minimum size of fillet weld for  $\frac{9}{16}$  in thick material is

$$w_{\min} = \frac{1}{4} \text{ in}$$

Use a weld size of

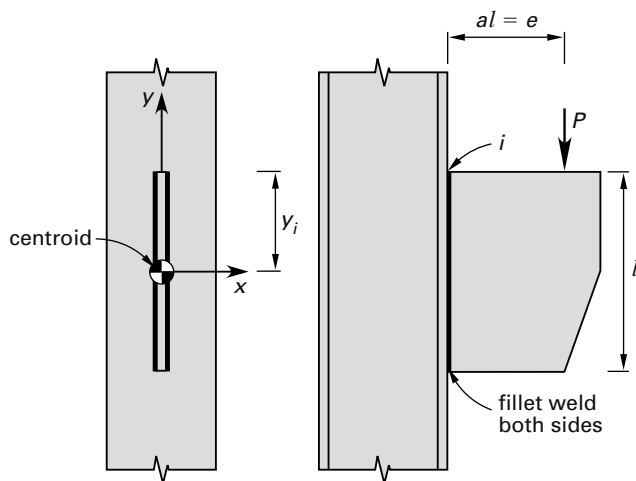
$$w_{\min} = \frac{1}{4} \text{ in}$$

### Weld Group Eccentrically Loaded Normal to the Faying Surface, ASD Method

Eccentrically loaded bolt groups of the type shown in Fig. 4.20 may be conservatively designed by means of the elastic vector analysis technique, assuming unit size of weld. For a total length of weld  $\bar{l}$ , the vertical force per linear inch of weld caused by the applied load  $P$  is

$$\begin{aligned} V_P &= \frac{P}{\bar{l}} \\ &= \frac{P}{2l} \end{aligned}$$

Figure 4.20 Weld Group Eccentrically Loaded Normal to the Faying Surface, ASD Method



The moment of inertia about the  $x$ -axis is

$$\begin{aligned} I_x &= \frac{2l^3}{12} \\ &= l^3/6 \end{aligned}$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$\begin{aligned} H_e &= \frac{P e y_i}{I_x} \\ &= \frac{3 P e}{l^2} \end{aligned}$$

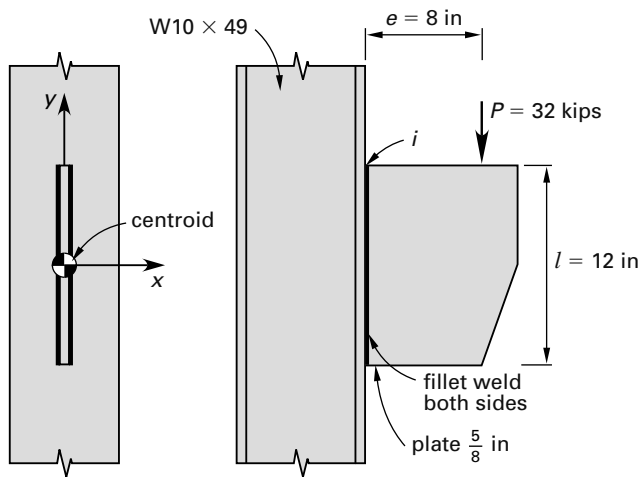
The resultant force at point  $i$  is

$$R = \sqrt{V_P^2 + H_e^2}$$

The instantaneous center of rotation method may also be used to analyze weld groups eccentrically loaded normal to the faying surface. AISC Table 8-4 provides a means of designing weld groups by this method.

### Example 4.19

Determine the required size of E70XX fillet weld in the welded gusset plate shown in the illustration. Use the elastic unit area method, and compare with the instantaneous center of rotation method.



### Solution

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\begin{aligned}\bar{l} &= 2l \\ &= 24 \text{ in}\end{aligned}$$

The vertical force per linear inch of weld, caused by the applied load  $P$ , is

$$\begin{aligned}V_P &= \frac{P}{2l} = \frac{32 \text{ kips}}{24 \text{ in}} \\ &= 1.3 \text{ kips/in}\end{aligned}$$

The moment of inertia about the  $x$ -axis is

$$\begin{aligned}I_x &= \frac{2l^3}{12} = \frac{(2)(12 \text{ in})^3}{12} \\ &= 288 \text{ in}^4/\text{in}\end{aligned}$$

The horizontal force at point  $i$ , caused by the eccentricity  $e$ , is

$$\begin{aligned}H_e &= \frac{Pe y_i}{I_x} = \frac{(32 \text{ kips})(8 \text{ in})(6 \text{ in})}{288 \frac{\text{in}^4}{\text{in}}} \\ &= 5.3 \text{ kips/in}\end{aligned}$$

The resultant force at point  $i$  is

$$\begin{aligned}R &= \sqrt{V_P^2 + H_e^2} \\ &= \sqrt{\left(1.3 \frac{\text{kips}}{\text{in}}\right)^2 + \left(5.3 \frac{\text{kips}}{\text{in}}\right)^2} \\ &= 5.46 \text{ kips/in}\end{aligned}$$

The required fillet weld size per  $1/16$  inch is

$$\begin{aligned}D &= \frac{R}{q} \\ &= \frac{5.46 \frac{\text{kips}}{\text{in}}}{0.928 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}} \\ &= 5.9 \text{ sixteenths}\end{aligned}$$

Use a weld size of

$$w = \frac{3}{8} \text{ in}$$

From AISC Table 8-4, for values of  $a = 0.67$  and  $k = 0$ , the coefficient  $C$  is given as

$$C = 1.83$$

For E70XX electrodes, the correction factor for electrode strength is obtained from Table 4.2 as

$$C_1 = 1.0$$

The required weld size is

$$\begin{aligned}D &= \frac{\Omega P}{C C_1 l} \\ &= \frac{(2)(32 \text{ kips})}{(1.83)(1.0)(12 \text{ in})} \\ &= 2.9 \text{ sixteenths}\end{aligned}$$

The flange thickness of the W10 x 49 is

$$\begin{aligned}t_f &= 0.560 \text{ in} \\ &\approx \frac{9}{16} \text{ in}\end{aligned}$$

From AISC Table J2.4, the minimum size of fillet weld for  $9/16$  in thick material is

$$w_{\min} = \frac{1}{4} \text{ in}$$

Use a weld size of

$$w_{\min} = \frac{1}{4} \text{ in}$$

## 4. COMPOSITE BEAMS

### Nomenclature

|           |                                                  |                 |
|-----------|--------------------------------------------------|-----------------|
| $a$       | depth of compression block                       | in              |
| $A_c$     | area of concrete slab within the effective width | in <sup>2</sup> |
| $A_{ctr}$ | concrete transformed area in compression         | in <sup>2</sup> |
| $A_s$     | cross-sectional area of structural steel         | in <sup>2</sup> |
| $b$       | effective concrete flange width                  | in              |

|           |                                                                                              |                      |
|-----------|----------------------------------------------------------------------------------------------|----------------------|
| $C_{con}$ | compressive force in a concrete slab at ultimate load                                        | kips                 |
| $d$       | depth of steel beam                                                                          | in                   |
| $f'_c$    | specified compressive strength of the concrete                                               | kips/in <sup>2</sup> |
| $f_u$     | compressive stress in the concrete stress block                                              | kips/in <sup>2</sup> |
| $F_y$     | specified minimum yield stress of the structural steel section                               | kips/in <sup>2</sup> |
| $h$       | overall height of the transformed composite section                                          | in                   |
| $L$       | span length                                                                                  | ft                   |
| $M$       | applied bending moment                                                                       | in-lbf               |
| $M_n$     | nominal flexural strength of member                                                          | in-kips or ft-kips   |
| $n$       | number of shear connectors between point of maximum positive moment and point of zero moment | —                    |
| $Q_n$     | nominal shear strength of single shear connector                                             | kips                 |
| $s$       | beam spacing                                                                                 | ft or in             |
| $t_c$     | actual slab thickness                                                                        | in                   |
| $T_{stl}$ | tensile force in steel at ultimate load                                                      | kips                 |
| $V'$      | total factored horizontal shear between point of maximum moment and point of zero moment     | kips                 |
| $y$       | moment arm between centroids of tensile force and compressive force                          | in                   |
| $Y_{con}$ | distance from top of steel beam to top of concrete                                           | in                   |
| $Y_1$     | distance from top of steel beam to plastic neutral axis                                      | in                   |
| $Y_2$     | distance from top of steel beam to concrete flange force                                     | in                   |

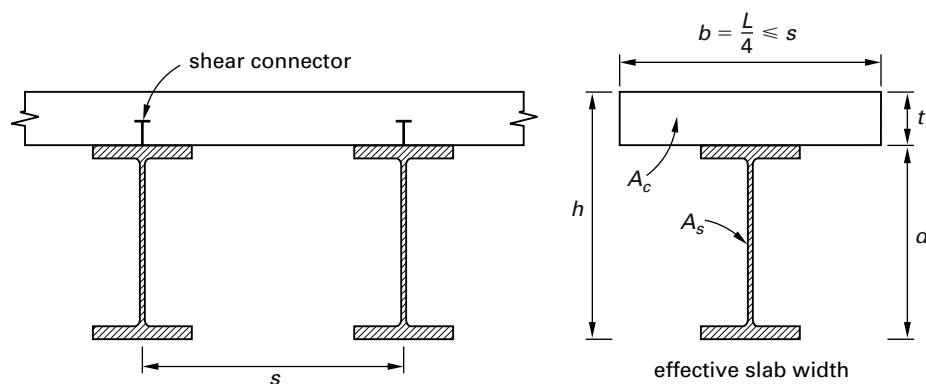
### Symbols

|            |                                                                                            |      |
|------------|--------------------------------------------------------------------------------------------|------|
| $\sum Q_n$ | summation of $Q_n$ between point of maximum moment and point of zero moment on either side | kips |
|------------|--------------------------------------------------------------------------------------------|------|

### General Considerations

As shown in Fig. 4.21, a composite beam consists of a concrete slab acting integrally with a steel beam to resist the applied loads. Shear connectors are welded to the top flange of the beam to provide composite action

Figure 4.21 Composite Section



between the slab and the beam. Because the compressive stress in the concrete slab is not uniform over the width of the slab, an effective width is assumed in calculations. In accordance with AISC Sec. I3.1, the effective width of the concrete slab on either side of the beam center line is defined as the lesser of

- one-eighth of the beam span
- one-half of the beam spacing
- the distance to the edge of the slab

### Composite Beam Design, LRFD Method

In accordance with AISC Sec. I1.1a, a rectangular stress block with depth  $a$  is formed in the concrete slab at the ultimate load. The stress in the stress block is

$$f_u = 0.85f'_c$$

The compressive force in the concrete slab is

$$C_{con} = baf_u \\ = 0.85f'_c ba$$

As shown in Fig. 4.22, when the stress block depth is less than the depth of the slab, the plastic neutral axis is located at the top of the steel beam. The beam has fully yielded in tension at the yield stress  $F_y$ . The force in the steel beam is

$$T_{stl} = F_y A_s$$

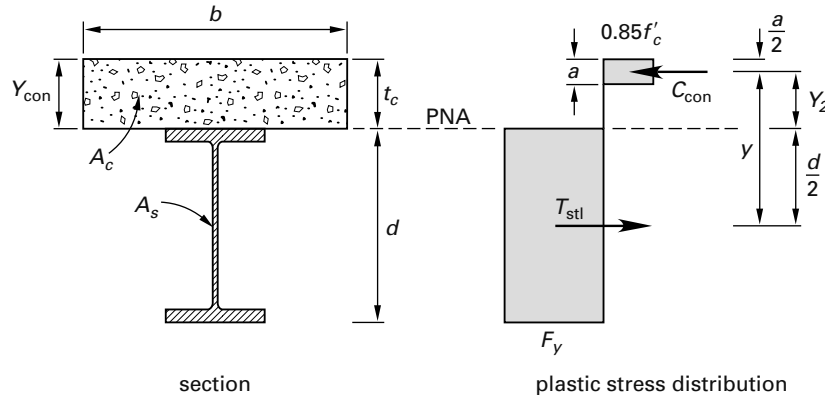
Equating horizontal forces gives

$$T_{stl} = C_{con} \\ F_y A_s = 0.85f'_c ba$$

The depth of the stress block, when sufficient shear connectors are provided to ensure full composite action, is

$$a = \frac{F_y A_s}{0.85f'_c b}$$

Figure 4.22 Fully Composite Beam at Ultimate Load



The distance from top of steel beam to the centroid of the compressive force is

$$Y_2 = Y_{con} - \frac{a}{2}$$

The moment arm between the centroids of the tensile and compressive forces is

$$y = Y_2 + \frac{d}{2}$$

The nominal flexural capacity of the fully composite section is

$$\begin{aligned} M_n &= T_{stl}y \\ &= T_{stl} \left( Y_2 + \frac{d}{2} \right) \end{aligned}$$

The design flexural capacity is

$$\phi M_n = 0.9 M_n$$

For given values of  $Y_2$  and  $\sum Q_n$ , AISC Table 3-19 provides values of  $\phi M_n$  for a range of W sections and for the three cases

- the plastic neutral axis located in the steel beam
- the plastic neutral axis located at the top of the steel beam
- shear connectors inadequate to provide full composite action

For all three cases the depth of the stress block is

$$a = \frac{\sum Q_n}{0.85f'_c b}$$

$\sum Q_n$  is the smaller of

- $0.85f'_c A_c$ , plastic neutral axis is located in the steel beam
- $F_y A_s$ , plastic neutral axis is located at the top of the steel beam
- $nQ_n$ , shear connectors are inadequate to provide full composite action

Composite beams may be constructed using either of two methods. In shored construction, the steel beam is propped before casting the slab, to eliminate stresses in the steel beam caused by the deposited concrete. In unshored construction, the slab is cast without propping the steel beam. For both shored and unshored beams, the composite section is designed to support the total factored loads, coming from all dead and live loads. For unshored construction, the steel beam alone must be adequate to support all loads applied before the concrete has attained 75% of its required strength.

#### Example 4.20

A simply supported composite beam consists of a concrete slab with a depth of  $t_c = Y_{con} = 4$  in, directly supported on a W21  $\times$  57 grade 50 steel beam. The beam spacing is  $s = 9$  ft, and the span is  $L = 30$  ft. The slab consists of normal weight concrete with a strength of  $f'_c = 4000$  lbf/in<sup>2</sup>. Determine the design moment capacity of the beam if full composite action is provided.

#### Solution

The effective width  $b$  of the concrete slab, in accordance with AISC Sec. I3.1, is the lesser of

$$\begin{aligned} s &= (9 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) = 108 \text{ in} \\ \frac{L}{4} &= \frac{(30 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{4} \\ &= 90 \text{ in} \quad [\text{governs}] \end{aligned}$$

For full composite action with the plastic neutral axis located at the top of the steel beam

$$\begin{aligned} \sum Q_n &= F_y A_s \\ &= \left( 50 \frac{\text{kips}}{\text{in}^2} \right) (16.7 \text{ in}^2) \\ &= 835 \text{ kips} \end{aligned}$$



The distance from the top of the steel beam to the plastic neutral axis is

$$Y_1 = 0 \text{ in}$$

The depth of the stress block is

$$\begin{aligned} a &= \frac{F_y A_s}{0.85 f'_c b} \\ &= \frac{835 \text{ kips}}{(0.85) \left( 4 \frac{\text{kips}}{\text{in}^2} \right) (90 \text{ in})} \\ &= 2.73 \text{ in} \end{aligned}$$

The distance from the top of the steel beam to the line of action of the concrete slab force is

$$\begin{aligned} Y_2 &= Y_{\text{con}} - \frac{a}{2} = 4 \text{ in} - \frac{2.73 \text{ in}}{2} \\ &= 2.64 \text{ in} \end{aligned}$$

By interpolation from AISC Table 3-19, for  $Y_1 = 0$  and  $Y_2 = 2.64 \text{ in}$ ,

$$\phi M_n = 827 \text{ ft-kips}$$

### Composite Beam Design, ASD Method

In accordance with AISC Sec. I1.1a, a rectangular stress block with depth  $a$  is formed in the concrete slab at the ultimate load. The stress in the stress block is

$$f_u = 0.85 f'_c$$

The compressive force in the concrete slab is

$$\begin{aligned} C_{\text{con}} &= b a f_u \\ &= 0.85 f'_c b a \end{aligned}$$

As shown in Fig. 4.22, when the stress block depth is less than the depth of the slab, the plastic neutral axis is located at the top of the steel beam. The beam has fully yielded in tension at the yield stress  $F_y$ . The force in the steel beam is

$$T_{\text{stl}} = F_y A_s$$

Equating horizontal forces gives

$$\begin{aligned} T_{\text{stl}} &= C_{\text{con}} \\ F_y A_s &= 0.85 f'_c b a \end{aligned}$$

The depth of the stress block, when sufficient shear connectors are provided to ensure full composite action, is

$$a = \frac{F_y A_s}{0.85 f'_c b}$$

The distance from top of steel beam to the centroid of the compressive force is

$$Y_2 = Y_{\text{con}} - \frac{a}{2}$$

The moment arm between the centroids of the tensile and compressive forces is

$$y = Y_2 + \frac{d}{2}$$

The nominal flexural capacity of the fully composite section is

$$\begin{aligned} M_n &= T_{\text{stl}} y \\ &= T_{\text{stl}} \left( Y_2 + \frac{d}{2} \right) \end{aligned}$$

The allowable flexural capacity is

$$\frac{M_n}{\Omega} = \frac{M_n}{1.67}$$

For given values of  $Y_2$  and  $\sum Q_n$ , AISC Table 3-19 provides values of  $M_n/\Omega$  for a range of W sections and for the three cases

- the plastic neutral axis located in the steel beam
- the plastic neutral axis located at the top of the steel beam
- shear connectors inadequate to provide full composite action

For all three cases the depth of the stress block is

$$a = \frac{\sum Q_n}{0.85 f'_c b}$$

$\sum Q_n$  is the smaller of

- $0.85 f'_c A_c$ , plastic neutral axis is located in the steel beam
- $F_y A_s$ , plastic neutral axis is located at the top of the steel beam
- $n Q_n$ , shear connectors are inadequate to provide full composite action

Composite beams may be constructed using either of two methods. In shored construction, the steel beam is propped before casting the slab, to eliminate stresses in the steel beam caused by the deposited concrete. In unshored construction, the slab is cast without propping the steel beam. For both shored and unshored beams, the composite section is designed to support the total factored loads, coming from all dead and live loads. For unshored construction, the steel beam alone must be adequate to support all loads applied before the concrete has attained 75% of its required strength.

**Example 4.21**

A simply supported composite beam consists of a concrete slab with a depth of  $t_c = Y_{\text{con}} = 4$  in, directly supported on a W21  $\times$  57 grade 50 steel beam. The beam spacing is  $s = 9$  ft, and the span is  $L = 30$  ft. The slab consists of normal weight concrete with a strength of  $f'_c = 4000$  lbf/in<sup>2</sup>. Determine the allowable moment capacity of the beam if full composite action is provided.

*Solution*

The effective width  $b$  of the concrete slab is given by AISC Sec. I3.1 as the lesser of

$$\begin{aligned} s &= (9 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \\ &= 108 \text{ in} \\ \frac{L}{4} &= \frac{(30 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{4} \\ &= 90 \text{ in} \quad [\text{governs}] \end{aligned}$$

For full composite action with the plastic neutral axis located at the top of the steel beam

$$\begin{aligned} \sum Q_n &= F_y A_s \\ &= \left( 50 \frac{\text{kips}}{\text{in}^2} \right) (16.7 \text{ in}^2) \\ &= 835 \text{ kips} \end{aligned}$$

The distance from the top of the steel beam to the plastic neutral axis is

$$Y_1 = 0 \text{ in}$$

The depth of the stress block is

$$\begin{aligned} a &= \frac{F_y A_s}{0.85 f'_c b} \\ &= \frac{835 \text{ kips}}{(0.85) \left( 4 \frac{\text{kips}}{\text{in}^2} \right) (90 \text{ in})} \\ &= 2.73 \text{ in} \end{aligned}$$

The distance from the top of the steel beam to the line of action of the concrete slab force is

$$\begin{aligned} Y_2 &= Y_{\text{con}} - \frac{a}{2} = 4 \text{ in} - \frac{2.73 \text{ in}}{2} \\ &= 2.64 \text{ in} \end{aligned}$$

By interpolation from AISC Table 3-19, for  $Y_1 = 0$  and  $Y_2 = 2.64$  in, the allowable moment capacity is

$$M_n / \Omega = 550 \text{ ft-kips}$$

**Shear Connection Design, LRFD Method**

Shear connectors are provided to transfer the horizontal shear force across the interface. The nominal shear strengths  $Q_n$  of shear stud connectors are given in AISC Table 3-21. The required number of connectors may be uniformly distributed between the point of maximum moment and the support on either side, with the total horizontal shear being determined by the lesser value of

$$\begin{aligned} V' &= 0.85 f'_c A_c \\ V' &= F_y A_s \end{aligned}$$

To provide complete shear connection and full composite action, the required number of connectors on either side of the point of maximum moment is

$$n = \frac{V'}{Q_n}$$

If a smaller number of connectors is provided, only partial composite action can be achieved, and the nominal flexural strength of the composite member is reduced. The number of shear connectors placed between a concentrated load and the nearest support shall be sufficient to develop the moment required at the load point.

**Example 4.22**

Determine the number of  $\frac{5}{8}$  in diameter, shear stud connectors required in the composite beam of Ex. 4.20 to provide full composite action. The beam carries a uniformly distributed load.

*Solution*

The total horizontal shear resisted by the connectors is the lesser value obtained from AISC Sec. I3.2d as

$$\begin{aligned} V' &= 0.85 f'_c A_c \\ &= (0.85) \left( \frac{4000 \frac{\text{lbf}}{\text{in}^2}}{1000 \frac{\text{lbf}}{\text{kip}}} \right) (90 \text{ in}) (4 \text{ in}) \\ &= 1224 \text{ kips} \\ V' &= F_y A_s \\ &= 835 \text{ kips} \quad [\text{from Ex. 4.20; governs}] \end{aligned}$$

The nominal shear strength of a  $2\frac{1}{2}$  in long,  $\frac{5}{8}$  in diameter stud connector in normal weight 4000 lbf/in<sup>2</sup> concrete is obtained from AISC Table 3-21 as

$$Q_n = 18.1 \text{ kips}$$

For full composite action, the total number of studs required on the beam is

$$2n = \frac{2V_h}{Q_n} = \frac{(2)(835 \text{ kips})}{18.1 \text{ kips}} = 92$$

Provide 92 studs in pairs at  $7\frac{3}{4}$  in centers with end distances of  $5\frac{5}{8}$  in.

The minimum longitudinal spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 6d = (6)\left(\frac{5}{8} \text{ in}\right) \\ &= 3.75 \text{ in} \\ &< 7\frac{3}{4} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

The maximum longitudinal spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 8t = (8)(4 \text{ in}) \\ &= 32 \text{ in} \\ &< 7\frac{3}{4} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

The minimum transverse spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 4d = (4)\left(\frac{5}{8} \text{ in}\right) \\ &= 2.5 \text{ in} \end{aligned}$$

Space the connectors 4 in apart transversely.

### Shear Connection Design, ASD Method

Shear connectors are provided to transfer the horizontal shear force across the interface. The strengths,  $Q_n$ , of shear stud connectors are given in AISC Table 3-21. The required number of connectors may be uniformly distributed between the point of maximum moment and the support on either side, with the total horizontal shear being determined by the lesser value of

$$\begin{aligned} V' &= 0.85f'_c A_c \\ V' &= F_y A_s \end{aligned}$$

To provide complete shear connection and full composite action, the required number of connectors on either side of the point of maximum moment is

$$n = \frac{V'}{Q_n}$$

If a smaller number of connectors is provided, only partial composite action can be achieved, and the nominal flexural strength of the composite member is reduced. The number of shear connectors placed between a concentrated load and the nearest support must be sufficient to develop the moment required at the load point.

### Example 4.23

Determine the number of  $\frac{5}{8}$  in diameter shear stud connectors required in the composite beam of Ex. 4.21 to provide full composite action. The beam carries a uniformly distributed load.

#### Solution

The total horizontal shear resisted by the connectors is the lesser value obtained from AISC Sec. I3.2d as

$$\begin{aligned} V' &= 0.85f'_c A_c \\ &= \frac{(0.85)\left(4000 \frac{\text{lb}}{\text{in}^2}\right)(4 \text{ in})(90 \text{ in})}{1000 \frac{\text{lb}}{\text{kip}}} \\ &= 1224 \text{ kips} \\ V' &= F_y A_s \\ &= 835 \text{ kips} \quad [\text{from Ex. 4.21; governs}] \end{aligned}$$

The nominal shear strength of a  $2\frac{1}{2}$  in long,  $\frac{5}{8}$  in diameter stud connector in normal weight 4000 lbf/in<sup>2</sup> concrete is obtained from AISC Table 3-21 as

$$Q_n = 18.1 \text{ kips}$$

For full composite action, the total number of studs required on the beam is

$$2n = \frac{2V_h}{Q_n} = \frac{(2)(835 \text{ kips})}{18.1 \text{ kips}} = 92$$

Provide 92 studs in pairs at  $7\frac{3}{4}$  in centers with end distances of  $5\frac{5}{8}$  in.

The minimum longitudinal spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 6d = (6)\left(\frac{5}{8} \text{ in}\right) \\ &= 3.75 \text{ in} \\ &< 7\frac{3}{4} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

The maximum longitudinal spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 8t = (8)(4 \text{ in}) \\ &= 32 \text{ in} \\ &> 7\frac{3}{4} \text{ in} \quad [\text{satisfactory}] \end{aligned}$$

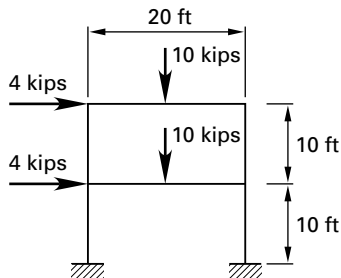
The minimum transverse spacing requirement is given by AISC Sec. I3.2d as

$$\begin{aligned} s &= 4d = (4)\left(\frac{5}{8} \text{ in}\right) \\ &= 2.5 \text{ in} \end{aligned}$$

Space the connectors 4 in apart transversely.

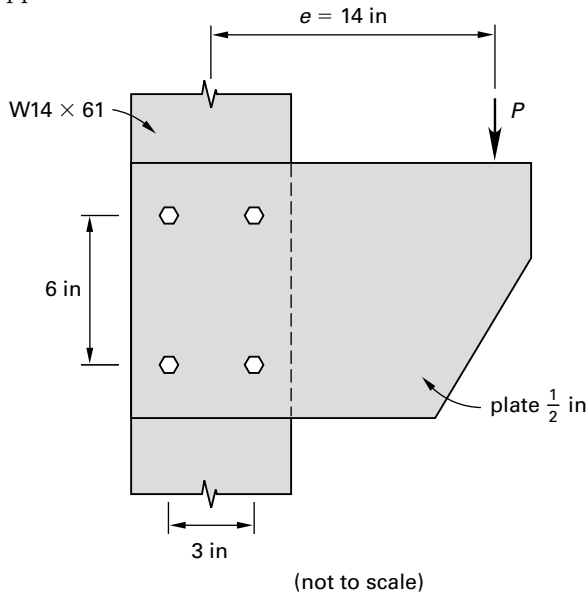
## PRACTICE PROBLEMS

1. The rigid frame shown in the illustration is fabricated from members of a uniform section in grade 50 steel. For the factored loading indicated, ignoring the member self weight and assuming adequate lateral support is provided, what is most nearly the required plastic moment of resistance?



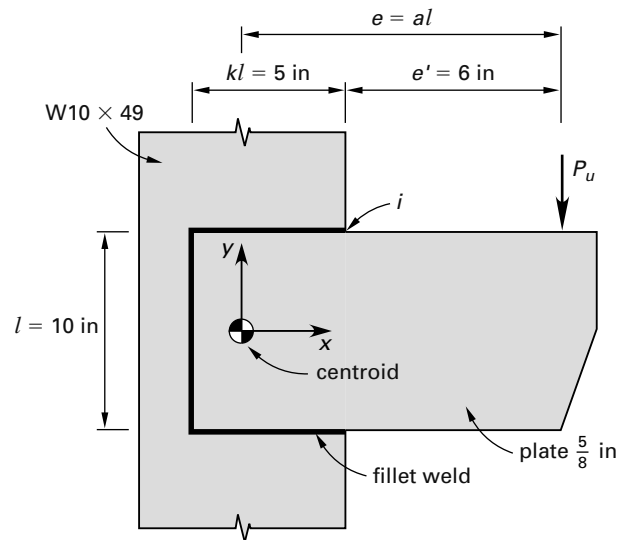
- (A) 24 ft-kips
- (B) 32 ft-kips
- (C) 40 ft-kips
- (D) 48 ft-kips

2. The bolts in the bracket shown are  $\frac{3}{4}$  in diameter A325 bearing-type with threads included in the shear plane. Using the ASD method, what is most nearly the maximum allowable load  $P$  that the bracket can support?



- (A) 9.54 kips
- (B) 10.1 kips
- (C) 11.3 kips
- (D) 12.5 kips

3. In the welded bracket shown, the size of E70XX fillet weld indicated is  $\frac{1}{4}$  in. Using the instantaneous center of rotation method, what is most nearly the maximum factored force  $P_u$  that the bracket can support?



- (A) 48 kips
- (B) 51 kips
- (C) 54 kips
- (D) 60 kips

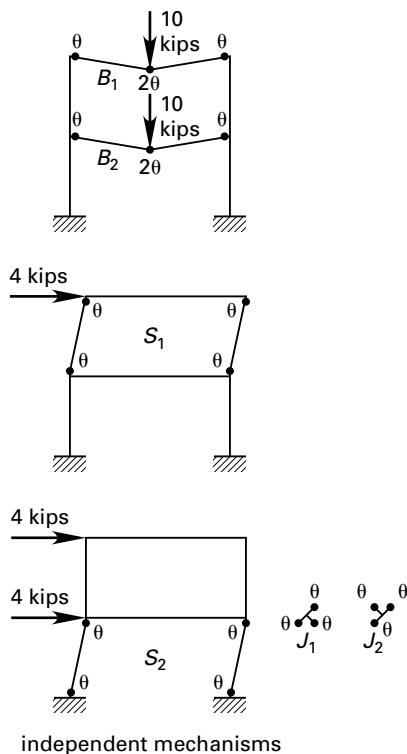
4. A simply supported composite beam consists of a concrete slab with a depth of  $t_c = Y_{con} = 4$  in, directly supported on a W21 x 57 grade 50 steel beam. The beam spacing is  $s = 9$  ft, and the span is  $L = 30$  ft. The slab consists of normal weight concrete with a strength of  $f'_c = 4000$  lbf/in<sup>2</sup>. Using the LRFD method, what is most nearly the design moment capacity of the beam if 60 number  $\frac{5}{8}$  in diameter, shear stud connectors are provided along the length of the beam?

- (A) 736 ft-kips
- (B) 749 ft-kips
- (C) 768 ft-kips
- (D) 783 ft-kips

# SOLUTIONS

1. The number of possible independent collapse mechanisms is

$$m_i = p - D = 12 - 6 = 6$$



The six possible independent collapse mechanisms shown in the illustration are  $B_1$ ,  $B_2$ ,  $S_1$ ,  $S_2$ ,  $J_1$ , and  $J_2$  mechanisms. Applying a virtual displacement to each of these mechanisms in turn, and equating internal and external work, provides the following equations.

For beam mechanism  $B_1$ ,

$$4M_p = (10 \text{ kips})(10 \text{ ft})$$

$$M_p = 25 \text{ ft-kips}$$

For beam mechanism  $B_2$ ,

$$M_p = 25 \text{ ft-kips}$$

For sway mechanism  $S_1$ ,

$$4M_p = (4 \text{ kips})(10 \text{ ft})$$

$$M_p = 10 \text{ ft-kips}$$

For sway mechanism  $S_2$ ,

$$4M_p = (8 \text{ kips})(10 \text{ ft})$$

$$M_p = 20 \text{ ft-kips}$$

For joint mechanism  $J_1$ ,

$$3M_p = 0$$

For joint mechanism  $J_2$ ,

$$3M_p = 0$$

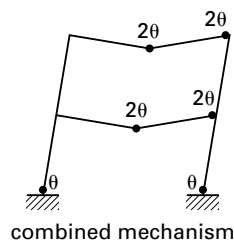
For the combined mechanism  $B_1 + B_2 + S_1 + S_2 + J_1 + J_2$  shown in the illustration,

$$10M_p = 100 \text{ ft-kips} + 100 \text{ ft-kips} + 40 \text{ ft-kips}$$

$$+ 80 \text{ ft-kips} + 0 + 0$$

$$= 320 \text{ ft-kips}$$

$$M_p = 32 \text{ ft-kips} \quad [\text{governs}]$$



The answer is (B).

2. The relevant details obtained from the illustration follow. The number of bolts in one vertical row is

$$n = 2$$

The bolt pitch is

$$b = 6 \text{ in}$$

The lateral distance between bolts is

$$D = 3 \text{ in}$$

The eccentricity is

$$e = 14 \text{ in}$$

From AISC Table 7-8, the coefficient  $C$  is

$$C = 0.90$$

Shear controls. From AISC Table 7-1, the design shear strength of a  $\frac{3}{4}$  in diameter, A325N bolt in a standard hole, in single shear with threads included in the shear plane, is

$$P_v = 10.6 \text{ kips}$$

The allowable load, based on the instantaneous center of rotation method, is

$$\begin{aligned} P &= P_v C = (10.6 \text{ kips})(0.90) \\ &= 9.54 \text{ kips} \end{aligned}$$

**The answer is (A).**

**3.** Relevant details obtained from the illustration are

$$\begin{aligned} l &= 10 \text{ in} \\ e' &= 6 \text{ in} \\ kl &= 5 \text{ in} \\ k &= 0.5 \end{aligned}$$

From AISC Table 8-8,

$$\begin{aligned} \bar{x} &= 0.125l = (0.125)(10 \text{ in}) \\ &= 1.25 \text{ in} \\ a &= \frac{e' + kl - \bar{x}}{l} \\ &= \frac{6 \text{ in} + 5 \text{ in} - 1.25 \text{ in}}{10 \text{ in}} \\ &= 0.98 \text{ in} \end{aligned}$$

From AISC Table 8-8, for values of  $a = 0.98$  and  $k = 0.5$ , the coefficient  $C$  is given as

$$C = 1.70$$

For E70XX electrodes, the correction factor for electrode strength is obtained from AISC Table 8-3 as

$$C_1 = 1.0$$

The maximum factored load that the bracket can support is

$$\begin{aligned} P_u &= \phi C C_1 l D \\ &= (0.75)(1.70)(1.0)(10 \text{ in})(4 \text{ sixteenths}) \\ &= 51 \text{ kips} \end{aligned}$$

**The answer is (B).**

**4.** The effective width  $b$  of the concrete slab, in accordance with AISC Sec. I3.1, is the lesser of

$$\begin{aligned} s &= (9 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \\ &= 108 \text{ in} \\ \frac{L}{4} &= \frac{(30 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{4} \\ &= 90 \text{ in} \quad [\text{governs}] \end{aligned}$$

If the plastic neutral axis is located at the top of the steel beam

$$\begin{aligned} \sum Q_n &= F_y A_s \\ &= \left( 50 \frac{\text{kips}}{\text{in}^2} \right) (16.7 \text{ in}^2) \\ &= 835 \text{ kips} \end{aligned}$$

The nominal shear strength of a  $2\frac{1}{2}$  in long,  $\frac{5}{8}$  in diameter shear stud connector in normal weight 4000 lbf/in<sup>2</sup> concrete is obtained from AISC Table 3-21 as

$$Q_n = 18.1 \text{ kips}$$

For 60 shear stud connectors, the shear capacity is

$$\begin{aligned} \sum Q_n &= 30 Q_n \\ &= 543 \text{ kips} \quad [\text{governs}] \end{aligned}$$

The depth of the stress block is

$$\begin{aligned} a &= \frac{\sum Q_n}{0.85 f'_c b} \\ &= \frac{543 \text{ kips}}{(0.85) \left( 4 \frac{\text{kips}}{\text{in}^2} \right) (90 \text{ in})} \\ &= 1.77 \text{ in} \end{aligned}$$

The distance from the top of the steel beam to the line of action of the concrete slab force is

$$\begin{aligned} Y_2 &= Y_{\text{con}} - \frac{a}{2} = 4 \text{ in} - \frac{1.77 \text{ in}}{2} \\ &= 3.12 \text{ in} \end{aligned}$$

Interpolate from AISC Table 3-19, for  $Y_2 = 3.12$  in and  $\sum Q_n = 543$  kips,

$$\phi M_n = 783 \text{ ft-kips}$$

**The answer is (D).**



# 5

# Design of Wood Structures

|                                     |      |
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## 1. DESIGN PRINCIPLES

### Nomenclature

|             |                                                         |                     |
|-------------|---------------------------------------------------------|---------------------|
| $b$         | breadth of rectangular bending member                   | in                  |
| $C_b$       | bearing area factor                                     | —                   |
| $C_c$       | curvature factor for structural glued laminated members | —                   |
| $C_D$       | load duration factor                                    | —                   |
| $C_F$       | size factor for sawn lumber                             | —                   |
| $C_{fu}$    | flat use factor                                         | —                   |
| $C_i$       | incising factor                                         | —                   |
| $C_L$       | beam stability factor                                   | —                   |
| $C_M$       | wet service factor                                      | —                   |
| $C_P$       | column stability factor                                 | —                   |
| $C_r$       | repetitive member factor for dimension lumber           | —                   |
| $C_t$       | temperature factor                                      | —                   |
| $C_V$       | volume factor for structural glued laminated timber     | —                   |
| $d$         | beam depth                                              | in                  |
| $E$         | reference modulus of elasticity                         | lbf/in <sup>2</sup> |
| $E'$        | adjusted modulus of elasticity                          | lbf/in <sup>2</sup> |
| $E_{\min}$  | reference modulus of elasticity for beam stability      | lbf/in <sup>2</sup> |
| $E'_{\min}$ | adjusted modulus of elasticity for beam stability       | lbf/in <sup>2</sup> |
| $F_b$       | reference bending design value                          | lbf/in <sup>2</sup> |
| $F'_b$      | adjusted bending design value                           | lbf/in <sup>2</sup> |
| $F_c$       | reference compression design value parallel to grain    | lbf/in <sup>2</sup> |
| $F'_c$      | adjusted compression design value parallel to grain     | lbf/in <sup>2</sup> |
| $l_b$       | length of bearing parallel to the grain of the wood     | in                  |
| $L$         | span length of bending member                           | ft or in            |
| $R$         | radius of curvature of inside face of laminations       | in                  |
| $t$         | thickness of laminations                                | in                  |
| $x$         | species parameter for volume factor                     | —                   |

### Glossary

The NDS Supplement (Supp.)<sup>1</sup> provides reference design values for the various species and types of wood products that are available. The types of wood products covered in the supplement are the following.

- Table 4A: visually graded dimension lumber, all species except Southern Pine
- Table 4B: visually graded Southern Pine dimension lumber
- Table 4C: mechanically graded dimension lumber
- Table 4D: visually graded timbers
- Table 4E: visually graded decking
- Table 4F: non-North American visually graded dimension lumber
- Table 5A: structural glued laminated softwood timber, stressed primarily in bending
- Table 5B: structural glued laminated softwood timber, stressed primarily in axial tension or compression
- Table 5C: structural glued laminated hardwood timber, stressed primarily in bending
- Table 5D: structural glued laminated hardwood timber, stressed primarily in axial tension or compression

A description and terminology for the available wood products follows.

**Decking.** Decking consists of solid sawn lumber or glued laminated members, 2 in to 4 in nominal thickness and 4 in or more wide. It is usually single tongue and groove, 2 in thick, and it may be double tongue and groove, 3 in or 4 in thick.

**Dimension lumber.** Dimension lumber consists of solid sawn lumber members, 2 in to 4 in nominal thickness and 2 in or more wide.

**Dressed size.** The dimensions of a lumber member after surfacing with a planing machine are usually  $\frac{1}{2}$  in to  $\frac{3}{4}$  in less than the nominal size.

**Grade.** Wood products are classified with respect to strength in accordance with specific grading rules.

**Joist.** A joist is a lumber member, 2 in to 4 in nominal thickness, and 5 in or wider. A joist is typically loaded

<sup>1</sup>American Forest and Paper Association, 2005 (See References and Codes)



**Table 5.1** Applicability of Adjustment Factors

| adjustment factor                 | sawn<br>lumber | glued<br>laminated | $F_b$ | $F_t$ | $F_v$ | $F_{c\perp}$ | $F_c$ | $E$ | $E_{min}$ |
|-----------------------------------|----------------|--------------------|-------|-------|-------|--------------|-------|-----|-----------|
| $C_F$ size                        | yes            | no                 | √     | √     | —     | —            | √     | —   | —         |
| $C_r$ repetitive member           | yes            | no                 | √     | —     | —     | —            | —     | —   | —         |
| $C_i$ incising                    | yes            | no                 | √     | √     | √     | √            | √     | √   | √         |
| $C_V$ volume <sup>a</sup>         | no             | yes                | √     | —     | —     | —            | —     | —   | —         |
| $C_c$ curvature                   | no             | yes                | √     | —     | —     | —            | —     | —   | —         |
| $C_D$ load duration               | yes            | yes                | √     | √     | √     | —            | √     | —   | —         |
| $C_M$ wet service                 | yes            | yes                | √     | √     | √     | √            | √     | √   | √         |
| $C_b$ bearing area                | yes            | yes                | —     | —     | —     | √            | —     | —   | —         |
| $C_t$ temperature                 | yes            | yes                | √     | √     | √     | √            | √     | √   | √         |
| $C_{fu}$ flat use                 | yes            | yes                | √     | —     | —     | —            | —     | —   | —         |
| $C_L$ beam stability <sup>a</sup> | yes            | yes                | √     | —     | —     | —            | —     | —   | —         |
| $C_P$ column stability            | yes            | yes                | —     | —     | —     | —            | √     | —   | —         |
| $C_T$ buckling                    | yes            | no                 | —     | —     | —     | —            | —     | —   | √         |

<sup>a</sup>When applied to glued laminated members, only the lesser value of  $C_L$  or  $C_V$  is applicable.

on the narrow face, and used as framing in floors and roofs.

**Lumber.** A lumber member is cut to size in the sawmill and surfaced in a planing machine, and not further processed.

**Mechanically graded lumber.** Mechanically graded lumber is dimension lumber that has been individually evaluated in a testing machine. Load is applied to the piece of lumber, the deflection measured, and the modulus of elasticity calculated. The strength characteristics of the lumber are directly related to the modulus of elasticity, and can be determined. A visual check is also made on the lumber to detect visible flaws.

**Nominal size.** Lumber is specified by its nominal, or undressed, size. The finished size of a member after dressing is normally  $\frac{1}{2}$  in to  $\frac{3}{4}$  in smaller than the original size. Thus, a 2 in nominal  $\times$  4 in nominal member has actual dimensions of  $1\frac{1}{2}$  in  $\times$   $3\frac{1}{2}$  in.

**Structural glued laminated timber.** Glued laminated members, or glulams, are built up from wood laminations bonded together with adhesives. The grain of all laminations is parallel to the length of the beam and the laminations are typically  $1\frac{1}{2}$  in thick.

**Timbers.** Timbers are lumber members of nominal 5 in  $\times$  5 in or larger.

**Visually stress-graded lumber.** Lumber members that have been graded visually to detect flaws and defects, and to assess the inherent strength of the member.

**Wood structural panel.** A wood structural panel is manufactured from veneers or wood strands bonded together with adhesives. Examples are plywood, oriented strand board, and composite panels.

## General Design Requirements

The allowable design values for a wood member depend on the application of the member and on the service conditions under which it is utilized. The reference design values for sawn lumber are tabulated in NDS Supp. Tables 4A–F. For glued laminated members, design values are found in NDS Supp. Tables 5A–D. These reference design values are applicable to normal conditions of service and normal load duration. To determine the relevant design values for other conditions of service, the reference design values are multiplied by adjustment factors, specified in NDS<sup>2</sup> Sec. 2.3. A summary of adjustment factors follows. The applicability of each factor to the reference design values and to sawn lumber or glued laminated timber is given in NDS Tables 4.3.1 and 5.3.1, and summarized in Table 5.1.

## Adjustment Factors Applicable Only to Sawn Lumber

### Size Factor, $C_F$

The size factor is applicable to visually graded dimension lumber, visually graded timbers, and visually graded decking. It is not applied to mechanically graded dimension lumber. The reference design values for bending, tension, and compression are multiplied by the size factor,  $C_F$ , to give the appropriate design values. For visually graded dimension lumber, 2 in to 4 in thick, values of the size factor are given in NDS Supp. Tables 4A, 4B, and 4F. For visually graded decking, values of the size factor are given in NDS Supp. Table 4E. For visually graded timbers exceeding 12 in depth and 5 in thickness, the size factor is specified in NDS Sec. 4.3.6 as

$$C_F = \left(\frac{12}{d}\right)^{1/9} \quad [\text{NDS 4.3-1}]$$

$$\leq 1.0$$

<sup>2</sup>American Forest and Paper Association, 2005 (See References and Codes)

**Repetitive Member Factor,  $C_r$** 

The repetitive member factor is applicable to visually graded dimension lumber and mechanically graded timbers. The design values for visually graded decking in NDS Supp. Table 4E already incorporate the applicable repetitive member factor. The reference design value for bending is multiplied by the repetitive member factor,  $C_r$ , when three or more sawn lumber elements, not more than 4 in thick and spaced not more than 24 in apart, are joined by a transverse load distributing element. The value of the repetitive member factor is given in NDS Supp. Tables 4A–C and 4F, and in NDS Sec. 4.3.9, as

$$C_r = 1.15$$

**Incising Factor,  $C_i$** 

Values of the incising factor,  $C_i$ , for a prescribed incising pattern are provided in NDS Sec. 4.3.8. These values are applicable to all reference design values for all sawn lumber. The prescribed incising pattern consists of incisions made parallel to the grain at a maximum depth of 0.4 in, a maximum length of  $\frac{3}{8}$  in, and at a density of 1100/ft<sup>2</sup>.

**Example 5.1**

Selected Douglas Fir-Larch, visually graded 3 × 6 decking is incised with the prescribed incising pattern. Determine the allowable bending stress. The repetitive member factor is applicable.

*Solution*

From NDS Table 4.3.8, the applicable incising factor is

$$C_i = 0.80$$

From NDS Supp. Table 4E, the applicable size factor for a 3 in thick member is

$$C_F = 1.04$$

From NDS Supp. Table 4E, the reference design value for bending, including the appropriate repetitive member factor, is

$$F_b C_r = 2000 \frac{\text{lb}}{\text{in}^2}$$

The allowable bending design value is

$$\begin{aligned} F'_b &= C_i C_F F_b C_r \\ &= (0.8)(1.04) \left( 2000 \frac{\text{lb}}{\text{in}^2} \right) \\ &= 1700 \text{ lb}/\text{in}^2 \end{aligned}$$

**Adjustment Factors Applicable Only to Glued Laminated Timber****Volume Factor,  $C_V$** 

The volume factor,  $C_V$ , is applicable to the reference design value for bending for glued laminated timber. The volume factor is defined in NDS Sec. 5.3.6 as

$$C_V = \left( \frac{1291.5}{bdL} \right)^{1/x} \quad [\text{NDS 5.3-1}]$$

$$\leq 1.0$$

$L$  is in feet, and  $b$  and  $d$  are in inches.

$$\begin{aligned} x &= 20 \quad [\text{Southern Pine}] \\ &= 10 \quad [\text{all other species}] \end{aligned}$$

The volume factor is not applied simultaneously with the beam stability factor,  $C_L$ . The lesser of these two factors is applicable.

**Curvature Factor,  $C_c$** 

The curvature factor,  $C_c$ , is applicable to the reference design value for bending for glued laminated timber. This factor accounts for residual stresses in curved, glued laminated members. The curvature factor is specified in NDS Sec. 5.3.8 as

$$C_c = 1 - (2000) \left( \frac{t}{R} \right)^2 \quad [\text{NDS 5.3-2}]$$

$$\begin{aligned} \frac{t}{R} &\leq \frac{1}{100} \quad [\text{hardwoods and Southern Pine}] \\ &\leq \frac{1}{125} \quad [\text{other softwoods}] \end{aligned}$$

The curvature factor shall not apply to design values in the straight portion of the member, regardless of curvature elsewhere.

**Example 5.2**

A glued laminated curved beam of stress class 24F-1.7E western species with 1.5 in thick laminations has a radius of curvature of 30 ft, a width of  $6\frac{3}{4}$  in, and a depth of 30 in. The beam has continuous lateral support, and is simply supported over a span of 40 ft. Determine the allowable design value in bending.

*Solution*

The basic design value for bending, tabulated in NDS Supp. Table 5A, is

$$F_b = 2400 \frac{\text{lb}}{\text{in}^2}$$

The beam is provided with continuous lateral support, giving a stability factor of

$$C_L = 1.0$$

The volume factor governs, and it is

$$\begin{aligned} C_V &= \left( \frac{1291.5}{bdL} \right)^{1/x} \quad [\text{NDS 5.3-1}] \\ &= \left( \frac{1291.5 \text{ in}^2\text{-ft}}{(6.75 \text{ in})(30 \text{ in})(40 \text{ ft})} \right)^{1/10} \\ &= 0.832 \end{aligned}$$

The curvature factor is

$$\begin{aligned} C_c &= 1 - 2000 \left( \frac{t}{R} \right)^2 \quad [\text{NDS 5.3-2}] \\ &= 1 - (2000) \left( \frac{1.5 \text{ in}}{(30 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)} \right)^2 \\ &= 0.965 \end{aligned}$$

The adjusted bending stress is

$$\begin{aligned} F'_b &= F_b C_V C_c \\ &= \left( 2400 \frac{\text{lbf}}{\text{in}^2} \right) (0.832) (0.965) \\ &= 1900 \text{ lbf/in}^2 \end{aligned}$$

### Adjustment Factors Applicable to Both Sawn Lumber and Glued Laminated Members

#### Load Duration Factor, $C_D$

Normal load duration is equivalent to applying the maximum allowable load to a member for a period of 10 years. For loads of shorter duration, a member has the capacity to sustain higher loads, and the basic design values are multiplied by the load duration factor,  $C_D$ . The load duration factor is applicable to all basic design values with the exception of compression perpendicular to the grain and modulus of elasticity. The load duration factor for the shortest duration load in a combination of loads is applicable for that load combination. Values of the load duration factor are given in NDS Table 2.3.2, and are summarized in Table 5.2.

**Table 5.2** Load Duration Factors

| design load             | load duration | $C_D$ |
|-------------------------|---------------|-------|
| dead load               | permanent     | 0.90  |
| occupancy live load     | 10 years      | 1.00  |
| snow load               | 2 months      | 1.15  |
| construction load       | 7 days        | 1.25  |
| wind or earthquake load | 10 minutes    | 1.60  |
| impact load             | impact        | 2.0   |

### Example 5.3

The axial stresses produced in a Douglas Fir-Larch, select structural,  $6 \times 14$  visually graded timber post for the applicable load combinations follow.

|                                 |                          |
|---------------------------------|--------------------------|
| dead load                       | 700 lbf/in <sup>2</sup>  |
| dead load + floor live load     | 1000 lbf/in <sup>2</sup> |
| dead load + 0.75                |                          |
| × (floor live load + snow load) | 1350 lbf/in <sup>2</sup> |

The post is fully braced in all directions. Determine whether the post is adequate.

#### Solution

The reference design value for compression parallel to the grain is obtained from NDS Supp. Table 4D as

$$F_c = 1150 \text{ lbf/in}^2$$

The load duration factor from NDS Table 2.3.2 is

$$\begin{aligned} C_D &= 0.9 \quad [\text{dead load}] \\ &= 1.0 \quad [\text{floor live load}] \\ &= 1.15 \quad [\text{snow load}] \end{aligned}$$

Applying the appropriate adjustment factors, the allowable axial stress for the dead load condition is

$$\begin{aligned} F'_c &= C_D F_c \\ &= (0.9) \left( 1150 \frac{\text{lbf}}{\text{in}^2} \right) \\ &= 1035 \frac{\text{lbf}}{\text{in}^2} \\ &> 700 \text{ lbf/in}^2 \quad [\text{satisfactory}] \end{aligned}$$

Applying the appropriate adjustment factors, the allowable axial stress for the dead load + floor live load condition is

$$\begin{aligned} F'_c &= C_D F_c \\ &= (1.0) \left( 1150 \frac{\text{lbf}}{\text{in}^2} \right) \\ &= 1150 \frac{\text{lbf}}{\text{in}^2} \\ &> 1000 \text{ lbf/in}^2 \quad [\text{satisfactory}] \end{aligned}$$

Applying the appropriate adjustment factors, the allowable axial stress for the dead load + 0.75 (floor live load + snow load) condition is

$$\begin{aligned} F'_c &= C_D F_c \\ &= (1.15) \left( 1150 \frac{\text{lbf}}{\text{in}^2} \right) \\ &= 1323 \frac{\text{lbf}}{\text{in}^2} \\ &< 1350 \text{ lbf/in}^2 \quad [\text{unsatisfactory}] \end{aligned}$$

The post is inadequate.

**Wet Service Factor,  $C_M$** 

The basic design values for sawn lumber and glued laminated timber are applicable to members that are used under dry conditions of service, irrespective of the moisture content at the time of construction. Dry conditions of service are defined as a moisture content not exceeding 19% for sawn lumber or 16% for glued laminated timber. When the moisture content of sawn lumber exceeds 19%, the basic design values are multiplied by the wet service factors,  $C_M$ , given in NDS Supp. Tables 4A–F. When the moisture content of glued laminated members exceeds 16%, the wet service factors,  $C_M$ , given in NDS Supp. Tables 5A–D are applicable.

**Bearing Area Factor  $C_b$** 

Reference compression design values perpendicular to the grain are applicable to bearings at the end of a member, and to bearings not less than 6 in long at any other location. For bearings less than 6 in long and not less than 3 in from the end of a member, the reference design values for compression perpendicular to the grain are multiplied by the bearing area factor,  $C_b$ . This is specified in NDS Sec. 3.10.4 as

$$C_b = \frac{l_b + 0.375}{l_b} \quad [\text{NDS 3.10-2}]$$

The bearing length,  $l_b$ , measured parallel to the grain is the diameter of the washer for tightened bolts and lag screws.

**Temperature Factor,  $C_t$** 

The temperature stability factor is applicable to all reference design values for members exposed to sustained temperatures exceeding 100°F. It is specified by NDS Sec. 2.3.3 and tabulated in NDS Table 2.3.3.

**Example 5.4**

A 4 × 8 visually graded, select structural, Southern Pine, sawn lumber purlin has a moisture content exceeding 19%, and is subjected to sustained temperatures between 125° and 150°F. The governing load combination is dead load plus live load plus wind load. The purlin is incised with the prescribed incising pattern. Determine the allowable design value in shear.

*Solution*

The reference design value for shear is tabulated in NDS Supp. Table 4B as

$$F_v = 175 \frac{\text{lbf}}{\text{in}^2}$$

The applicable adjustment factors for shear stress are the following. The load duration factor is

$$C_D = 1.6 \quad [\text{NDS Table 2.3.2}]$$

The wet service factor is

$$C_M = 0.97 \quad [\text{NDS Supp. Table 4B}]$$

The temperature factor for wet conditions is

$$C_t = 0.5 \quad [\text{NDS Table 2.3.3}]$$

The incising factor is

$$C_i = 0.80 \quad [\text{NDS Table 4.3.8}]$$

The adjusted shear stress is

$$\begin{aligned} F'_v &= F_v C_D C_M C_t C_i \\ &= \left(175 \frac{\text{lbf}}{\text{in}^2}\right)(1.6)(0.97)(0.50)(0.80) \\ &= 109 \text{ lbf/in}^2 \end{aligned}$$

**Flat Use Factor,  $C_{fu}$** 

As specified in NDS Sec. 4.3.7, the reference bending design value for sawn lumber members, loaded on their wide face, may be multiplied by the flat use factor,  $C_{fu}$ . The adjustment factors are given in NDS Supp. Tables 4A and 4B for visually graded dimension lumber, and in NDS Supp. Table 4C for machine graded dimension lumber. Design values for visually graded decking in NDS Supp. Table 4E already incorporate the applicable flat use factor.

As specified in NDS Sec. 5.3.7, the reference bending design value for glued laminated members loaded parallel to the wide faces of the laminations may be multiplied by the flat use factor,  $C_{fu}$ . The adjustment factors for glued laminated members are given in NDS Supp. Tables 5A–D.

**Example 5.5**

A glued laminated beam of combination 24F-V10 western species with 1.5 in thick laminations has a width of 6<sup>3</sup>/<sub>4</sub> in, and a depth of 30 in. The beam has a moisture content exceeding 16%, and is subjected to sustained temperatures between 100°F and 125°F. The governing load combination is dead load plus live load plus wind load. Determine the allowable design value for the modulus of elasticity about the  $x$ - $x$  axis.

### Solution

The reference design value for the modulus of elasticity about the  $x$ - $x$  axis is tabulated in NDS Supp. Table 5A, Expanded Combinations, as

$$E_x = 1.8 \times 10^6 \frac{\text{lb}}{\text{in}^2}$$

The applicable adjustment factors for the modulus of elasticity are  $C_M$ , for wet service conditions, and  $C_t$ , for elevated temperature in wet conditions.

$$C_M = 0.833 \quad [\text{NDS Supp. Table 5A}]$$

$$C_t = 0.9 \quad [\text{NDS Table 2.3.3}]$$

The adjusted modulus of elasticity is

$$\begin{aligned} E'_x &= E_x C_M C_t \\ &= \left(1.8 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)(0.833)(0.90) \\ &= 1.3 \times 10^6 \text{ lb}/\text{in}^2 \end{aligned}$$

## 2. DESIGN FOR FLEXURE

### Nomenclature

|             |                                                                                                                    |                    |
|-------------|--------------------------------------------------------------------------------------------------------------------|--------------------|
| $b$         | breadth of rectangular bending member                                                                              | in                 |
| $C_L$       | beam stability factor                                                                                              | —                  |
| $d$         | beam depth                                                                                                         | in                 |
| $E$         | reference modulus of elasticity                                                                                    | lb/in <sup>2</sup> |
| $E'$        | adjusted modulus of elasticity                                                                                     | lb/in <sup>2</sup> |
| $E_{\min}$  | reference modulus of elasticity for beam stability                                                                 | lb/in <sup>2</sup> |
| $E'_{\min}$ | adjusted modulus of elasticity for beam stability                                                                  | lb/in <sup>2</sup> |
| $F$         | ratio of $F_{bE}$ to $F_b^*$                                                                                       | —                  |
| $F_b$       | reference bending design value                                                                                     | lb/in <sup>2</sup> |
| $F_b^*$     | reference bending design value multiplied by all applicable adjustment factors except $C_{fu}$ , $C_V$ , and $C_L$ | lb/in <sup>2</sup> |
| $F'_b$      | adjusted bending design value                                                                                      | lb/in <sup>2</sup> |
| $F_{bE}$    | critical buckling design value for bending members                                                                 | lb/in <sup>2</sup> |
| $l_e$       | effective span length of bending member                                                                            | ft or in           |
| $l_u$       | laterally unsupported length of beam                                                                               | ft or in           |
| $L$         | span length of bending member                                                                                      | ft or in           |
| $R_B$       | slenderness ratio of bending member                                                                                | —                  |

### Beam Stability Factor, $C_L$

The beam stability factor,  $C_L$ , is applicable to the reference bending design value for sawn lumber and glued laminated members. In accordance with NDS Sec. 3.3.3,  $C_L = 1.0$  when the depth of a bending member does not exceed its breadth, or when the compression edge of a

bending member is provided with continuous lateral restraint, and the ends are restrained against rotation.

Similarly,  $C_L = 1.0$  when sawn lumber beams are supported in accordance with NDS Sec. 4.4.1, and any of the following conditions are met.

- $d/b \leq 2$
- $2 < d/b \leq 4$ , and full depth bracing is provided at the ends of the member
- $4 < d/b \leq 5$ , and the compression edge is continuously restrained
- $5 < d/b \leq 6$ , and the compression edge is continuously restrained with full depth bracing provided at intervals not exceeding 8 ft
- $6 < d/b \leq 7$ , and both edges are continuously restrained

For glued laminated members,  $C_L$  is not applied simultaneously with the volume factor,  $C_V$ , and the lesser of these two factors is applicable. The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}} \quad [\text{NDS 3.3-6}]$$

$$F_b^* = F_b C_D C_M C_t C_F C_r C_c$$

$$F_{bE} = \frac{1.20 E'_{\min}}{R_B^2}$$

The adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M C_t$$

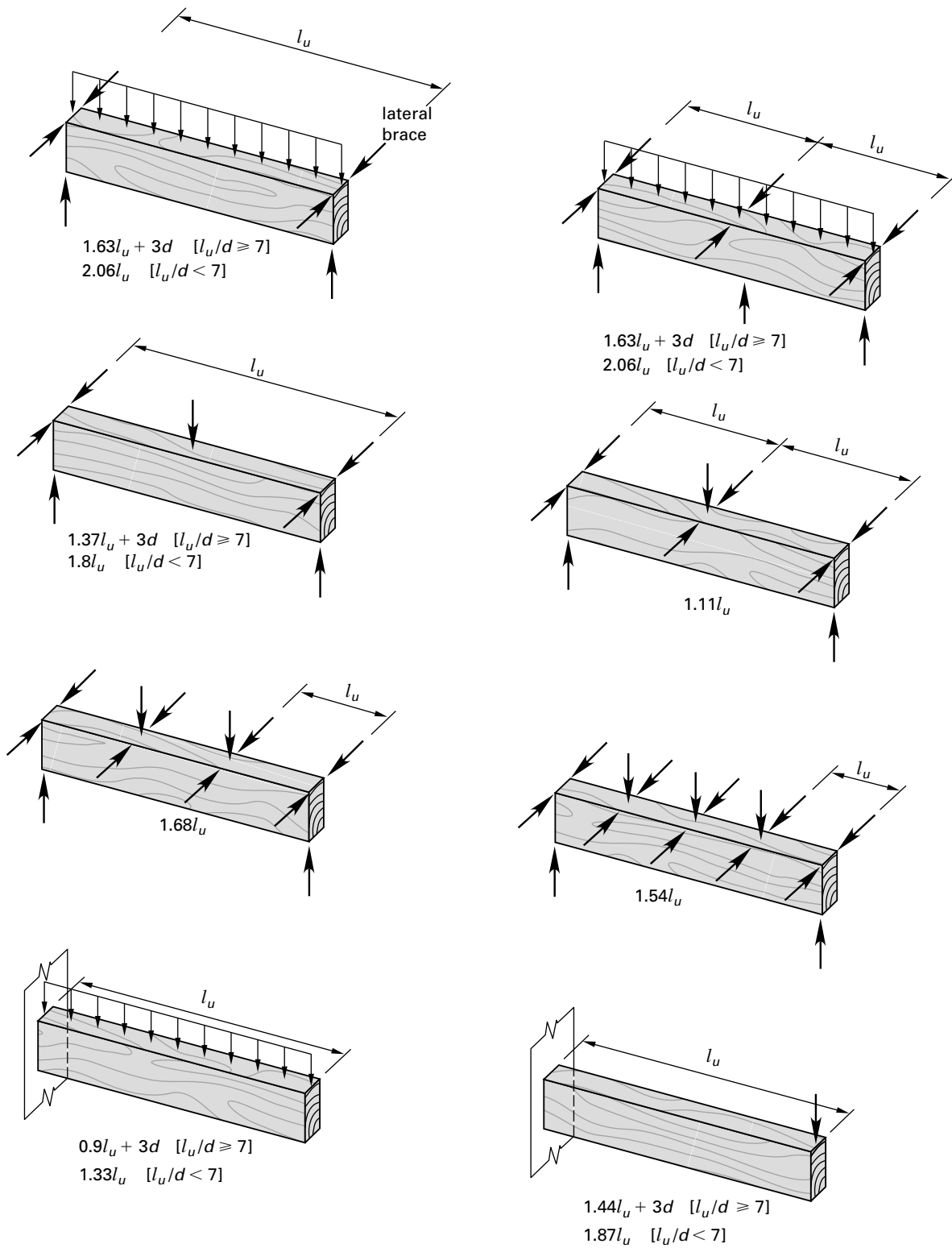
The slenderness ratio is

$$\begin{aligned} R_B &= \sqrt{\frac{l_e d}{b^2}} \quad [\text{NDS 3.3-5}] \\ &\leq 50 \end{aligned}$$

The effective span length,  $l_e$ , is determined in accordance with NDS Table 3.3.3. The value of  $l_e$  depends on the load configuration and the distance between lateral restraints,  $l_u$ . Typical values for  $l_e$  are given in Fig. 5.1.

### Flexural Design of Sawn Lumber

When designing sawn lumber members for bending, both the size factor,  $C_F$ , and the stability factor,  $C_L$ , must be applied. The design span is defined in NDS Sec. 3.2.1 as clear span plus half the bearing length at each end.

Figure 5.1 Typical Values of Effective Length,  $l_e$ 

**Example 5.6**

A visually graded, no. 1 and btr., Douglas Fir-Larch  $4 \times 12$  joist is simply supported over a span of 20 ft. The governing load combination is a uniformly distributed dead load plus live load plus snow load, and the joist is laterally braced at midspan. Determine the allowable design value in bending.

*Solution*

The reference design values for bending and the modulus of elasticity for beam stability calculations, tabulated in NDS Supp. Table 4A, are

$$F_b = 1200 \frac{\text{lb}}{\text{in}^2}$$

$$E_{\min} = 0.66 \times 10^6 \frac{\text{lb}}{\text{in}^2}$$

The applicable adjustment factors for modulus of elasticity are the following. The wet service factor is

$$C_M = 1.0 \quad [\text{NDS Supp. Table 4A}]$$

The temperature factor is

$$C_t = 1.0 \quad [\text{NDS Table 2.3.3}]$$

The incising factor is

$$C_i = 1.0 \quad [\text{NDS Table 4.3.8}]$$

The adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M C_t C_i$$

$$= \left(0.66 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right) (1.0)(1.0)(1.0)$$

$$= 0.66 \times 10^6 \text{ lb}/\text{in}^2$$

The distance between lateral restraints is

$$l_u = \frac{20 \text{ ft}}{2}$$

$$= 10 \text{ ft}$$

$$\frac{l_u}{d} = \frac{(10 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{11.25 \text{ in}}$$

$$= 10.7$$

$$> 7$$

For a uniformly distributed load and an  $l_u/d$  ratio greater than 7, the effective length is obtained from Fig. 5.1 as

$$l_e = 1.63l_u + 3d$$

$$= (1.63)(10 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right) + (3)(11.25 \text{ in})$$

$$= 229.4 \text{ in}$$

The slenderness ratio is given by NDS Sec. 3.3.3 as

$$R_B = \sqrt{\frac{l_e d}{b^2}} \quad [\text{NDS 3.3-5}]$$

$$= \sqrt{\frac{(229.4 \text{ in})(11.25 \text{ in})}{(3.5 \text{ in})^2}}$$

$$= 14.51$$

$$< 50 \quad [\text{satisfies NDS Sec. 3.3.3}]$$

The critical buckling design value is

$$F_{bE} = \frac{1.20E'_{\min}}{R_B^2}$$

$$= \frac{(1.20) \left(0.66 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)}{(14.51)^2}$$

$$= 3762 \text{ lb}/\text{in}^2$$

The applicable adjustment factors for flexure are  $C_F$ , the size factor for a  $4 \times 12$  member, and  $C_D$ , the load duration factor for snow load.

$$C_F = 1.1 \quad [\text{NDS Supp. Table 4A}]$$

$$C_D = 1.15 \quad [\text{NDS Table 2.3.2}]$$

The reference flexural design value, multiplied by all applicable adjustment factors except  $C_L$ , is

$$F_b^* = F_b C_F C_D$$

$$= \left(1200 \frac{\text{lb}}{\text{in}^2}\right) (1.1)(1.15)$$

$$= 1518 \text{ lb}/\text{in}^2$$

The ratio of  $F_{bE}$  to  $F_b^*$  is

$$F = \frac{F_{bE}}{F_b^*}$$

$$= \frac{3762 \frac{\text{lb}}{\text{in}^2}}{1518 \frac{\text{lb}}{\text{in}^2}}$$

$$= 2.48$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 2.48}{1.9} - \sqrt{\left(\frac{1.0 + 2.48}{1.9}\right)^2 - \frac{2.48}{0.95}}$$

$$= 0.969$$



The allowable flexural design value is

$$\begin{aligned} F_b &= C_L C_F C_D F_b^* \\ &= C_L F_b^* \\ &= (0.969) \left( 1518 \frac{\text{lb}}{\text{in}^2} \right) \\ &= 1500 \text{ lb}/\text{in}^2 \end{aligned}$$

### Flexural Design of Glued Laminated Timber Members

When designing glued laminated timber members for bending, both the volume factor,  $C_V$ , and the stability factor,  $C_L$ , must be determined. However, only the lesser of these two factors is applied in determining the allowable design value in bending.

#### Example 5.7

A glued laminated beam of combination 24F-V10 western species with 1.5 in thick laminations has a width of  $6\frac{3}{4}$  in, and a depth of 30 in. Tension laminations are provided as specified in NDS Table 5A, Expanded Combinations, Footnote 2. The beam is simply supported over a span of 40 ft, and is laterally braced at midspan. The governing load combination is a uniformly distributed dead load plus live load plus snow load. Determine the allowable design value in bending.

#### Solution

The reference design values for bending and the modulus of elasticity, tabulated in NDS Supp. Table 5A, Expanded Combinations, are

$$\begin{aligned} F_b &= 2400 \text{ lb}/\text{in}^2 \\ E_{x,\min} &= 0.93 \times 10^6 \text{ lb}/\text{in}^2 \\ E_{y,\min} &= 0.78 \times 10^6 \text{ lb}/\text{in}^2 \end{aligned}$$

The applicable adjustment factors for the modulus of elasticity are  $C_M$ , the wet service factor, and  $C_t$ , the temperature factor.

$$\begin{aligned} C_M &= 1.0 \quad [\text{NDS Supp. Table 5A}] \\ C_t &= 1.0 \quad [\text{NDS Table 2.3.3}] \end{aligned}$$

The adjusted modulus of elasticity is

$$\begin{aligned} E'_{\min} &= E'_{y,\min} \\ &= E_{y,\min} C_M C_t \\ &= \left( 0.78 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right) (1.0) (1.0) \\ &= 0.78 \times 10^6 \text{ lb}/\text{in}^2 \end{aligned}$$

The distance between lateral restraints is

$$\begin{aligned} l_u &= \frac{40 \text{ ft}}{2} \\ &= 20 \text{ ft} \\ \frac{l_u}{d} &= \frac{(20 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{30 \text{ in}} \\ &= 8.0 \\ &> 7 \end{aligned}$$

For a uniformly distributed load and an  $l_u/d$  ratio greater than 7, the effective length is obtained from Fig. 5.1 as

$$\begin{aligned} l_e &= 1.63 l_u + 3d \\ &= (1.63)(20 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) + (3)(30 \text{ in}) \\ &= 481 \text{ in} \end{aligned}$$

The slenderness ratio is given by NDS Sec. 3.3.3 as

$$\begin{aligned} R_B &= \sqrt{\frac{l_e d}{b^2}} \\ &= \sqrt{\frac{(481 \text{ in})(30 \text{ in})}{(6.75 \text{ in})^2}} \\ &= 17.80 \\ &< 50 \quad [\text{satisfies NDS Sec. 3.3.3}] \end{aligned}$$

The critical buckling design value is

$$\begin{aligned} F_{bE} &= \frac{1.20 E'_{\min}}{R_B^2} \\ &= \frac{(1.20) \left( 0.78 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right)}{(17.80)^2} \\ &= 2954 \text{ lb}/\text{in}^2 \end{aligned}$$

The reference flexural design value, multiplied by all applicable adjustment factors except  $C_L$  and  $C_V$ , is

$$\begin{aligned} F_b^* &= F_b C_D \\ &= \left( 2400 \frac{\text{lb}}{\text{in}^2} \right) (1.15) \\ &= 2760 \text{ lb}/\text{in}^2 \end{aligned}$$

The ratio of  $F_{bE}$  to  $F_b^*$  is

$$\begin{aligned} F &= \frac{F_{bE}}{F_b^*} \\ &= \frac{2954 \frac{\text{lb}}{\text{in}^2}}{2760 \frac{\text{lb}}{\text{in}^2}} \\ &= 1.07 \end{aligned}$$



The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 1.07}{1.9} - \sqrt{\left(\frac{1.0 + 1.07}{1.9}\right)^2 - \frac{1.07}{0.95}}$$

$$= 0.843$$

From Ex. 5.2, the volume factor is

$$C_V = 0.832$$

$$< C_L$$

The volume factor governs.

The allowable flexural design value is

$$F'_b = C_V C_D F_b$$

$$= (0.832)(1.15) \left( 2400 \frac{\text{lb}}{\text{in}^2} \right)$$

$$= 2300 \text{ lb/in}^2$$

### 3. DESIGN FOR COMPRESSION

#### Nomenclature

|          |                                                                                                                     |                         |
|----------|---------------------------------------------------------------------------------------------------------------------|-------------------------|
| $A$      | area of cross section                                                                                               | $\text{in}^2$           |
| $c$      | column parameter                                                                                                    | —                       |
| $C_{m1}$ | moment magnification factor for biaxial bending and axial compression, $1.0 - f_c/F_{cE1}$                          | —                       |
| $C_{m2}$ | moment magnification factor for biaxial bending and axial compression, $1.0 - f_c/F_{cE2} - (f_{b1}/F_{bE})^2$      | —                       |
| $C_{m3}$ | moment magnification factor for axial compression and flexure with load applied to narrow face, $1.0 - f_c/F_{cE1}$ | —                       |
| $C_{m4}$ | moment magnification factor for axial compression and flexure with load applied to wide face, $1.0 - f_c/F_{cE2}$   | —                       |
| $C_{m5}$ | moment magnification factor for biaxial bending, $1.0 - (f_{b1}/F_{bE})^2$                                          | —                       |
| $C_P$    | column stability factor                                                                                             | —                       |
| $d$      | least dimension of rectangular compression member                                                                   | in                      |
| $d_1$    | dimension of wide face                                                                                              | in                      |
| $d_2$    | dimension of narrow face                                                                                            | in                      |
| $E$      | tabulated modulus of elasticity                                                                                     | $\text{lb}/\text{in}^2$ |
| $E'$     | allowable modulus of elasticity                                                                                     | $\text{lb}/\text{in}^2$ |
| $F'$     | ratio of $F_{cE}$ to $F_c^*$                                                                                        | —                       |

|              |                                                                                                                                              |                         |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| $F_{bE}$     | critical buckling design value for bending member, $1.20E'_{\min}/R_B^2$                                                                     | $\text{lb}/\text{in}^2$ |
| $F'_{b1}$    | allowable bending design value for load applied to the narrow face including adjustment for the slenderness ratio                            | $\text{lb}/\text{in}^2$ |
| $F'_{b2}$    | allowable bending design value for load applied to the wide face including adjustment for the slenderness ratio                              | $\text{lb}/\text{in}^2$ |
| $F_c^*$      | reference compressive design value multiplied by all applicable adjustment factors except $C_P$                                              | $\text{lb}/\text{in}^2$ |
| $F'_c$       | allowable compression design value including adjustment for the largest slenderness ratio                                                    | $\text{lb}/\text{in}^2$ |
| $F_{cE}$     | critical buckling design value                                                                                                               | $\text{lb}/\text{in}^2$ |
| $F_{cE1}$    | critical buckling design value in plane of bending for load applied to the narrow face, $0.822E'_{\min}/(l_{e1}/d_1)^2$                      | $\text{lb}/\text{in}^2$ |
| $F_{cE2}$    | critical buckling design value in plane of bending for load applied to the wide face, $0.822E'_{\min}/(l_{e2}/d_2)^2$                        | $\text{lb}/\text{in}^2$ |
| $f_{b1}$     | actual edgewise bending stress for load applied to the narrow face                                                                           | $\text{lb}/\text{in}^2$ |
| $f_{b2}$     | actual flatwise bending stress for load applied to the wide face                                                                             | $\text{lb}/\text{in}^2$ |
| $f_c$        | actual compression stress parallel to grain                                                                                                  | $\text{lb}/\text{in}^2$ |
| $K_e$        | buckling length coefficient for compression members                                                                                          | —                       |
| $L$          | span                                                                                                                                         | ft or in                |
| $l_1$        | distance between points of lateral support restraining buckling about the strong axis of compression member                                  | ft or in                |
| $l_2$        | distance between points of lateral support restraining buckling about the weak axis of compression member                                    | ft or in                |
| $l_e$        | effective length of compression member                                                                                                       | ft or in                |
| $l_{e1}$     | effective length between supports restraining buckling in plane of bending from load applied to narrow face of compression member, $K_e l_1$ | ft or in                |
| $l_{e2}$     | effective length between supports restraining buckling in plane of bending from load applied to wide face of compression member, $K_e l_2$   | ft or in                |
| $l_{e1}/d_1$ | slenderness ratio about the strong axis of compression member                                                                                | —                       |
| $l_{e2}/d_2$ | slenderness ratio about the weak axis of compression member                                                                                  | —                       |
| $P$          | total concentrated load or total axial load                                                                                                  | $\text{lb}$ or kips     |
| $S$          | section modulus                                                                                                                              | $\text{in}^3$           |
| $W$          | applied load                                                                                                                                 | $\text{lb}$             |

### Column Stability Factor, $C_P$

The column stability factor is applicable to the reference compression design values parallel to the grain for sawn lumber and glued laminated members. In accordance with NDS Sec. 3.7.1,  $C_L = 1.0$  when a compression member is provided with continuous lateral restraint in all directions. For other conditions, the column stability factor is specified by NDS Sec. 3.7.1 as

$$C_P = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}} \quad [\text{NDS 3.7-1}]$$

The reference compression design value, multiplied by all applicable adjustment factors except  $C_P$ , is

$$F_c^* = F_c C_D C_M C_t C_i C_F$$

The critical buckling design value is

$$F_{cE} = \frac{0.822 E'_{\min}}{\left(\frac{l_e}{d}\right)^2}$$

The adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M C_t C_i$$

The column parameter is

$$\begin{aligned} c &= 0.8 \quad [\text{sawn lumber}] \\ &= 0.9 \quad [\text{glue laminated timber}] \end{aligned}$$

### Axial Load Only

The effective length of a column is defined in NDS App. G as the length of the column that is assumed to buckle in the shape of a sine wave. The effective length is dependent on the lateral translation and fixity at the ends of the column. For various standard column types, the effective length may be determined by multiplying the distance between lateral supports by a buckling length coefficient. Values of the buckling length coefficient,  $K_e$ , for various restraint conditions are given in NDS App. G Table G1, and summarized in Fig. 5.2. The effective length is specified in NDS Sec. 3.7.1.2 as

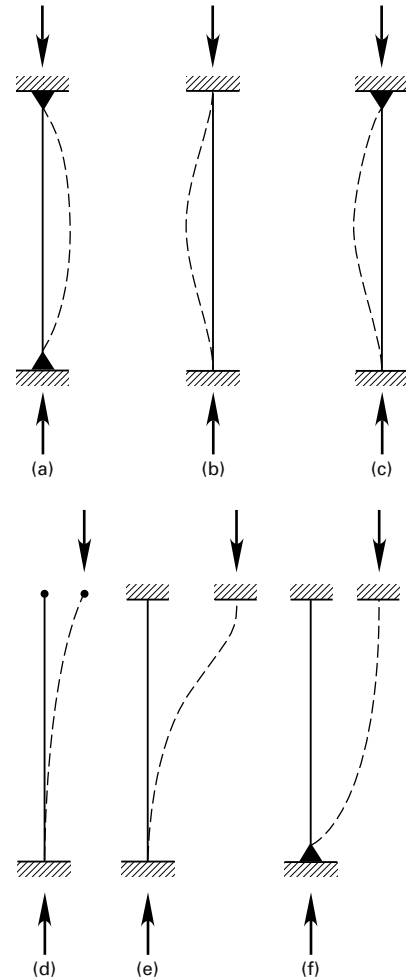
$$l_e = K_e l$$

The slenderness ratio is defined in NDS Sec. 3.7.1.3 as

$$\frac{l_e}{d} \leq 50$$

Figure 5.2 Buckling Length Coefficients

| illus. | end conditions                                                         | $K_e$       |        |
|--------|------------------------------------------------------------------------|-------------|--------|
|        |                                                                        | theoretical | design |
| (a)    | both ends pinned                                                       | 1           | 1.00   |
| (b)    | both ends built in                                                     | 0.5         | 0.65   |
| (c)    | one end pinned, one end built in                                       | 0.7         | 0.8    |
| (d)    | one end built in, one end free                                         | 2           | 2.10   |
| (e)    | one end built in, one end fixed against rotation but free to translate | 1           | 1.20   |
| (f)    | one end pinned, one end fixed against rotation but free to translate   | 2           | 2.40   |

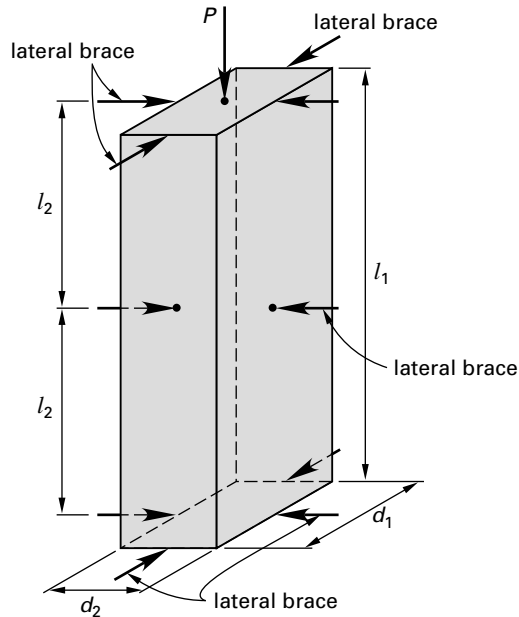


As shown in Fig. 5.3, where the distances between lateral supports about the  $x$ - $x$  axis and the  $y$ - $y$  axis are different, two values of the slenderness ratio are obtained,  $l_{e1}/d_1$  and  $l_{e2}/d_2$ , and the larger of these governs.

The applicable allowable compression design value,  $F'_c$ , is governed by the maximum slenderness ratio of the column. The maximum allowable axial load on a column is

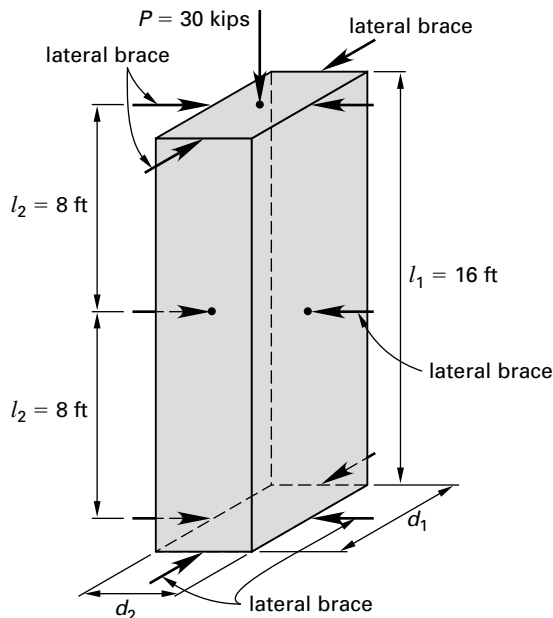
$$P_{al} = A F'_c$$

Figure 5.3 Slenderness Ratios for a Rectangular Column



### Example 5.8

A Douglas Fir-Larch, select structural,  $6 \times 14$  visually graded timber post supports an axial load of 30 kips, as shown in the illustration. The post is 16 ft long, and is pinned at each end. The governing load combination consists of dead load plus live load plus snow load, and the moisture content exceeds 19%. The column is laterally braced at midheight about the weak axis, and the self-weight of the column and bracing members may be neglected. Determine whether the member is adequate.



### Solution

The reference design values for compression parallel to the grain are obtained from NDS Supp. Table 4D as

$$F_c = 1150 \frac{\text{lb}}{\text{in}^2}$$

$$E_{\min} = 0.58 \times 10^6 \text{ lb}/\text{in}^2$$

The size factor is obtained from NDS Supp. Table 4D as

$$C_F = 1.00 \quad [\text{for } F_c]$$

$$= 1.00 \quad [\text{for } E]$$

The load duration factor is obtained from NDS Table 2.3.2 as

$$C_D = 1.15 \quad [\text{snow load}]$$

The wet service factor is obtained from NDS Supp. Table 4D as

$$C_M = 0.91 \quad [\text{for } F_c]$$

$$= 1.00 \quad [\text{for } E]$$

For pinned ended support conditions, the buckling length coefficient is obtained from NDS App. G Table G1 as

$$K_e = 1.0$$

The slenderness ratio about the strong axis is

$$\frac{K_e l_1}{d_1} = \frac{(1.0)(16 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{13.5 \text{ in}} = 14.22$$

The slenderness ratio about the weak axis is

$$\frac{K_e l_2}{d_2} = \frac{(1.0)(8 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}} = 17.45 \quad [\text{governs}]$$

The adjusted modulus of elasticity for stability calculations is

$$\begin{aligned} E'_{\min} &= E_{\min} C_M C_F \\ &= \left(0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)(1.0)(1.0) \\ &= 0.58 \times 10^6 \text{ lb}/\text{in}^2 \end{aligned}$$

The reference compression design value, multiplied by all applicable adjustment factors except  $C_P$ , is

$$\begin{aligned} F_c^* &= F_c C_M C_F C_D \\ &= \left(1150 \frac{\text{lb}}{\text{in}^2}\right)(0.91)(1.00)(1.15) \\ &= 1203 \text{ lb}/\text{in}^2 \end{aligned}$$

The critical buckling design value is

$$\begin{aligned} F_{cE2} &= \frac{0.822 E'_{\min}}{\left(\frac{l_{e2}}{d_2}\right)^2} \\ &= \frac{(0.822) \left(0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)}{(17.45)^2} \\ &= 1566 \text{ lb}/\text{in}^2 \end{aligned}$$

The ratio of  $F_{cE2}$  to  $F_c^*$  is

$$\begin{aligned} F' &= \frac{F_{cE2}}{F_c^*} \\ &= \frac{1566 \frac{\text{lb}}{\text{in}^2}}{1203 \frac{\text{lb}}{\text{in}^2}} \\ &= 1.30 \end{aligned}$$

The column parameter is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8 \quad [\text{sawn lumber}]$$

The column stability factor is specified by NDS Sec. 3.7.1.5 as

$$\begin{aligned} C_P &= \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}} \\ &= \frac{1.0 + 1.30}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 1.30}{(2)(0.8)}\right)^2 - \frac{1.30}{0.8}} \\ &= 0.77 \end{aligned}$$

The allowable compression design value parallel to the grain is

$$\begin{aligned} F'_c &= F_c C_M C_F C_D C_P \\ &= F_c^* C_P \\ &= \left(1203 \frac{\text{lb}}{\text{in}^2}\right)(0.77) \\ &= 926 \text{ lb}/\text{in}^2 \end{aligned}$$

The actual compressive stress on the column is

$$\begin{aligned} f_c &= \frac{P}{A} \\ &= \frac{30,000 \text{ lb}}{74.25 \text{ in}^2} \\ &= 404 \text{ lb}/\text{in}^2 \\ &< F'_c \end{aligned}$$

The column is adequate.

### Combined Axial Compression and Flexure

Members subjected to combined compression and flexural stresses from axial and transverse loading must satisfy the interaction equation given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m1}} + \frac{f_{b2}}{F'_{b2} C_{m2}} \leq 1.00 \quad [\text{NDS 3.9-3}]$$

For bending load applied to the narrow face of the member and concentric axial compression load, the interaction equation reduces to

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m3}} \leq 1.00$$

For bending load applied to the wide face of the member and concentric axial compression load, the equation reduces to

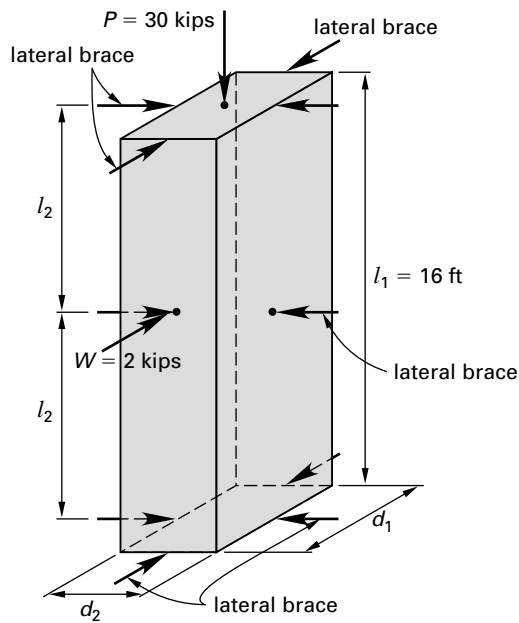
$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b2}}{F'_{b2} C_{m4}} \leq 1.00$$

For bending loads applied to the narrow and wide faces of the member and no concentric axial load, the equation reduces to

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2} C_{m5}} \leq 1.00$$

### Example 5.9

A Douglas Fir-Larch, select structural,  $6 \times 14$  visually graded timber post supports an axial load of 30 kips. A lateral load of 2 kips is applied to the narrow face of the column, as shown in the illustration. The post is 16 ft long, pinned at each end. The governing load combination consists of dead load plus live load plus snow load, and the moisture content exceeds 19%. The column is laterally braced at midheight about the weak axis, and the self-weight of the column and bracing members may be neglected. Determine whether the member is adequate.



### Solution

The relevant details from Ex. 5.8 are the following.

$$F_c = 1150 \frac{\text{lbf}}{\text{in}^2}$$

$$E_{\min} = 0.58 \times 10^6 \text{ lbf/in}^2$$

The size factor for axial load and modulus of elasticity is

$$C_F = 1.00$$

The load duration factor is

$$C_D = 1.15 \quad [\text{snow load}]$$

The wet service factor is

$$C_M = 0.91 \quad [\text{for } F_c]$$

$$= 1.00 \quad [\text{for } E]$$

The buckling length coefficient is

$$K_e = 1.0$$

The slenderness ratio about the strong axis is

$$\frac{K_e l_1}{d_1} = 14.22$$

The adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = 0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2}$$

$$F'_c = 926 \frac{\text{lbf}}{\text{in}^2}$$

$$f_c = 404 \frac{\text{lbf}}{\text{in}^2}$$

The reference design value for bending is obtained from NDS Supp. Table 4D as

$$F_b = 1500 \frac{\text{lbf}}{\text{in}^2}$$

The distance between lateral restraints for bending instability is

$$l_u = \frac{16 \text{ ft}}{2}$$

$$= 8 \text{ ft}$$

For a concentrated load at midspan and with lateral restraint at midspan, the effective length is obtained from Fig. 5.1 as

$$l_e = 1.11 l_u$$

$$= (1.11)(8 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)$$

$$= 106.56 \text{ in}$$

The slenderness ratio for bending is given by NDS Sec. 3.3.3 as

$$R_B = \sqrt{\frac{l_e d_1}{d_2^2}}$$

$$= \sqrt{\frac{(106.56 \text{ in})(13.5 \text{ in})}{(5.5 \text{ in})^2}}$$

$$= 6.90$$

$$< 50 \quad [\text{satisfies NDS Sec. 3.3.3}]$$

The critical buckling design value is

$$F_{bE} = \frac{1.20 E'_{\min}}{R_B^2}$$

$$= \frac{(1.20) \left( 0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \right)}{(6.90)^2}$$

$$= 14,619 \text{ lbf/in}^2$$

The applicable wet service factor for flexure, obtained from NDS Supp. Table 4D, is

$$C_M = 1.00$$

The size factor for flexure is obtained from NDS Supp. Table 4D as

$$C_F = \left( \frac{12}{d} \right)^{1/9}$$

$$= \left( \frac{12}{13.5 \text{ in}} \right)^{1/9}$$

$$= 0.99$$

The basic flexural design value, multiplied by all applicable adjustment factors except  $C_L$ , is

$$\begin{aligned} F_b^* &= F_b C_M C_F C_D \\ &= \left(1500 \frac{\text{lb}}{\text{in}^2}\right)(1.00)(0.99)(1.15) \\ &= 1708 \text{ lb}/\text{in}^2 \end{aligned}$$

The ratio of  $F_{bE}$  to  $F_b^*$  is

$$\begin{aligned} F &= \frac{F_{bE}}{F_b^*} \\ &= \frac{14,619 \frac{\text{lb}}{\text{in}^2}}{1708 \frac{\text{lb}}{\text{in}^2}} \\ &= 8.56 \end{aligned}$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$\begin{aligned} C_L &= \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}} \\ &= \frac{1.0 + 8.56}{1.9} - \sqrt{\left(\frac{1.0 + 8.56}{1.9}\right)^2 - \frac{8.56}{0.95}} \\ &= 0.99 \end{aligned}$$

The allowable flexural design value for the load applied to the narrow face is

$$\begin{aligned} F'_{b1} &= F_b C_M C_F C_D C_L \\ &= \left(1708 \frac{\text{lb}}{\text{in}^2}\right)(0.99) \\ &= 1690 \text{ lb}/\text{in}^2 \end{aligned}$$

The actual edgewise bending stress is

$$\begin{aligned} f_{b1} &= \frac{WL}{4S} \\ &= \frac{(2000 \text{ lb})(16 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(4)(167 \text{ in}^3)} \\ &= 575 \text{ lb}/\text{in}^2 \end{aligned}$$

The critical buckling design value in the plane of bending for load applied to the narrow face is

$$\begin{aligned} F_{cE1} &= \frac{0.822 E'_{\min}}{\left(\frac{l_{e1}}{d_1}\right)^2} \\ &= \frac{(0.822)\left(0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)}{(14.22)^2} \\ &= 2358 \text{ lb}/\text{in}^2 \end{aligned}$$

The moment magnification factor for axial compression and flexure with load applied to the narrow face is

$$\begin{aligned} C_{m3} &= 1.0 - \frac{f_c}{F_{cE1}} \\ &= 1.0 - \frac{404 \frac{\text{lb}}{\text{in}^2}}{2358 \frac{\text{lb}}{\text{in}^2}} \\ &= 0.83 \end{aligned}$$

The interaction equation for bending load applied to the narrow face of the member and concentric axial compression load is given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m3}} \leq 1.0$$

The left-hand side of the expression is

$$\begin{aligned} \left(\frac{404 \frac{\text{lb}}{\text{in}^2}}{926 \frac{\text{lb}}{\text{in}^2}}\right)^2 + \frac{575 \frac{\text{lb}}{\text{in}^2}}{\left(1690 \frac{\text{lb}}{\text{in}^2}\right)(0.83)} &= 0.190 + 0.410 \\ &= 0.60 \\ &< 1.0 \quad [\text{satisfactory}] \end{aligned}$$

The post is adequate.

#### 4. DESIGN FOR TENSION

##### Nomenclature

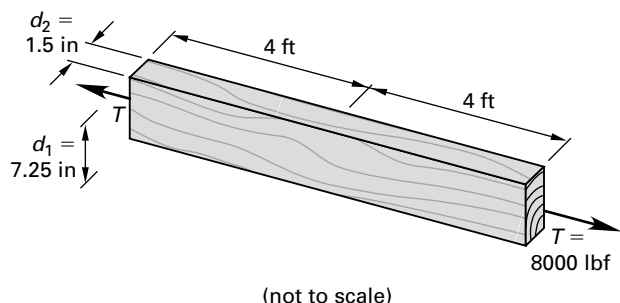
|            |                                                                                             |                    |
|------------|---------------------------------------------------------------------------------------------|--------------------|
| $A$        | area of cross section                                                                       | —                  |
| $C_D$      | load duration factor                                                                        | —                  |
| $C_F$      | size factor for sawn lumber                                                                 | —                  |
| $C_i$      | incising factor                                                                             | —                  |
| $C_m$      | wet service factor                                                                          | —                  |
| $C_t$      | temperature factor                                                                          | —                  |
| $E$        | reference modulus of elasticity                                                             | lb/in <sup>2</sup> |
| $E'$       | allowable modulus of elasticity                                                             | lb/in <sup>2</sup> |
| $f_t$      | actual tension stress parallel to grain                                                     | lb/in <sup>2</sup> |
| $F$        | ratio of $F_{bE}$ to $F_b^*$                                                                | —                  |
| $F_b^*$    | reference bending design value multiplied by all applicable adjustment factors except $C_L$ | lb/in <sup>2</sup> |
| $F_b^{**}$ | reference bending design value multiplied by all applicable adjustment factors except $C_V$ | lb/in <sup>2</sup> |
| $F_{bE}$   | critical buckling design value for bending members                                          | —                  |
| $F_t$      | reference tension design value parallel to grain                                            | lb/in <sup>2</sup> |
| $F'_t$     | allowable tension design value parallel to grain                                            | lb/in <sup>2</sup> |
| $l_e$      | effective length of compression member                                                      | ft                 |
| $l_u$      | laterally unsupported length of beam                                                        | ft                 |
| $R_B$      | slenderness ratio of bending member                                                         | —                  |
| $T$        | tensile force on member                                                                     | lb                 |

## Axial Tension

The reference design values for tension parallel to the grain are tabulated in the NDS Supplements. Allowable design values are obtained by multiplying basic values by the applicable adjustment factors. As specified in NDS Sec. 3.8.2, tension perpendicular to the grain is to be avoided.

### Example 5.10

The select structural  $2 \times 8$  Douglas Fir-Larch bottom chord of a truss is axially loaded in tension as shown in the illustration. The governing load combination consists of dead load plus live load plus snow load, and the moisture content exceeds 19%. The self-weight of the chord may be neglected. At the end connections, the net area is  $A_{\text{net}} = 9.3 \text{ in}^2$ . Determine whether the member is adequate.



#### Solution

The reference design value for tension, tabulated in NDS Supp. Table 4A, is

$$F_t = 1000 \frac{\text{lbf}}{\text{in}^2}$$

The size factor for tensile load is obtained from NDS Supp. Table 4A as

$$C_F = 1.20$$

The load duration factor is obtained from NDS Table 2.3.2 as

$$C_D = 1.5 \quad [\text{snow load}]$$

The wet service factor for tensile load is obtained from NDS Supp. Table 4A as

$$C_M = 1.00$$

The allowable tension design value parallel to grain is

$$\begin{aligned} F'_t &= F_t C_M C_F C_D \\ &= \left(1000 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.2)(1.15) \\ &= 1380 \text{ lbf/in}^2 \end{aligned}$$

The actual tension stress on the net area of the chord is

$$\begin{aligned} f_{t,\text{net}} &= \frac{T}{A_{\text{net}}} \\ &= \frac{8000 \text{ lbf}}{9.3 \text{ in}^2} \\ &= 860 \frac{\text{lbf}}{\text{in}^2} \\ &< F'_t \quad [\text{satisfactory}] \end{aligned}$$

The member is adequate.

## Combined Axial Tension and Flexure

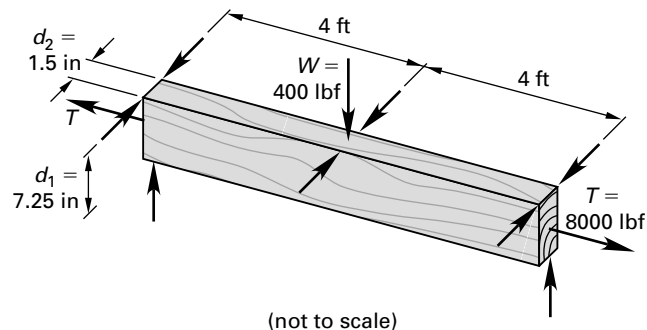
Members subjected to combined tension and flexural stresses caused by axial and transverse loading must satisfy the two expressions given in NDS Sec. 3.9.1 as

$$\frac{f_t}{F'_t} + \frac{f_b}{F'_b} \leq 1.0 \quad [\text{NDS 3.9-1}]$$

$$\frac{f_b - f_t}{F''_b} \leq 1.0 \quad [\text{NDS 3.9-2}]$$

### Example 5.11

The select structural  $2 \times 8$  Douglas Fir-Larch bottom chord of a truss is loaded as shown in the illustration. The governing load combination consists of dead load plus live load plus snow load, and the moisture content exceeds 19%. The chord is laterally braced at midlength about the weak axis, and the self-weight of the chord and bracing members may be neglected. Determine whether the member is adequate.



#### Solution

The relevant details from Ex. 5.10 follow. The allowable tension design value parallel to the grain is

$$F'_t = 1380 \frac{\text{lbf}}{\text{in}^2}$$

The actual tension stress on the gross area of the chord is

$$\begin{aligned} f_t &= (f_{t,\text{net}}) \left( \frac{A_{\text{net}}}{A_{\text{gross}}} \right) \\ &= \left( 860 \frac{\text{lbf}}{\text{in}^2} \right) \left( \frac{9.3 \text{ in}^2}{10.88 \text{ in}^2} \right) \\ &= 735 \text{ lbf/in}^2 \end{aligned}$$

The reference design values for bending and the modulus of elasticity for stability calculations, tabulated in NDS Supp. Table 4A, are

$$\begin{aligned} F_b &= 1500 \frac{\text{lbf}}{\text{in}^2} \\ E_{\text{min}} &= 0.69 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \end{aligned}$$

The applicable adjustment factors for the modulus of elasticity are the following. The wet service factor is

$$C_M = 0.9 \quad [\text{NDS Supp. Table 4A}]$$

The temperature factor is

$$C_t = 1.0 \quad [\text{NDS Table 2.3.3}]$$

The incising factor is

$$C_i = 1.0 \quad [\text{NDS Table 4.3.8}]$$

The adjusted modulus of elasticity for stability calculations is

$$\begin{aligned} E'_{\text{min}} &= E_{\text{min}} C_M C_t C_i \\ &= \left( 0.69 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \right) (0.9)(1.0)(1.0) \\ &= 0.62 \times 10^6 \text{ lbf/in}^2 \end{aligned}$$

For a concentrated load with lateral restraint, both at midspan, the effective length is obtained from Fig. 5.1 as

$$\begin{aligned} l_e &= 1.11 l_u \\ &= (1.11)(4 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \\ &= 53.28 \text{ in} \end{aligned}$$

The slenderness ratio is given by NDS Sec. 3.3.3 as

$$\begin{aligned} R_B &= \sqrt{\frac{l_e d_1}{d_2^2}} \\ &= \sqrt{\frac{(53.28 \text{ in})(7.25 \text{ in})}{(1.5 \text{ in})^2}} \\ &= 13.10 \\ &< 50 \quad [\text{satisfies criteria of NDS Sec. 3.3.3}] \end{aligned}$$

The critical buckling design value is

$$\begin{aligned} F_{bE} &= \frac{1.20 E'_{\text{min}}}{R_B^2} \\ &= \frac{(1.20) \left( 0.62 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \right)}{(13.10)^2} \\ &= 4335 \text{ lbf/in}^2 \end{aligned}$$

The applicable adjustment factors for bending are the following. The wet service factor is

$$C_M = 0.85 \quad [\text{NDS Supp. Table 4A}]$$

The size factor is

$$C_F = 1.2 \quad [\text{NDS Supp. Table 4A}]$$

The load duration factor is

$$C_D = 1.15 \quad [\text{NDS Table 2.3.2}]$$

The reference flexural design value, multiplied by all applicable adjustment factors except  $C_L$ , is

$$\begin{aligned} F_b^* &= F_b C_M C_F C_D \\ &= \left( 1500 \frac{\text{lbf}}{\text{in}^2} \right) (0.85)(1.2)(1.15) \\ &= 1760 \text{ lbf/in}^2 \end{aligned}$$

The ratio of  $F_{bE}$  to  $F_b^*$  is

$$\begin{aligned} F &= \frac{F_{bE}}{F_b^*} \\ &= \frac{4335 \frac{\text{lbf}}{\text{in}^2}}{1760 \frac{\text{lbf}}{\text{in}^2}} \\ &= 2.46 \end{aligned}$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$\begin{aligned} C_L &= \frac{1.0 + F}{1.9} - \sqrt{\left( \frac{1.0 + F}{1.9} \right)^2 - \frac{F}{0.95}} \\ &= \frac{1 + 2.46}{1.9} - \sqrt{\left( \frac{1 + 2.46}{1.9} \right)^2 - \frac{2.46}{0.95}} \\ &= 0.97 \end{aligned}$$

The reference flexural design value, multiplied by all applicable adjustment factors except  $C_V$ , is

$$\begin{aligned} F_b^{**} &= F_b C_M C_F C_D C_L \\ &= F_b^* C_L \\ &= \left( 1760 \frac{\text{lbf}}{\text{in}^2} \right) (0.97) \\ &= 1707 \text{ lbf/in}^2 \end{aligned}$$



The actual edgewise bending stress is

$$f_{b1} = \frac{WL}{4S}$$

$$= \frac{(400 \text{ lbf})(8 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{(4)(13.14 \text{ in}^3)}$$

$$= 731 \text{ lbf/in}^2$$

Substituting in the two expressions given in NDS Sec. 3.9.1 gives

$$\frac{f_t}{F_t'} + \frac{f_{b1}}{F_b'} = \frac{735 \frac{\text{lbf}}{\text{in}^2}}{1380 \frac{\text{lbf}}{\text{in}^2}} + \frac{731 \frac{\text{lbf}}{\text{in}^2}}{1760 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.533 + 0.415$$

$$= 0.948$$

$$< 1.0 \quad [\text{satisfactory}]$$

$$\frac{f_{b1} - f_t}{F_b^{**}} = \frac{731 \frac{\text{lbf}}{\text{in}^2} - 735 \frac{\text{lbf}}{\text{in}^2}}{1707 \frac{\text{lbf}}{\text{in}^2}}$$

$$= -0.002$$

$$< 1.0 \quad [\text{satisfactory}]$$

The chord is adequate.

## 5. DESIGN FOR SHEAR

### Nomenclature

|        |                                                                                                                                                |                     |
|--------|------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| $C_D$  | load duration factor                                                                                                                           | —                   |
| $C_M$  | wet service factor                                                                                                                             | —                   |
| $C_t$  | temperature factor                                                                                                                             | —                   |
| $d$    | depth of unnotched bending member                                                                                                              | in                  |
| $d_e$  | depth of member, less the distance from the unloaded edge of the member to the nearest edge of the nearest split ring or shear plate connector | in                  |
| $d_e$  | depth of member, less the distance from the unloaded edge of the member to the center of the nearest bolt or lag screw                         | in                  |
| $d_n$  | depth of member remaining at a notch                                                                                                           | in                  |
| $e$    | distance a notch extends past the inner edge of a support                                                                                      | in                  |
| $f_v$  | actual shear stress parallel to grain                                                                                                          | lbf/in <sup>2</sup> |
| $F_v$  | tabulated shear design value parallel to grain                                                                                                 | lbf/in <sup>2</sup> |
| $F_v'$ | allowable shear design value parallel to grain                                                                                                 | lbf/in <sup>2</sup> |
| $I$    | moment of inertia                                                                                                                              | in <sup>4</sup>     |
| $l_n$  | length of notch                                                                                                                                | in                  |
| $Q$    | statical moment of an area about the neutral axis                                                                                              | in <sup>3</sup>     |
| $V$    | shear force                                                                                                                                    | lbf or kips         |

|        |                                           |             |
|--------|-------------------------------------------|-------------|
| $V_r'$ | allowable design shear                    | lbf or kips |
| $W$    | concentrated load                         | kips        |
| $W'$   | equivalent concentrated load              | kips        |
| $x$    | distance from face of support to the load | in          |

### General Requirements Applicable to Sawn Lumber and Glued Laminated Members

The shear stress parallel to the grain in a flexural member is given in NDS Sec. 3.4.2 as

$$f_v = \frac{VQ}{Ib} \quad [\text{NDS 3.4-1}]$$

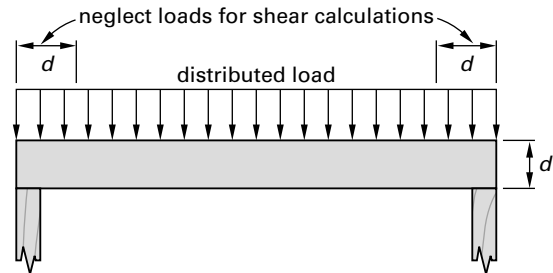
For a rectangular beam, this reduces to

$$f_v = \frac{3V}{2bd} \quad [\text{NDS 3.4-2}]$$

$$= \frac{1.5V}{bd}$$

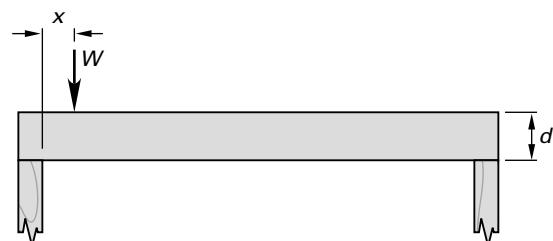
For a beam with distributed loads, the manner of calculating the shear force is specified in NDS Sec. 3.4.3.1, and illustrated in Fig. 5.4. In determining the shear force on a beam, uniformly distributed loads applied to the top of the beam within a distance from either support equal to the depth of the beam, are ignored.

Figure 5.4 Shear Caused by Distributed Loads



For a beam with concentrated loads, the manner of calculating the shear force is specified in NDS Sec. 3.4.3.1, and illustrated in Fig. 5.5. Concentrated loads within a distance from either support equal to the depth of the beam are multiplied by  $x/d$ , where  $x$  is the distance from the support to the load, to give an equivalent shear force.

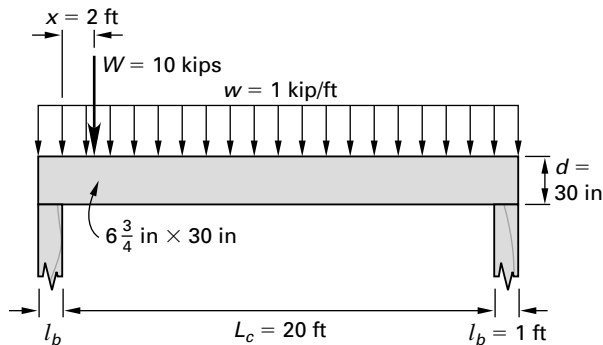
Figure 5.5 Shear Caused by Concentrated Loads



To facilitate the selection of glued laminated beam sections, tables are available<sup>3</sup> that provide shear and bending capacities of sections. For lumber joists, tables are available<sup>4</sup> that assist in the selection of a joist size for various span and live load combinations.

### Example 5.12

A glued laminated beam of combination 24F-V10 western species with a width of  $6\frac{3}{4}$  in, and a depth of 30 in, is loaded as shown in the illustration. The load consists of dead load plus live load, and the distributed load of 1.0 kip/ft includes the self-weight of the beam. The moisture content exceeds 16%. Determine whether the beam is adequate.



#### Solution

The reference design value for shear is tabulated in NDS Supp. Table 5A, Expanded Combinations, as

$$F_v = 215 \frac{\text{lb}}{\text{in}^2}$$

The applicable adjustment factor for shear stress is the wet service factor.

$$C_M = 0.875 \quad [\text{NDS Supp. Table 5A}]$$

The adjusted shear stress is

$$\begin{aligned} F'_v &= F_v C_M = \left(215 \frac{\text{lb}}{\text{in}^2}\right)(0.875) \\ &= 188 \text{ lb}/\text{in}^2 \end{aligned}$$

The clear span is obtained from the illustration as

$$L_c = 20 \text{ ft}$$

The bearing length is obtained from the illustration as

$$l_b = 1.0 \text{ ft}$$

The design span is defined in NDS Sec. 3.2.1 as

$$\begin{aligned} L &= L_c + l_b = 20 \text{ ft} + 1.0 \text{ ft} \\ &= 21 \text{ ft} \end{aligned}$$

The shear at the left support caused by the distributed load is given by NDS Sec. 3.4.3.1(a) as

$$\begin{aligned} V_D &= \frac{w(L_c - 2x)}{2} = \frac{\left(1.0 \frac{\text{kips}}{\text{ft}}\right)(16 \text{ ft})}{2} \\ &= 8 \text{ kips} \end{aligned}$$

The concentrated load,  $W$ , is less than a distance,  $d$ , from the face of the left support. In accordance with NDS Sec. 3.4.3.1, this is equivalent to a load of

$$\begin{aligned} W' &= \frac{Wx}{d} = \frac{(10 \text{ kips})(24 \text{ in})}{30 \text{ in}} \\ &= 8 \text{ kips} \end{aligned}$$

The shear at the left support caused by the equivalent load is

$$\begin{aligned} V_C &= \frac{W' \left(L - x - \frac{l_b}{2}\right)}{L} \\ &= \frac{(8 \text{ kips}) \left(21 \text{ ft} - 2 \text{ ft} - \frac{1 \text{ ft}}{2}\right)}{21 \text{ ft}} \\ &= 7.05 \text{ kips} \end{aligned}$$

The total shear at the left support is

$$\begin{aligned} V &= V_D + V_C \\ &= 8 \text{ kips} + 7.05 \text{ kips} \\ &= 15.05 \text{ kips} \end{aligned}$$

The shear stress parallel to the grain is given by NDS Sec. 3.4.2 as

$$\begin{aligned} f_v &= \frac{1.5 V}{bd} \quad [\text{NDS 3.4-2}] \\ &= \frac{(1.5)(15,050 \text{ lb})}{(6.75 \text{ in})(30 \text{ in})} \\ &= 111 \text{ lb}/\text{in}^2 \\ &< F'_v \quad [\text{satisfactory}] \end{aligned}$$

The beam is adequate.

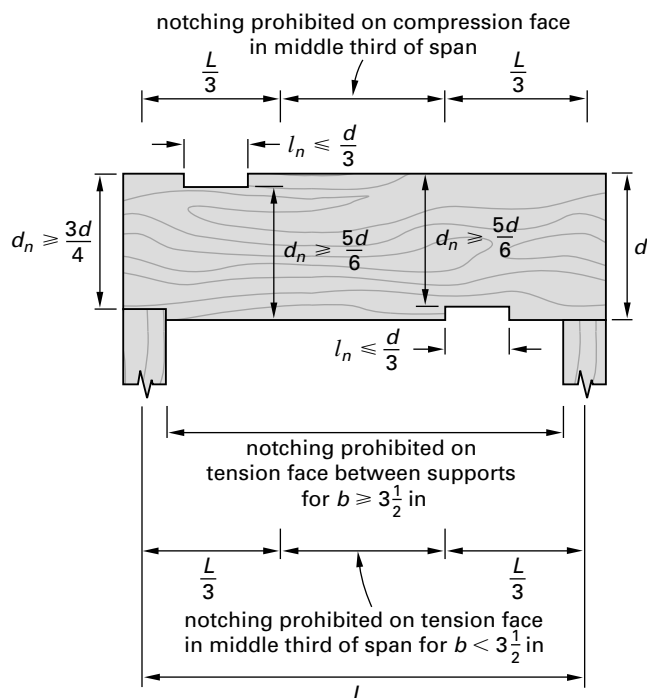
### Notched Beams

Notches reduce the shear capacity of a beam. NDS Sec. 4.4.3 imposes restrictions on their size and location in sawn lumber members, as shown in Fig. 5.6. For glued laminated members, similar restrictions are specified in NDS Sec. 5.4.4, shown in Fig. 5.7.

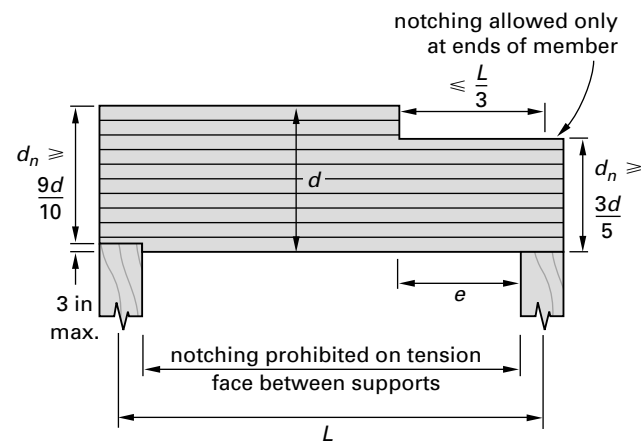
<sup>3</sup>American Plywood Association, 2007 (See References and Codes)

<sup>4</sup>Western Wood Products Association, 1992 (See References and Codes)

**Figure 5.6** Notches in Sawn Lumber Beams



**Figure 5.7** Notches in Glued Laminated Beams



The allowable design shear at a notch on the tension side of a beam is given by NDS Sec. 3.4.3.2(a) as

$$V'_r = \left( \frac{F'_v b d_n}{1.5} \right) \left( \frac{d_n}{d} \right)^2 \quad [\text{NDS 3.4-3}]$$

When  $e$ , the distance the notch extends past the inner edge of the support, is less than or equal to  $d_n$ , the shear stress at the notch on the compression side of the beam is given by NDS Sec. 3.4.3.2(e) as

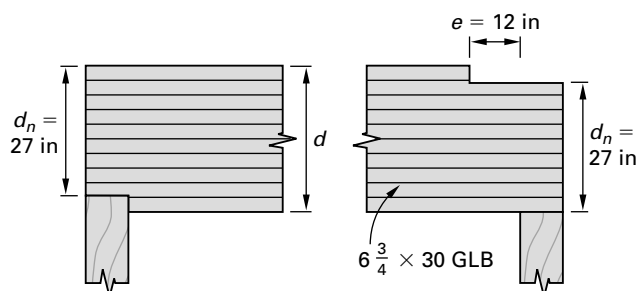
$$V'_r = \left( \frac{F'_v b}{1.5} \right) \left( d - \left( \frac{d - d_n}{d_n} \right) e \right) \quad [\text{NDS 3.4-5}]$$

When  $e$  is greater than  $d_n$ ,

$$V'_r = \frac{F'_v b d_n}{1.5}$$

### Example 5.13

A glued laminated beam of combination 24F-V10 western species, with a width of  $6\frac{3}{4}$  in and a depth of 30 in, is notched as shown in the illustration. The beam has a moisture content exceeding 16%, and is subjected to sustained temperatures between 100°F and 125°F. The governing load combination is dead load plus live load plus snow load. Determine the maximum allowable shear force at each support.



### Solution

The reference design value for shear is tabulated in NDS Supp. Table 5A, Expanded Combinations, as

$$F_v = 215 \frac{\text{lb}}{\text{in}^2}$$

The applicable adjustment factors for shear are the following. The wet service factor is

$$C_M = 0.875 \quad [\text{NDS Supp. Table 5A}]$$

The temperature factor for wet conditions is

$$C_t = 0.7 \quad [\text{NDS Table 2.3.3}]$$

The load duration factor is

$$C_D = 1.15$$

After allowing for the notch, as required by Table 5A, Expanded Combinations, Footnote 3, the adjusted shear stress is

$$\begin{aligned} F'_v &= 0.72 F_v C_M C_t C_D \\ &= (0.72) \left( 215 \frac{\text{lb}}{\text{in}^2} \right) (0.875) (0.70) (1.15) \\ &= 109 \text{ lb/in}^2 \end{aligned}$$

At the left support, the allowable shear force is

$$\begin{aligned} V'_r &= \left( \frac{F'_v b d_n}{1.5} \right) \left( \frac{d_n}{d} \right)^2 \quad [\text{NDS 3.4-3}] \\ &= \left( \frac{\left( 109 \frac{\text{lb}}{\text{in}^2} \right) (6.75 \text{ in}) (27 \text{ in})}{1.5} \right) \left( \frac{27 \text{ in}}{30 \text{ in}} \right)^2 \\ &= 10,727 \text{ lbf} \end{aligned}$$

At the right support,  $e$  is less than  $d_n$ , and the allowable shear force is

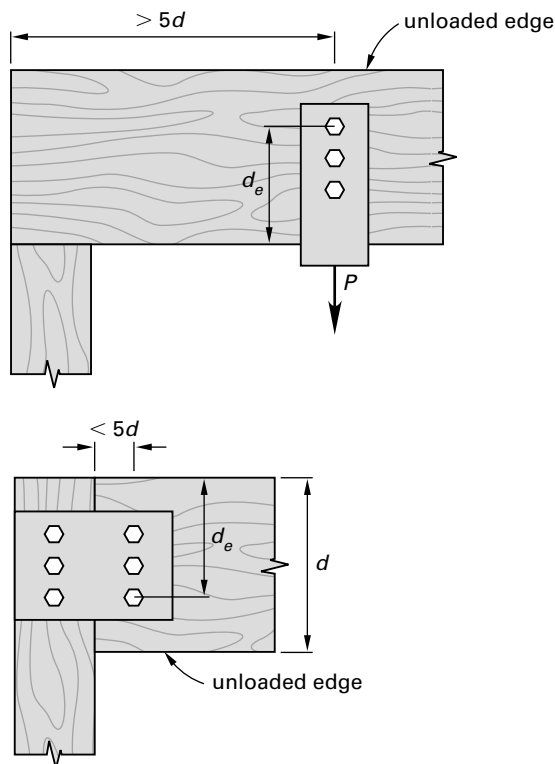
$$\begin{aligned} V'_r &= \left( \frac{F'_v b}{1.5} \right) \left( d - \left( \frac{d - d_n}{d_n} \right) e \right) \quad [\text{NDS 3.4-5}] \\ &= \left( \frac{\left( 109 \frac{\text{lb}}{\text{in}^2} \right) (6.75 \text{ in})}{1.5} \right) \\ &\quad \times \left( 30 \text{ in} - \left( \frac{30 \text{ in} - 27 \text{ in}}{27 \text{ in}} \right) (12 \text{ in}) \right) \\ &= 14,061 \text{ lbf} \end{aligned}$$

### Shear at Connections

For a connection less than five times the depth of the member from its end, shown in Fig. 5.8, the allowable design shear is given by NDS Sec. 3.4.3.3 as

$$V'_r = \left( \frac{F'_v b d_e}{1.5} \right) \left( \frac{d_e}{d} \right)^2 \quad [\text{NDS 3.4-6}]$$

Figure 5.8 Bolted Connections

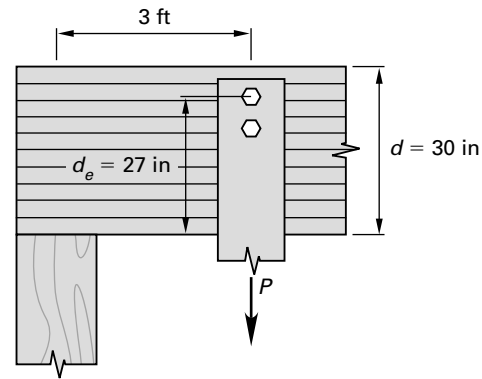


When the connection is at least five times the depth of the member from its end, the allowable design shear is

$$V'_r = \frac{F'_v b d_e}{1.5} \quad [\text{NDS 3.4-7}]$$

### Example 5.14

A glued laminated beam of combination 24F-V10 (24F-1.7E) western species with a width of  $6\frac{3}{4}$  in and a depth of 30 in is loaded with a bolted hanger connection, as shown in the illustration. The beam has a moisture content exceeding 16%. The governing load combination is dead load plus live load. Determine the maximum allowable shear force at the hanger connection.



### Solution

The reference design value for shear is tabulated in NDS Supp. Table 5A as

$$F_v = 215 \frac{\text{lb}}{\text{in}^2}$$

The applicable adjustment factor for shear is the wet service factor.

$$C_M = 0.875 \quad [\text{NDS Supp. Table 5A}]$$

The adjusted shear stress is

$$\begin{aligned} F'_v &= F_v C_M \\ &= \left( 215 \frac{\text{lb}}{\text{in}^2} \right) (0.875) \\ &= 188 \text{ lb/in}^2 \end{aligned}$$

The hanger connection is less than  $5d$  from the end of the beam, and from NDS Eq. (3.4-6) the allowable shear force is

$$\begin{aligned} F'_r &= \left( \frac{b d_e F'_v}{1.5} \right) \left( \frac{d_e}{d} \right)^2 \\ &= \left( \frac{(6.75 \text{ in}) (27 \text{ in}) \left( 188 \frac{\text{lb}}{\text{in}^2} \right)}{1.5} \right) \left( \frac{27 \text{ in}}{30 \text{ in}} \right)^2 \\ &= 18,502 \text{ lbf} \end{aligned}$$

# 6. DESIGN OF CONNECTIONS

## Nomenclature

|              |                                                                                                                                          |                 |
|--------------|------------------------------------------------------------------------------------------------------------------------------------------|-----------------|
| $a$          | center to center spacing between adjacent rows of fasteners                                                                              | in              |
| $a_e$        | minimum edge distance with load parallel to grain                                                                                        | in              |
| $a_p$        | minimum end distance with load parallel to grain                                                                                         | in              |
| $a_q$        | minimum end distance with load perpendicular to grain                                                                                    | in              |
| $A$          | area of cross section                                                                                                                    | in <sup>2</sup> |
| $A_m$        | gross cross-sectional area of main wood member(s)                                                                                        | in <sup>2</sup> |
| $A_n$        | net area of member                                                                                                                       | in <sup>2</sup> |
| $A_s$        | sum of gross cross-sectional areas of side member(s)                                                                                     | in <sup>2</sup> |
| $C_d$        | penetration depth factor for connections                                                                                                 | —               |
| $C_D$        | load duration factor                                                                                                                     | —               |
| $C_{di}$     | diaphragm factor for nailed connections                                                                                                  | —               |
| $C_{eg}$     | end grain factor for connections                                                                                                         | —               |
| $C_g$        | group action factor for connections                                                                                                      | —               |
| $C_M$        | wet service factor                                                                                                                       | —               |
| $C_{st}$     | metal side plate factor for 4 in shear plate connections                                                                                 | —               |
| $C_{tn}$     | toe-nail factor for nailed connections                                                                                                   | —               |
| $C_{\Delta}$ | geometry factor for connections                                                                                                          | —               |
| $d$          | pennyweight of nail or spike                                                                                                             | —               |
| $d_e$        | effective depth of member at a connection                                                                                                | in              |
| $D$          | diameter                                                                                                                                 | in              |
| $e_p$        | minimum edge distance, unloaded edge                                                                                                     | in              |
| $e_q$        | minimum edge distance, loaded edge                                                                                                       | in              |
| $g$          | gage of screw                                                                                                                            | —               |
| $l_m$        | length of bolt in wood main member                                                                                                       | in              |
| $l_s$        | total length of bolt in wood side member(s)                                                                                              | in              |
| $L$          | length of nail                                                                                                                           | in              |
| $n$          | number of fasteners in a row                                                                                                             | —               |
| $N$          | reference lateral design value at an angle, $\theta$ , to the grain for a single split ring connector unit or shear plate connector unit | lbf             |
| $N'$         | allowable lateral design value at an angle, $\theta$ , to the grain for a single split ring connector unit or shear plate connector unit | lbf             |
| $p$          | depth of fastener penetration into wood member                                                                                           | in              |
| $P$          | reference lateral design value parallel to grain for a single split ring connector unit or shear plate connector unit                    | lbf             |
| $P'$         | allowable lateral design value parallel to grain for a single split ring connector unit or shear plate connector unit                    | lbf             |

|               |                                                                                                                                                                               |                       |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| $Q$           | reference lateral design value perpendicular to grain for a single split ring connector unit or shear plate connector unit                                                    | lbf                   |
| $Q'$          | allowable lateral design value perpendicular to grain for a single split ring connector or shear plate connector unit                                                         | lbf                   |
| $s$           | center to center spacing between adjacent fasteners in a row                                                                                                                  | in                    |
| $s_p$         | minimum spacing                                                                                                                                                               | in                    |
| $t_m$         | thickness of main member                                                                                                                                                      | in                    |
| $t_s$         | thickness of side member                                                                                                                                                      | in                    |
| $T$           | tensile force on member                                                                                                                                                       | lbf                   |
| $W$           | reference withdrawal design value for fastener                                                                                                                                | lbf/in of penetration |
| $W'$          | allowable withdrawal design value for fastener                                                                                                                                | lbf/in of penetration |
| $Z$           | reference lateral design value for a single fastener connection                                                                                                               | lbf                   |
| $Z'$          | allowable lateral design value for a single fastener connection                                                                                                               | lbf                   |
| $Z_{  }$      | reference lateral design value for a single bolt or lag screw connection with all wood members loaded parallel to grain                                                       | lbf                   |
| $Z_{\perp}$   | reference lateral design value for a single bolt or lag screw, wood-to-wood, wood-to-metal or wood-to-concrete connection with all wood members loaded perpendicular to grain | lbf                   |
| $Z'_{\alpha}$ | allowable design value for lag screw with load applied at an angle, $\alpha$ , to the wood surface                                                                            | lbf                   |
| $Z_{m\perp}$  | reference lateral design value for a single bolt or lag screw wood-to-wood connection with main member loaded perpendicular to grain and side member loaded parallel to grain | lbf                   |
| $Z_{s\perp}$  | reference lateral design value for a single bolt or lag screw wood-to-wood connection with main member loaded parallel to grain and side member loaded perpendicular to grain | lbf                   |

## Symbols

|          |                                                                                      |        |
|----------|--------------------------------------------------------------------------------------|--------|
| $\alpha$ | angle between wood surface and direction of applied load                             | deg    |
| $\gamma$ | load/slip modulus for a connection                                                   | lbf/in |
| $\theta$ | angle between direction of load and direction of grain (longitudinal axis of member) | deg    |

## Adjustment of Design Values

The allowable design values for wood fasteners depend on the type of fastener and on the service conditions. The reference design values, tabulated in NDS Secs. 10–13, are applicable to normal conditions of use and normal load duration. To determine the relevant design values for other conditions of service, the reference design values are multiplied by adjustment factors specified in NDS Sec. 10.3. A summary of adjustment factors follows. The applicability of each to the basic design values is given in NDS Table 10.3.1, and summarized in Table 5.3.

### Load Duration Factor, $C_D$

Normal load duration is equivalent to applying the maximum allowable load to a member for a period of 10 years. For loads of shorter duration, a member has the capacity to sustain higher loads, and the basic design values are multiplied by the load duration factor,  $C_D$ . With the exception of the impact load duration factor, which does not apply to fasteners, values of the load duration factor given in Table 5.2 are also applicable to connections.

### Wet Service Factor, $C_M$

Basic design values apply to fasteners in wood seasoned to a moisture content of 19% or less. When the moisture content of the member exceeds 19%, the adjustment factors given in NDS Table 10.3.3 are applicable.

### Temperature Factor, $C_t$

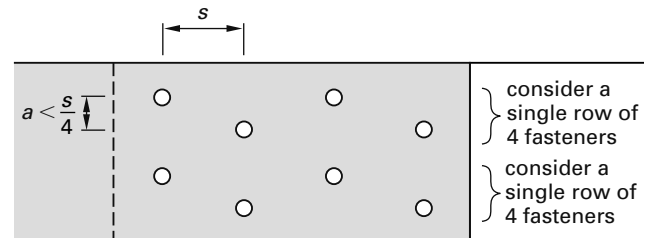
The temperature factor is applicable to all fasteners exposed to sustained temperatures up to 150°F. It is specified by NDS Table 10.3.4.

### Group Action Factor, $C_g$

The group action factors for various connection geometries and fastener types are given in NDS

Tables 10.3.6A–D. This factor is dependent on the ratio of the area of the side members in a connection to the area of the main member,  $A_s/A_m$ .  $A_m$  and  $A_s$  are calculated using gross areas without deduction for holes. When adjacent rows of fasteners are staggered, as shown in Fig. 5.9, the adjacent rows are considered as a single row.

Figure 5.9 Staggered Fasteners



### Geometry Factor, $C_\Delta$

The geometry factor applies to bolts, lag screws, split rings, and shear plates. The factor is applied, in accordance with NDS Sec. 11.5.1, to dowel-type fasteners when the end distance or spacing is less than the minimum required for full design value. The smallest geometry factor for any fastener in a group shall apply to the whole group. NDS Tables 11.5.1B and 11.5.1C stipulate the minimum end distance and spacing requirements for dowel-type fasteners.

The factor is applied, in accordance with NDS Sec. 12.3.2, to split ring and shear plate connectors when the end distance, edge distance, or spacing is less than the minimum required for full design value. The smallest geometry factor for any connector in a group shall apply to the whole group. NDS Table 12.3 specifies the minimum end distance, edge distance, spacing requirements, and geometry factors for split ring and shear plate connectors.

Table 5.3 Adjustment Factors for Fasteners

| adjustment factor                | bolts | lag screws |       | split rings and<br>shear plates |       | screws |       | nails |       |
|----------------------------------|-------|------------|-------|---------------------------------|-------|--------|-------|-------|-------|
| design value                     | $Z^a$ | $W^b$      | $Z^a$ | $P^c$                           | $Q^d$ | $W^b$  | $Z^a$ | $W^b$ | $Z^a$ |
| $C_D$ load duration factor       | ✓     | ✓          | ✓     | ✓                               | ✓     | ✓      | ✓     | ✓     | ✓     |
| $C_M$ wet service factor         | ✓     | ✓          | ✓     | ✓                               | ✓     | ✓      | ✓     | ✓     | ✓     |
| $C_t$ temperature factor         | ✓     | ✓          | ✓     | ✓                               | ✓     | ✓      | ✓     | ✓     | ✓     |
| $C_g$ group action factor        | ✓     | —          | ✓     | ✓                               | ✓     | —      | —     | —     | —     |
| $C_\Delta$ geometry factor       | ✓     | —          | ✓     | ✓                               | ✓     | —      | —     | —     | —     |
| $C_d$ penetration depth factor   | —     | —          | ✓     | ✓                               | ✓     | —      | ✓     | —     | ✓     |
| $C_{eg}$ end grain factor        | —     | ✓          | ✓     | —                               | —     | —      | ✓     | —     | ✓     |
| $C_{st}$ metal side plate factor | —     | —          | —     | ✓                               | —     | —      | —     | —     | —     |
| $C_{di}$ diaphragm factor        | —     | —          | —     | —                               | —     | —      | —     | —     | ✓     |
| $C_{tn}$ toe-nail factor         | —     | —          | —     | —                               | —     | —      | —     | ✓     | ✓     |

<sup>a</sup>lateral design value

<sup>b</sup>withdrawal design value

<sup>c</sup>parallel to grain design value

<sup>d</sup>perpendicular to grain design value

**Penetration Depth Factor,  $C_d$** 

The penetration depth factor applies to lag screws, split rings, shear plates, screws, and nails. The factor is applied in accordance with NDS Tables 11J–R and 12.2.3 when the penetration is less than the minimum specified for full design values.

**End Grain Factor,  $C_{eg}$** 

The end grain factor applies to lag screws, screws, and nails. The factor is applied in accordance with NDS Sec. 11.5.2 when the fastener is inserted in the end grain of a member.

**Metal Side Plate Factor,  $C_{st}$** 

The metal side plate factor is applicable to split rings and shear plates. The factor is applied in accordance with NDS Sec. 12.2.4 when metal side plates are used instead of wood side members.

**Diaphragm Factor,  $C_{di}$** 

The diaphragm factor applies to nails and spikes in accordance with NDS Sec. 11.5.3. When the fasteners are used in diaphragm construction, reference design values are multiplied by the diaphragm factor

$$C_{di} = 1.1$$

**Toe-Nail Factor,  $C_{tn}$** 

The toe-nail factor applies to nails and spikes in accordance with NDS Sec. 11.5.4. The reference withdrawal values for toe-nailed connections are multiplied by the toe-nail factor

$$C_{tn} = 0.67$$

The reference lateral design values for toe-nailed connections are multiplied by the toe-nail factor

$$C_{tn} = 0.83$$

**Example 5.15**

A select structural  $2 \times 8$  Douglas Fir-Larch tie is connected to a  $4 \text{ in} \times \frac{1}{4} \text{ in}$  steel gusset plate by a single row of eight  $\frac{7}{8}$  in diameter bolts in single shear. The governing load combination consists of dead load plus live load plus snow load, and the in-service moisture content exceeds 19%. The bolt spacing and end distance are 4 in. Determine the capacity of the connection.

*Solution*

From Ex. 5.10, the allowable tension design value for the tie, parallel to the grain, is

$$\begin{aligned} F'_t &= F_t C_M C_F C_D \\ &= \left(1000 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.2)(1.15) \\ &= 1380 \text{ lbf/in}^2 \end{aligned}$$

The allowable tension capacity of the Douglas Fir-Larch tie is

$$\begin{aligned} T &= F'_t A_n \\ &= F'_t \left( A - \left( D + \frac{1}{16} \right) b \right) \\ &= \left( 1380 \frac{\text{lbf}}{\text{in}^2} \right) \left( 10.88 \text{ in}^2 - \left( 0.875 \text{ in} + \frac{1}{16} \right) (1.5 \text{ in}) \right) \\ &= 13,074 \text{ lbf} \end{aligned}$$

The reference  $\frac{7}{8}$  in diameter bolt design value for a  $1\frac{1}{2}$  in thick member loaded parallel to the grain, with a steel side plate in single shear, is tabulated in NDS Table 11B as

$$Z_{||} = 1020 \text{ lbf}$$

The gross cross-sectional area of the main wood member is

$$A_m = 10.88 \text{ in}^2$$

The gross cross-sectional area of the steel gusset plate is

$$\begin{aligned} A_s &= (4 \text{ in})(0.25 \text{ in}) \\ &= 1.0 \text{ in}^2 \\ \frac{A_m}{A_s} &= \frac{10.88 \text{ in}^2}{1.0 \text{ in}^2} \\ &= 10.88 \end{aligned}$$

The specified minimum spacing between bolts for the full bolt design value is specified in NDS Table 11.5.1C as

$$\begin{aligned} s_{p,\text{full}} &= 4D \\ &= (4)(0.875 \text{ in}) \\ &= 3.5 \text{ in} \end{aligned}$$

The actual spacing is

$$\begin{aligned} s_p &= 4 \text{ in} \\ &> s_{p,\text{full}} \end{aligned}$$

The geometry factor for bolt spacing is given by NDS Sec. 11.5.1 as

$$C_\Delta = 1.0$$

The minimum end distance for the full bolt design value, for loading parallel to the grain, is specified in NDS Table 11.5.1B as

$$\begin{aligned} a_{p,\text{full}} &= 7D \\ &= (7)(0.875 \text{ in}) \\ &= 6.125 \text{ in} \end{aligned}$$



The minimum end distance for the reduced bolt design value, for loading parallel to the grain, is specified in NDS Table 11.5.1B as

$$\begin{aligned} a_{p,\text{red}} &= 3.5D \\ &= (3.5)(0.875 \text{ in}) \\ &= 3.063 \text{ in} \end{aligned}$$

The actual end distance is

$$\begin{aligned} a_p &= 4 \text{ in} \\ &< a_{p,\text{full}} \\ &> a_{p,\text{red}} \end{aligned}$$

The geometry factor for bolt end spacing is given by NDS Sec. 11.5.1 as

$$\begin{aligned} C_\Delta &= \frac{a_p}{a_{p,\text{full}}} \\ &= \frac{4 \text{ in}}{6.125 \text{ in}} \\ &= 0.653 \quad [\text{governs}] \end{aligned}$$

The additional applicable adjustment factors for the bolts are the following. The wet service factor from NDS Table 10.3.3 is

$$C_M = 0.7$$

The group action factor from NDS Table 10.3.6C for  $A_m/A_s = 10.88$  and  $A_m = 10.88 \text{ in}^2$  is

$$C_g = 0.58$$

The load duration factor from NDS Table 2.3.2 is

$$C_D = 1.15$$

The allowable lateral design value for 8 bolts is

$$\begin{aligned} T &= nZ_{\parallel} C_M C_g C_\Delta C_D \\ &= (8)(1020 \text{ lbf})(0.70)(0.58)(0.653)(1.15) \\ &= 2500 \text{ lbf} \quad [\text{governs}] \end{aligned}$$

## Bolted Connections

Bolt holes shall be a minimum of  $1/32$  in to a maximum of  $1/16$  in larger than the bolt diameter, as specified in NDS Sec. 11.1.2. A metal washer or plate is required between the wood and the nut, and between the wood and the bolt head. To ensure that the full design values of bolts are developed, spacing, edge, and end distances are specified in NDS Sec. 11.5. These distances are illustrated in Fig. 5.10.

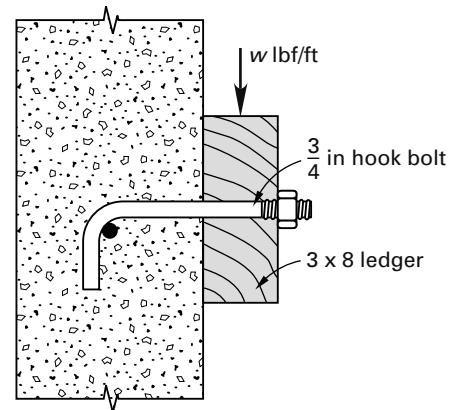
Reference design values for single shear and symmetric double shear connections are specified in NDS Sec. 11.3 and tabulated in NDS Table 11A for two sawn lumber

members of identical species; in NDS Table 11B for a sawn lumber member with a steel side plate; in NDS Table 11C for a glued laminated member with sawn lumber side member; in NDS Table 11D for a glued laminated member with a steel side plate; and in NDS Table 11E for connections to concrete.

Design values for double shear connections are specified in NDS Sec. 11.3 and tabulated in NDS Table 11F for three sawn lumber members of identical species; in NDS Table 11G for a sawn lumber member with steel side plates; in NDS Table 11H for a glued laminated member with sawn lumber side members; and in NDS Table 11I for a glued laminated member with steel side plates.

### Example 5.16

A  $3 \times 8$  select structural Douglas Fir-Larch ledger is attached to a concrete wall with  $3/4$  in bolts at 4 ft centers, as shown in the illustration. What is the maximum dead load plus live load that the ledger can support?



### Solution

The nominal  $3/4$  in diameter bolt design value for a  $2\frac{1}{2}$  in thick member loaded perpendicular to the grain and attached to a concrete wall is tabulated in NDS Table 11E as

$$Z_{\perp} = 800 \text{ lbf}$$

The minimum edge distance for the full bolt design value, for loading perpendicular to grain, is specified in NDS Table 11.5.1A as

$$\begin{aligned} e_{q,\text{full}} &= 4D \\ &= (4)(0.75 \text{ in}) \\ &= 3.0 \text{ in} \end{aligned}$$

The actual edge distance is

$$\begin{aligned} e_q &= \frac{7.25 \text{ in}}{2} \\ &= 3.625 \text{ in} \\ &> e_{q,\text{full}} \quad [\text{satisfactory}] \end{aligned}$$



The geometry factor is given by NDS Sec. 11.5.1 as

$$C_A = 1.0$$

The maximum load that the ledger can support is

$$\begin{aligned} w &= \frac{Z_{\perp}}{4} \\ &= \frac{800 \text{ lbf}}{4 \text{ ft}} \\ &= 200 \text{ lbf/ft} \end{aligned}$$

## Lag Screw Connections

In accordance with NDS Sec. 11.1.3, a clearance hole equal in diameter to the diameter of the shank, must be bored in the member for the full length of the unthreaded shank. A lead hole at least equal in length to the threaded portion of the screw must be provided. For wood of specific gravity in excess of 0.6, the lead hole diameter must equal 65–85% of the shank diameter. For wood of specific gravity of 0.5 or less, the lead hole diameter must equal 40–70% of the shank diameter. For wood of intermediate specific gravity, the lead hole diameter must

equal 60–75% of the shank diameter. The lag screw must be inserted into the lead hole by turning with a wrench. For lag screws of  $\frac{3}{8}$  in diameter or smaller, loaded primarily in withdrawal in wood with specific gravity of 0.5 or less, lead holes and clearance holes are not required.

## Lateral Design Values in Side Grain

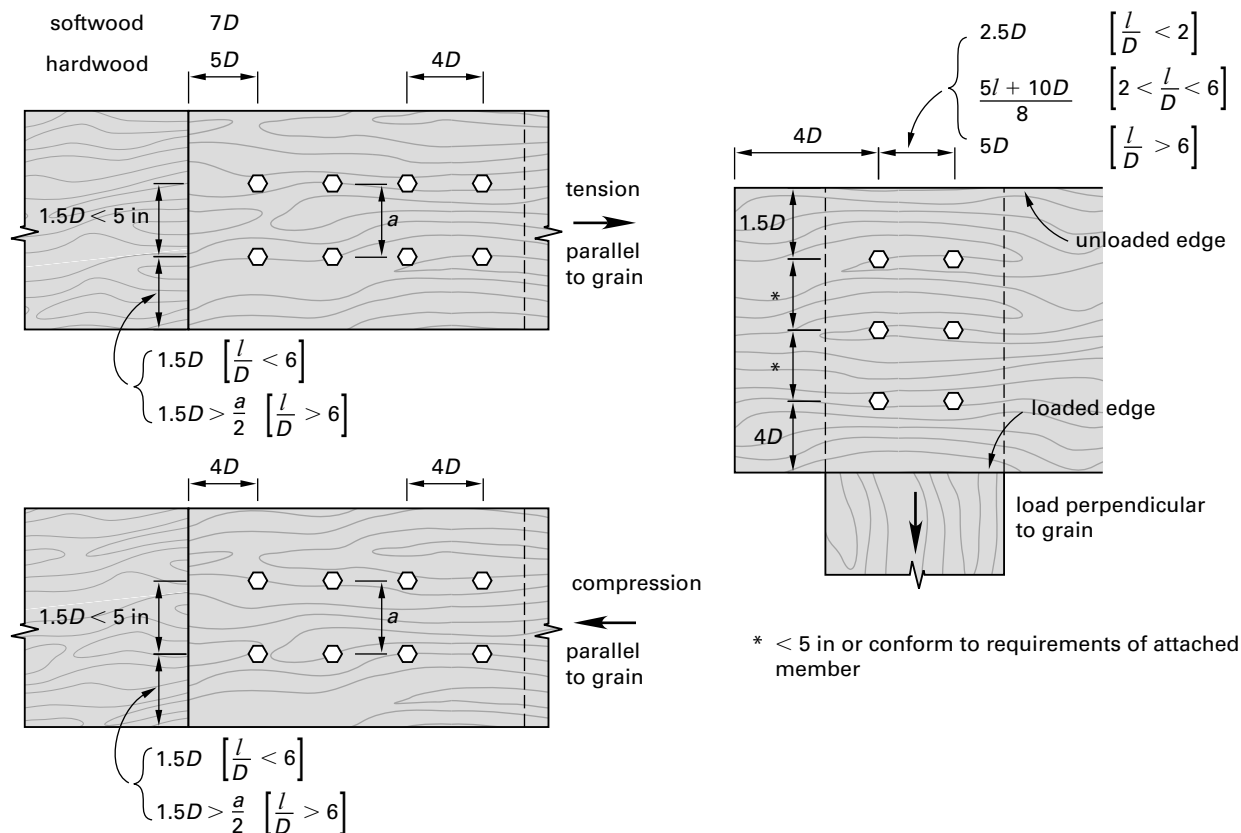
Minimum edge distances, end distances, spacing, and geometry factors are given in NDS Tables 11.5.1A–E, and are identical with those for bolts with a diameter equal to the shank diameter of the lag screw. The minimum allowable penetration of the lag screw into the member, not including the length of the tapered tip, is specified in NDS Sec. 11.1.3.6 as

$$p = 4D$$

As specified in NDS Table 11J, for full design values to be applicable, the depth of penetration shall not be less than

$$p = 8D$$

Figure 5.10 Bolt Spacing Requirements for Full Design Values



$l$  is the lesser of length of bolt in main member or total length of bolt in side member(s)  
 $D$  is the diameter of bolt

When the penetration is between  $4D$  and  $8D$ , the nominal design value is multiplied by the penetration factor, which is defined in NDS Table 11J as

$$C_d = \frac{p}{8D}$$

Reference design values for single shear connections are specified in NDS Sec. 11.3. They are tabulated in NDS Table 11J for connections with a wood side member, and in NDS Table 11K for connections with a steel side plate.

### Withdrawal Design Values in Side Grain Without Lateral Load

Minimum edge distance, end distance, and spacing, specified in NDS Table 11.5.1E, are

$$\text{edge distance} = 1.5D$$

$$\text{end distance} = 4D$$

$$\text{spacing} = 4D$$

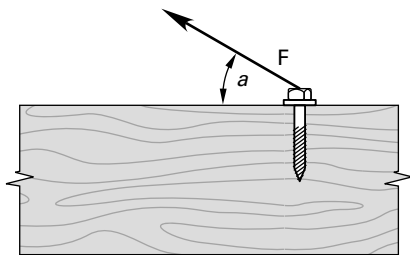
Withdrawal design values in pounds per inch of thread penetration, not including the length of tapered tip, are given in NDS Table 11.2A.

### Combined Lateral and Withdrawal Loads

When the load applied to a lag screw is at an angle,  $\alpha$ , to the wood surface, as shown in Fig. 5.11, the lag screw is subjected to combined lateral and withdrawal loading. In accordance with NDS Sec. 11.4.1, the design value is now determined by Hankinson's formula,

$$Z'_\alpha = \frac{W'pZ'}{W'p \cos^2 \alpha + Z' \sin^2 \alpha} \quad [\text{NDS 11.4-1}]$$

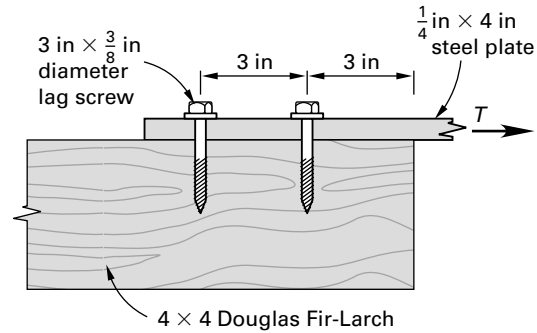
**Figure 5.11** Combined Lateral and Withdrawal Load



### Example 5.17

A  $\frac{1}{4}$  in thick steel plate is attached to a Douglas Fir-Larch member, as shown in the illustration, using two 3 in long  $\times \frac{3}{8}$  in diameter lag screws. The governing load combination consists of dead load plus live load, and the in-service moisture content exceeds 19%. The bolt spacing and end distance are 3 in. What is the

maximum allowable load that can be applied to the plate?



### Solution

From NDS Table 11K, the reference lateral design value for load applied parallel to the grain is

$$Z_{||} = 280 \text{ lbf}$$

The gross cross-sectional area of the  $4 \times 4$  main wood member is

$$A_m = 12.25 \text{ in}^2$$

The gross cross-sectional area of the steel plate is

$$A_s = (4 \text{ in})(0.25 \text{ in})$$

$$= 1.0 \text{ in}^2$$

$$\frac{A_m}{A_s} = \frac{12.25 \text{ in}^2}{1.0 \text{ in}^2}$$

$$= 12.25$$

The specified minimum spacing between lag screws for the full design value is specified in NDS Table 11.5.1C as

$$s_{p,\text{full}} = 4D$$

$$= (4)(0.375 \text{ in})$$

$$= 1.50 \text{ in}$$

The actual spacing is

$$s_p = 3 \text{ in}$$

$$> s_{p,\text{full}}$$

The geometry factor for lag screw spacing is given by NDS Sec. 11.5.1 as

$$C_\Delta = 1.0$$

The minimum end distance for the full lag screw design value, for loading parallel to grain, is specified in NDS Table 11.5.1B as

$$\begin{aligned} a_{p,\text{full}} &= 7D \\ &= (7)(0.375 \text{ in}) \\ &= 2.625 \text{ in} \end{aligned}$$

The actual end distance is

$$\begin{aligned} a_p &= 3 \text{ in} \\ &> a_{p,\text{full}} \end{aligned}$$

The geometry factor for lag screw end spacing is given by NDS Sec. 11.5.1 as

$$C_\Delta = 1.0$$

From NDS App. L, the penetration into the main member by the screw shank, plus the threaded length, less the length of the tapered tip is

$$\begin{aligned} p &= S + T - E - t_s \\ &= 1.0 \text{ in} + 1.781 \text{ in} - 0.25 \text{ in} \\ &= 2.531 \text{ in} \end{aligned}$$

From NDS Table 11K, the penetration factor is

$$\begin{aligned} C_d &= \frac{p}{8D} \\ &= \frac{2.531 \text{ in}}{(8)(0.375 \text{ in})} \\ &= 0.84 \end{aligned}$$

The additional applicable adjustment factors for the lag screws are the following. The wet service factor from NDS Table 10.3.3 is

$$C_M = 0.7$$

The group action factor from NDS Table 10.3.6C for  $A_m/A_s = 12.25$  and  $A_m = 12.25 \text{ in}^2$  is

$$C_g = 0.985$$

The load duration factor from NDS Table 2.3.2 is

$$C_D = 1.0$$

The allowable lateral design value for two lag screws is

$$\begin{aligned} T &= nZ_{||} C_M C_g C_\Delta C_D C_d \\ &= (2)(280 \text{ lbf})(0.70)(0.985)(1.0)(1.0)(0.84) \\ &= 320 \text{ lbf} \end{aligned}$$

### Split Ring and Shear Plate Connections

Dimensions for split ring and shear plate connectors are provided in NDS App. K. Edge and end distances,

spacing, and geometry factors,  $C_\Delta$ , for various sizes of split ring and shear plate connectors are specified in NDS Table 12.3. When lag screws are used instead of bolts, reference design values must, where appropriate, be multiplied by the penetration depth factors specified in NDS Table 12.2.3.

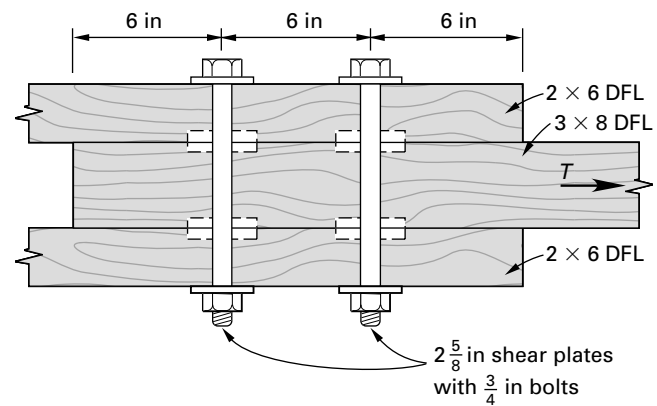
NDS Table 12.2.4 provides metal side plate factors,  $C_{st}$ , for 4 in shear plate connectors, loaded parallel to the grain, when metal side plates are substituted for wood side members. Group action factors,  $C_g$ , for 4 in split ring or shear plate connectors with wood side members are tabulated in NDS Table 10.3.6B. Group action factors,  $C_g$ , for 4 in shear plate connectors with steel side plates are tabulated in NDS Table 10.3.6D.

Reference design values for split ring connectors are provided in NDS Table 12.2A and, for shear plate connectors, in NDS Table 12.2B. When a load acts in the plane of the wood surface at an angle,  $\theta$ , to the grain, the allowable design value is given by NDS Sec. 12.2.5 as

$$N' = \frac{P'Q'}{P'\sin^2\theta + Q'\cos^2\theta} \quad [\text{NDS 12.2-1}]$$

### Example 5.18

The Douglas Fir-Larch select structural members shown in the illustration are connected with  $2\frac{5}{8}$  in shear plate connectors. The governing load combination consists of dead load plus live load, and the in-service moisture content exceeds 19%. The connector spacing and end distances are as shown. Determine the capacity of the connection.



### Solution

The reference  $2\frac{5}{8}$  in shear plate design value for the  $2\frac{1}{2}$  in thick main member of Group B species, with a connector on two faces, is tabulated in NDS Table 12.2B as

$$P_{\text{main}} = 2860 \text{ lbf}$$

The reference  $2^{5/8}$  in shear plate design value for a  $1\frac{1}{2}$  in thick side member of Group B species, with a connector on one face, is tabulated in NDS Table 12.2B as

$$P_{\text{side}} = 2670 \text{ lbf} \quad [\text{governs}]$$

The gross cross-sectional area of the  $3 \times 8$  main wood member is

$$A_m = 18.13 \text{ in}^2$$

The gross cross-sectional area of the two  $2 \times 6$  side members is

$$\begin{aligned} A_s &= (2)(8.25 \text{ in}^2) \\ &= 16.50 \text{ in}^2 \\ \frac{A_s}{A_m} &= \frac{16.50 \text{ in}^2}{18.13 \text{ in}^2} \\ &= 0.91 \end{aligned}$$

The group action factor from NDS Table 10.3.6B for  $A_s/A_m = 0.91$  and  $A_s = 16.50 \text{ in}^2$  is

$$C_g = 0.98$$

The minimum end distance for the full shear plate design value, for loading parallel to the grain, is specified in NDS Table 12.3 as

$$a_{p,\text{full}} = 5.5 \text{ in}$$

The actual edge distance is

$$\begin{aligned} a_p &= 6.0 \text{ in} \\ &> a_{p,\text{full}} \end{aligned}$$

The geometry factor is given by NDS Sec. 12.3.4 as

$$C_\Delta = 1.0$$

The specified minimum spacing for the full shear plate design value, for loading parallel to the grain, is given in NDS Table 12.3 as

$$s_{\text{full}} = 6.75 \text{ in}$$

The specified minimum spacing for the reduced shear plate design value, for loading parallel to the grain, is specified in NDS Table 12.3 as

$$s_{\text{red}} = 3.5 \text{ in}$$

The actual spacing is

$$\begin{aligned} s &= 6 \text{ in} \\ &< s_{\text{full}} \\ &> s_{\text{red}} \end{aligned}$$

The geometry factor for shear plate spacing is given by interpolation, as specified in NDS Sec. 12.3.5, as

$$\begin{aligned} C_\Delta &= 0.5 + \frac{(0.5)(6 \text{ in} - 3.5 \text{ in})}{6.75 \text{ in} - 3.5 \text{ in}} \\ &= 0.885 \quad [\text{governs}] \end{aligned}$$

The wet service factor is obtained from NDS Table 10.3.3 as

$$C_M = 0.7$$

The allowable design value for four shear plates is

$$\begin{aligned} T &= nP_{\text{side}}C_gC_\Delta C_M \\ &= (4)(2670 \text{ lbf})(0.98)(0.885)(0.7) \\ &= 6500 \text{ lbf} \end{aligned}$$

## Wood Screw Connections

In accordance with NDS Sec. 11.1.4.3, for screws loaded laterally in wood of specific gravity in excess of 0.6, a clearance hole approximately equal in diameter to the diameter of the shank must be bored in the member for the full length of the unthreaded shank. The lead hole receiving the threaded portion of the screw must have a diameter approximately equal to the wood screw root diameter. For wood of specific gravity not exceeding 0.6, a clearance hole approximately equal in diameter to  $\frac{7}{8}$  the diameter of the shank must be bored in the member for the full length of the unthreaded shank. The lead hole receiving the threaded portion of the screw must have a diameter approximately equal to  $\frac{7}{8}$  the diameter of the wood screw root diameter.

In accordance with NDS Sec. 11.1.4.2, for screws loaded in withdrawal in wood of specific gravity in excess of 0.6, a clearance hole approximately equal in diameter to 90% of the wood screw root diameter must be provided for the full length of the screw. For wood with specific gravity of 0.5 or less, lead holes are not required. For wood of intermediate specific gravity, the lead hole diameter must approximately equal 70% of the wood screw root diameter. The screw must be inserted into the lead hole by turning with a screw driver, not by driving with a hammer.

## Lateral Design Values in Side Grain

Recommended edge distances, end distances, and spacing are tabulated in NDS Commentary Table C11.1.4.7 for wood and steel side plates with and without pre-bored holes. Wood screws are not subject to the group action factor,  $C_g$ .

As specified in NDS Table 11L, for full design values to be applicable, the depth of penetration must not be less than

$$p_{\text{full}} = 10D$$

The minimum allowable penetration is given by NDS Sec. 11.1.4.6 as

$$p_{\min} = 6D$$

When the actual penetration,  $p$ , is between  $6D$  and  $10D$ , the nominal design value is multiplied by the penetration factor, which is defined in NDS Table 11L as

$$C_d = \frac{p}{p_{\text{full}}} \leq 1.0$$

Reference design values for single shear connections are specified in NDS Sec. 11.3, tabulated in NDS Table 11L for connections with a wood side member, and tabulated in NDS Table 11M for connections with a steel side plate.

### Withdrawal Design Values in Side Grain

Withdrawal design values in pounds per inch of thread penetration are tabulated in NDS Table 11.2B. The length of thread is specified in the NDS App. L as  $\frac{2}{3}$  the total screw length or four times the screw diameter, whichever is the greater.

Wood screws shall not be loaded in withdrawal from end grain of wood.

### Combined Lateral and Withdrawal Loads

When the load applied to a wood screw is at an angle,  $\alpha$ , to the wood surface, as shown in Fig. 5.11, the wood screw is subjected to combined lateral and withdrawal loading. The design value is determined by the Hankinson's formula, given in NDS Sec. 11.4.1 as

$$Z'_\alpha = \frac{W'pZ'}{W'p \cos^2 \alpha + Z' \sin^2 \alpha} \quad [\text{NDS 11.4-1}]$$

### Example 5.19

A  $2 \times 4$  select structural Douglas Fir-Larch collector is secured to the Douglas Fir-Larch top plate of a shear wall with a single row of eight 14 g  $\times 3\frac{1}{2}$  in wood screws. Edge distances, end distances, and spacing are sufficient to prevent splitting of the wood. Determine the maximum tensile force,  $T$ , due to seismic load, which can be resisted by the wood screws.

#### Solution

The diameter of a 14 g wood screw is obtained from NDS App. L as

$$D = 0.242 \text{ in}$$

The reference design value for a 14 g wood screw in a  $1\frac{1}{2}$  in Douglas Fir-Larch side member in single shear is tabulated in NDS Table 11L as

$$Z = 163 \text{ lbf}$$

As specified in NDS Table 11L, for full design values to be applicable, the depth of penetration shall not be less than

$$\begin{aligned} p_{\text{full}} &= 10D \\ &= (10)(0.242 \text{ in}) \\ &= 2.42 \text{ in} \end{aligned}$$

The minimum allowable penetration is given by NDS Sec. 11.1.4.6 as

$$\begin{aligned} p_{\min} &= 6D \\ &= (6)(0.242 \text{ in}) \\ &= 1.45 \text{ in} \end{aligned}$$

The actual penetration of the wood screw into the top plate of the shear wall is

$$\begin{aligned} p &= 3.5 \text{ in} - 1.5 \text{ in} \\ &= 2.0 \text{ in} \\ &< p_{\text{full}} \\ &> p_{\min} \end{aligned}$$

The penetration depth factor is

$$\begin{aligned} C_d &= \frac{p}{p_{\text{full}}} \\ &= \frac{2 \text{ in}}{2.42 \text{ in}} \\ &= 0.826 \end{aligned}$$

The load duration factor for seismic load is obtained from NDS Table 2.3.2 as

$$C_D = 1.60$$

The allowable lateral design value for eight screws is

$$\begin{aligned} T &= nZC_DC_d \\ &= (8)(163 \text{ lbf})(1.60)(0.826) \\ &= 1723 \text{ lbf} \end{aligned}$$

### Connections with Nails and Spikes

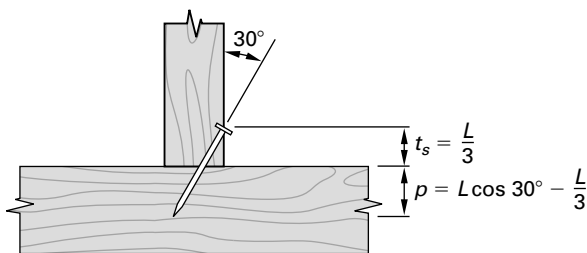
The NDS specifications apply to common nails and spikes, box nails, sinker nails, and threaded hardened-steel nails. The tabulated reference design values apply to nailed connections either with or without prebored holes. A prebored hole may be used to prevent splitting of the wood. For wood of specific gravity in excess of 0.6, the hole diameter must not exceed 90% of diameter of

the nail. For wood of specific gravity not exceeding 0.6, the hole diameter must not exceed 75% of the diameter of the nail.

As shown in Fig. 5.12 and specified in NDS Sec. 11.1.5, toe-nails are driven at an angle of approximately  $30^\circ$  with the face of the member, with the point of penetration approximately  $\frac{1}{3}$  the length of the nail from the member end. In accordance with NDS Commentary Sec. C11.5.4, the side member thickness,  $t_s$ , is taken as equal to this end distance, and

$$t_s = \frac{L}{3}$$

**Figure 5.12** Toe-Nail Connection



### Lateral Design Values in Side Grain

Recommended edge distances, end distances, and spacing are tabulated in NDS Commentary Table C11.1.5.6 for wood and steel side plates with and without pre-bored holes. Nails and spikes are not subject to the group action factor,  $C_g$ .

As specified in NDS Table 11N, for full design values to be applicable, the depth of penetration must not be less than

$$p_{\text{full}} = 10D$$

The minimum allowable penetration is given by NDS Table 11N as

$$p_{\text{min}} = 6D$$

When the actual penetration,  $p$ , is between  $6D$  and  $10D$  the reference design value is multiplied by the penetration factor, which is defined in NDS Table 11N as

$$C_d = \frac{p}{p_{\text{full}}} \leq 1.0$$

In accordance with NDS Sec. 11.5.3, reference design values for nails and spikes used in diaphragm construction must be multiplied by the diaphragm factor, which is

$$C_{di} = 1.1$$

In accordance with NDS Sec. 11.5.4, reference lateral design values for nails and spikes used in toe-nailed connections must be multiplied by the toe-nail factor, which is

$$C_{tn} = 0.83$$

Reference design values for single shear connections for two sawn lumber members of identical species are tabulated in NDS Table 11N.

Reference design values for single shear connections for a sawn lumber member with a steel side plate are tabulated in NDS Table 11P.

In accordance with NDS Sec. 11.3.7, the nominal double shear value for a three-member sawn lumber connection is twice the lesser of the nominal design values for each shear plane. The minimum allowable penetration into the side members is given by NDS Sec. 11.1.5.5 as

$$p_{\text{min}} = 6D$$

An exception is permitted when the side member is at least  $\frac{3}{8}$  in thick, and  $12d$  or smaller nails extend at least three diameters beyond the side member and are clinched.

### Withdrawal Design Values in Side Grain

Reference withdrawal design values in pounds per inch of penetration are tabulated in NDS Table 11.2C. In accordance with NDS Sec. 11.5.4, reference withdrawal design values for nails and spikes used in toe-nailed connections shall be multiplied by the toe-nail factor, which is

$$C_{tn} = 0.67$$

As specified in NDS Sec. 11.2.3.2, nails and spikes must not be loaded in withdrawal from end grain of wood.

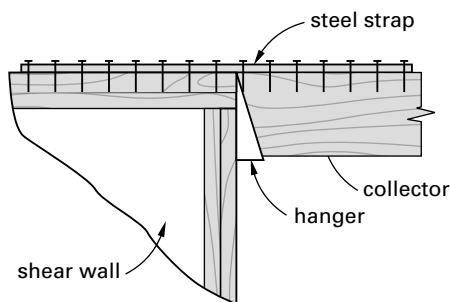
### Combined Lateral and Withdrawal Loads

When the load applied to a wood screw is at an angle,  $\alpha$ , to the wood surface, the wood screw is subjected to combined lateral and withdrawal loading. In accordance with NDS Sec. 11.4.2, the design value is determined by the interaction equation

$$Z'_\alpha = \frac{W'pZ'}{W'p \cos \alpha + Z' \sin \alpha} \quad [\text{NDS 11.4-2}]$$

### Example 5.20

A  $4 \times 8$  select structural Douglas Fir-Larch collector is secured to the Douglas Fir-Larch top plate of a shear wall with a 12 gage steel strap, as shown in the illustration. Fourteen  $16d$  common nails,  $2\frac{1}{2}$  in long, are provided on each side of the strap. Edge distances, end distances, and spacing are sufficient to prevent splitting of the wood. Determine the maximum tensile force,  $T$ , due to seismic load, which can be resisted by the nails.



### Solution

The diameter of a 16d common nail is obtained from NDS Table 11P as

$$D = 0.162 \text{ in}$$

The reference single shear design value for a 16d common nail in a Douglas Fir-Larch member with a 12 gauge side plate is tabulated in NDS Table 11P as

$$Z = 149 \text{ lbf}$$

As specified in NDS Table 11N, for full design values to be applicable, the depth of penetration shall not be less than

$$\begin{aligned} p_{\text{full}} &= 10D \\ &= (10)(0.162 \text{ in}) \\ &= 1.62 \text{ in} \end{aligned}$$

The actual penetration of the nails is

$$\begin{aligned} p &= 2.5 \text{ in} - 0.105 \text{ in} \\ &= 2.395 \text{ in} \\ &> p_{\text{full}} \end{aligned}$$

The penetration depth factor is

$$C_d = 1.0$$

The load duration factor for seismic load is obtained from NDS Table 2.3.2 as

$$C_D = 1.60$$

The allowable lateral design value for fourteen nails is

$$\begin{aligned} T &= nZC_DC_d \\ &= (14)(149 \text{ lbf})(1.60)(1.0) \\ &= 3340 \text{ lbf} \end{aligned}$$

### PRACTICE PROBLEMS

Problems 1–3 refer to a glued laminated beam of combination 24F-V4 (24F-1.8E) western species that has a width of  $6\frac{3}{4}$  in, and a depth of 24 in. The beam is simply supported over an effective span of 24 ft, and is laterally braced at 8 ft on center. The governing load combination is a uniformly distributed dead load plus live load, and the moisture content exceeds 16%.

1. What is most nearly the volume factor for the beam?

- (A) 0.75
- (B) 0.80
- (C) 0.85
- (D) 0.90

2. What is most nearly the stability factor?

- (A) 0.91
- (B) 0.95
- (C) 0.99
- (D) 1.0

3. What is most nearly the design value in bending?

- (A) 1728 lbf/in<sup>2</sup>
- (B) 1824 lbf/in<sup>2</sup>
- (C) 1901 lbf/in<sup>2</sup>
- (D) 1920 lbf/in<sup>2</sup>

Problems 4–6 refer to a 6 × 6 Douglas Fir-Larch no. 1 visually graded timber post that is 12 ft long and pinned at each end. The governing load combination consists of dead load plus live load, and the column is not braced laterally.

4. What is most nearly the stability factor of the post?

- (A) 0.52
- (B) 0.56
- (C) 0.64
- (D) 0.75

5. Neglecting the self-weight of the column, what is most nearly the maximum load that the post can support?

- (A) 15,125 lbf
- (B) 16,940 lbf
- (C) 19,360 lbf
- (D) 22,687 lbf



6. Determine whether the post is adequate to support an axial load of 5 kips and a horizontal wind load of 500 lbf applied on one face at midheight.

- (A) No, the interaction equation for bending and concentric axial compression is less than 1.0.
- (B) No, the interaction equation for bending and concentric axial compression is greater than 1.0.
- (C) Yes, the interaction equation for bending and concentric axial compression is less than 1.0.
- (D) Yes, the interaction equation for bending and concentric axial compression is greater than 1.0.

### SOLUTIONS

1. The volume factor is given by NDS Eq. (5.3-1) as

$$\begin{aligned} C_V &= \left( \frac{1291.5}{bdL} \right)^{1/x} \\ &= \left( \frac{1291.5 \text{ in}^2\text{-ft}}{(6.75 \text{ in})(24 \text{ in})(24 \text{ ft})} \right)^{1/10} \\ &= 0.90 \end{aligned}$$

**The answer is (D).**

2. The reference design values for bending and the modulus of elasticity, tabulated in NDS Supp. Table 5A, are

$$\begin{aligned} F_b &= 2400 \frac{\text{lbf}}{\text{in}^2} \\ E_{y,\min} &= 0.83 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \end{aligned}$$

The applicable adjustment factor for the modulus of elasticity is the wet service factor.

$$C_M = 0.833 \quad [\text{NDS Supp. Table 5A}]$$

The adjusted modulus of elasticity for stability calculations is

$$\begin{aligned} E'_{\min} &= E_{y,\min} \\ &= E_{y,\min} C_M \\ &= \left( 0.83 \times 10^6 \frac{\text{lbf}}{\text{in}^2} \right) (0.833) \\ &= 0.69 \times 10^6 \text{ lbf/in}^2 \end{aligned}$$

The distance between lateral restraints is

$$\begin{aligned} l_u &= 8 \text{ ft} \\ \frac{l_u}{d} &= \frac{(8 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{24 \text{ in}} \\ &= 4.0 \\ &< 7 \end{aligned}$$

For a uniformly distributed load and an  $l_u/d$  ratio less than 7, the effective length is obtained from NDS Table 3.3.3 as

$$\begin{aligned} l_e &= 2.06 l_u \\ &= (2.06)(8 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right) \\ &= 198 \text{ in} \end{aligned}$$



The slenderness ratio is given by NDS Sec. 3.3.3 as

$$\begin{aligned} R_B &= \sqrt{\frac{l_e d}{b_2}} \\ &= \sqrt{\frac{(198 \text{ in})(24 \text{ in})}{(6.75 \text{ in})^2}} \\ &= 10.21 \\ &< 50 \quad [\text{satisfies criteria of NDS Sec. 3.3.3}] \end{aligned}$$

The critical buckling design value is

$$\begin{aligned} F_{bE} &= \frac{1.20 E'_{\min}}{R_B^2} \\ &= \frac{(1.20) \left( 0.69 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right)}{(10.21)^2} \\ &= 7943 \text{ lb}/\text{in}^2 \end{aligned}$$

The reference flexural design value, multiplied by all applicable adjustment factors except  $C_V$  and  $C_L$ , is

$$\begin{aligned} F_b^* &= F_b C_M \\ &= \left( 2400 \frac{\text{lb}}{\text{in}^2} \right) (0.80) \\ &= 1920 \text{ lb}/\text{in}^2 \end{aligned}$$

The ratio of  $F_{bE}$  to  $F_b^*$  is

$$\begin{aligned} F &= \frac{F_{bE}}{F_b^*} \\ &= \frac{7943 \frac{\text{lb}}{\text{in}^2}}{1920 \frac{\text{lb}}{\text{in}^2}} \\ &= 4.14 \end{aligned}$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$\begin{aligned} C_L &= \frac{1.0 + F}{1.9} - \sqrt{\left( \frac{1.0 + F}{1.9} \right)^2 - \frac{F}{0.95}} \\ &= \frac{1.0 + 4.14}{1.9} - \sqrt{\left( \frac{1.0 + 4.14}{1.9} \right)^2 - \frac{4.14}{0.95}} \\ &= 0.98 \end{aligned}$$

**The answer is (C).**

**3.** From Prob. 2, the beam stability factor is

$$C_L = 0.98$$

From Prob. 1, the volume factor is

$$\begin{aligned} C_V &= 0.90 \\ &< C_L \end{aligned}$$

The volume factor governs.

The allowable flexural design value is

$$\begin{aligned} F'_b &= C_V F_b C_M \\ &= C_V F_b^* \\ &= (0.90) \left( 1920 \frac{\text{lb}}{\text{in}^2} \right) \\ &= 1728 \text{ lb}/\text{in}^2 \end{aligned}$$

**The answer is (A).**

**4.** The basic design values for compression parallel to the grain are obtained from NDS Supp. Table 4D as

$$\begin{aligned} F_c &= 1000 \frac{\text{lb}}{\text{in}^2} \\ E_{\min} &= 0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2} \end{aligned}$$

The size factor for a post from NDS Supp. Table 4D is

$$\begin{aligned} C_F &= 0.90 \quad [\text{modulus of elasticity for flexure}] \\ &= 1.00 \quad [\text{modulus of elasticity for compression}] \end{aligned}$$

For pinned-ended support conditions, the buckling length coefficient from NDS App. Table G1 is

$$K_e = 1.0$$

The slenderness ratio about both axes is

$$\begin{aligned} \frac{K_e l}{d} &= \frac{(1.0)(12 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{5.5 \text{ in}} \\ &= 26.18 \end{aligned}$$

The adjusted modulus of elasticity for compression for stability calculations is

$$\begin{aligned} E'_{\min} &= E_{\min} C_F \\ &= \left( 0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2} \right) (1.00) \\ &= 0.58 \times 10^6 \text{ lb}/\text{in}^2 \end{aligned}$$

The tabulated compression design value, multiplied by all applicable adjustment factors except  $C_P$ , is

$$\begin{aligned} F_c^* &= F_c C_F \\ &= \left( 1000 \frac{\text{lb}}{\text{in}^2} \right) (1.00) \\ &= 1000 \text{ lb}/\text{in}^2 \end{aligned}$$

The critical buckling design value is

$$\begin{aligned} F_{cE} &= \frac{0.822 E'_{\min}}{\left(\frac{l_e}{d}\right)^2} \\ &= \frac{(0.822) \left(0.58 \times 10^6 \frac{\text{lb}}{\text{in}^2}\right)}{(26.18)^2} \\ &= 696 \text{ lb}/\text{in}^2 \end{aligned}$$

The ratio of  $F_{cE}$  to  $F_c^*$  is

$$\begin{aligned} F' &= \frac{F_{cE}}{F_c^*} \\ &= \frac{696 \frac{\text{lb}}{\text{in}^2}}{1000 \frac{\text{lb}}{\text{in}^2}} \\ &= 0.696 \end{aligned}$$

The column parameter is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8 \quad [\text{sawn lumber}]$$

The column stability factor specified by NDS Sec. 3.7.1 is

$$\begin{aligned} C_p &= \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}} \\ &= \frac{1.0 + 0.696}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 0.696}{(2)(0.8)}\right)^2 - \frac{0.696}{0.8}} \\ &= 0.56 \end{aligned}$$

**The answer is (B).**

**5.** From Prob. 4, the column stability factor is

$$C_P = 0.56$$

The allowable compression design value parallel to the grain is

$$\begin{aligned} F'_c &= F_c C_F C_P \\ &= \left(1000 \frac{\text{lb}}{\text{in}^2}\right)(0.56) \\ &= 560 \text{ lb}/\text{in}^2 \end{aligned}$$

The allowable axial load on the column is

$$\begin{aligned} P_{al} &= A F'_c \\ &= (30.25 \text{ in}^2) \left(560 \frac{\text{lb}}{\text{in}^2}\right) \\ &= 16,940 \text{ lb} \end{aligned}$$

**The answer is (B).**

**6.** The reference design value for bending from NDS Supp. Table 4D is

$$F_b = 1200 \frac{\text{lb}}{\text{in}^2}$$

The applicable load duration factor for flexure from NDS Table 2.3.2 is

$$C_D = 1.6$$

The size factor for flexure from NDS Supp. Table 4D is

$$C_F = 0.74$$

The beam stability factor for a square section, given by NDS Sec. 3.3.3.1, is

$$C_L = 1.0$$

The allowable flexural design value is

$$\begin{aligned} F'_b &= F_b C_F C_D C_L \\ &= \left(1200 \frac{\text{lb}}{\text{in}^2}\right)(0.74)(1.6)(1.0) \\ &= 1421 \text{ lb}/\text{in}^2 \end{aligned}$$

The actual edgewise bending stress is

$$\begin{aligned} f_{b1} &= \frac{WL}{4S} \\ &= \frac{(500 \text{ lb})(12 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{(4)(27.73 \text{ in}^3)} \\ &= 649 \text{ lb}/\text{in}^2 \\ &< F'_b \quad [\text{satisfactory}] \end{aligned}$$

The allowable compression design value parallel to the grain, obtained from Prob. 5, is

$$F'_c = 560 \frac{\text{lb}}{\text{in}^2}$$

The actual compressive stress on the column is

$$\begin{aligned} f_c &= \frac{P}{A} \\ &= \frac{5000 \text{ lb}}{30.25 \text{ in}^2} \\ &= 165 \frac{\text{lb}}{\text{in}^2} \\ &< F'_c \quad [\text{satisfactory}] \end{aligned}$$

The distance between lateral restraints for bending instability is

$$\begin{aligned} l_u &= 12 \text{ ft} \\ \frac{l_u}{d} &= \frac{(12 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}} \\ &= 26 \\ &> 7 \end{aligned}$$

For a concentrated load at midheight and an  $l_u/d$  ratio greater than 7, the effective length from NDS Table 3.3.3 is

$$\begin{aligned} l_e &= 1.37l_u + 3d \\ &= (1.37)(12 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right) + (3)(5.5 \text{ in}) \\ &= 214 \text{ in} \end{aligned}$$

The adjusted modulus of elasticity for flexure is

$$\begin{aligned} E'_{\min} &= E_{\min} C_F \\ &= \left(0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right) (0.90) \\ &= 0.52 \times 10^6 \text{ lbf/in}^2 \end{aligned}$$

The critical buckling design value in the plane of bending is

$$\begin{aligned} F_{cE1} &= \frac{0.822 E'_{\min}}{\left(\frac{l_{e1}}{d_1}\right)^2} = \frac{(0.822) \left(0.52 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{214 \text{ in}}{5.5 \text{ in}}\right)^2} \\ &= 282 \text{ lbf/in}^2 \\ &> f_c \quad [\text{satisfactory}] \end{aligned}$$

The moment magnification factor for axial compression and flexure is

$$\begin{aligned} C_{m3} &= 1.0 - \frac{f_c}{F_{cE1}} = 1.0 - \frac{165 \frac{\text{lbf}}{\text{in}^2}}{282 \frac{\text{lbf}}{\text{in}^2}} \\ &= 0.41 \end{aligned}$$

The interaction equation for bending and concentric axial compression is given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m3}} \leq 1.0$$

The left-hand side of the expression is

$$\begin{aligned} \left(\frac{165 \frac{\text{lbf}}{\text{in}^2}}{560 \frac{\text{lbf}}{\text{in}^2}}\right)^2 + \frac{649 \frac{\text{lbf}}{\text{in}^2}}{\left(1421 \frac{\text{lbf}}{\text{in}^2}\right) (0.41)} &= 0.087 + 1.114 \\ &= 1.201 \\ &> 1.0 \\ &[\text{unsatisfactory}] \end{aligned}$$

The post is inadequate.

**The answer is (B).**

# 6

# Design of Reinforced Masonry

|                                                 |      |
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## 1. DESIGN PRINCIPLES

### Nomenclature

|       |                                                                                               |                 |
|-------|-----------------------------------------------------------------------------------------------|-----------------|
| $b_s$ | width of support                                                                              | in              |
| $d$   | effective depth, distance from extreme compression fiber to centroid of tension reinforcement | in              |
| $D$   | dead load                                                                                     | kips or kips/ft |
| $E$   | earthquake load                                                                               | kips or kips/ft |
| $F$   | load caused by fluids                                                                         | kips or kips/ft |
| $h$   | overall depth of beam                                                                         | in              |
| $H$   | load caused by lateral earth pressure                                                         | kips or kips/ft |
| $l$   | clear span length of beam between supports                                                    | ft              |
| $l_e$ | effective span length of beam                                                                 | ft              |
| $l_s$ | distance between centers of supports                                                          | ft              |
| $L$   | live load                                                                                     | kips or kips/ft |
| $M_D$ | service level bending moment caused by applied loads                                          | ft-kips         |
| $M_s$ | service level bending moment caused by member self-weight                                     | ft-kips         |
| $Q$   | required strength                                                                             | kips or ft-kips |
| $R$   | rain load                                                                                     | kips or kips/ft |
| $S$   | snow load                                                                                     | kips or kips/ft |
| $T$   | self-straining force                                                                          | kips or kips/ft |
| $V$   | unfactored shear force at section under consideration                                         | kips            |
| $V_D$ | shear force caused by dead loads                                                              | kips            |
| $V_L$ | shear force caused by live loads                                                              | kips            |
| $V_s$ | service level shear force caused by member self-weight                                        | kips            |
| $w$   | distributed load                                                                              | lb/ft           |
| $W$   | wind load                                                                                     | kips or kips/ft |

### Symbols

|        |                           |   |
|--------|---------------------------|---|
| $\phi$ | strength reduction factor | — |
|--------|---------------------------|---|

## General Design Requirements

Allowable stress design is the method of proportioning structural members so that the allowable stress of the material is not exceeded under the action of design loads calculated using ASD load combinations.

The design of masonry structures using allowable stress design is permitted by IBC<sup>1</sup> Sec. 2107.1. Design must comply with MSJC<sup>2</sup> Chs. 1 and 2, with the exception of Secs. 2.1.2, 2.1.9.7, and 2.1.9.7.1.1. Hence, the load combinations in MSJC Ch. 2 are not adopted.

## Load Combinations

Where the legally adopted building code does not provide load combinations, the combinations specified in ASCE<sup>3</sup> Sec. 2.4.1 must be used. The governing load condition for dead load and fluid pressure is given by ASCE Sec. 2.4.1 load combination 1 as

$$Q = D + F$$

The governing load condition when the structure is subjected to maximum values of occupancy live loads is given by load combination 2 as

$$Q = D + F + T + L + H$$

The governing load condition when the structure is subjected to maximum values of roof live load, rainwater, or snow load is given by load combination 3 as

$$Q = D + H + F + (L_r \text{ or } S \text{ or } R)$$

The governing load condition when the structure is subjected to simultaneous values of occupancy live loads, roof live load, rainwater, or snow load is given by load combination 4 as

$$Q = D + H + F + 0.75(L + T) + 0.75(L_r \text{ or } S \text{ or } R)$$

The governing load condition when the structure is subjected to simultaneous values of occupancy live loads, roof live load, rainwater, or snow load plus wind

<sup>1</sup>International Code Council, 2009 (See References and Codes)

<sup>2</sup>American Concrete Institute, 2008 (See References and Codes)

<sup>3</sup>American Society of Civil Engineers, 2005 (See References and Codes)

load or seismic load increasing the effects of dead load is given by load combination 6 as

$$Q = D + H + F + 0.75(W \text{ or } 0.75E) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$$

The governing load condition when the structure is subjected to maximum values of seismic load or wind load, which increase the effects of dead load, is given by load combination 5 as

$$Q = D + H + F + (W + 0.7E)$$

The governing load condition when the structure is subjected to maximum values of wind load, which oppose the effects of dead load, is given by load combination 7 as

$$Q = 0.6D + W + H$$

The governing load condition when the structure is subjected to maximum values of seismic load, which oppose the effects of dead load, is given by load combination 8 as

$$Q = 0.6D + 0.7E + H$$

No increase in allowable stresses is permitted with these load combinations.

### Effective Span Length of Beams

In accordance with MSJC Sec. 1.13, the effective span length of a simply supported beam is defined as the clear span plus the depth of the member, but it is not to exceed the distance between the centers of the supports. As shown in Fig. 6.1(b) and (c),

$$l_e = l + h \leq l_s$$

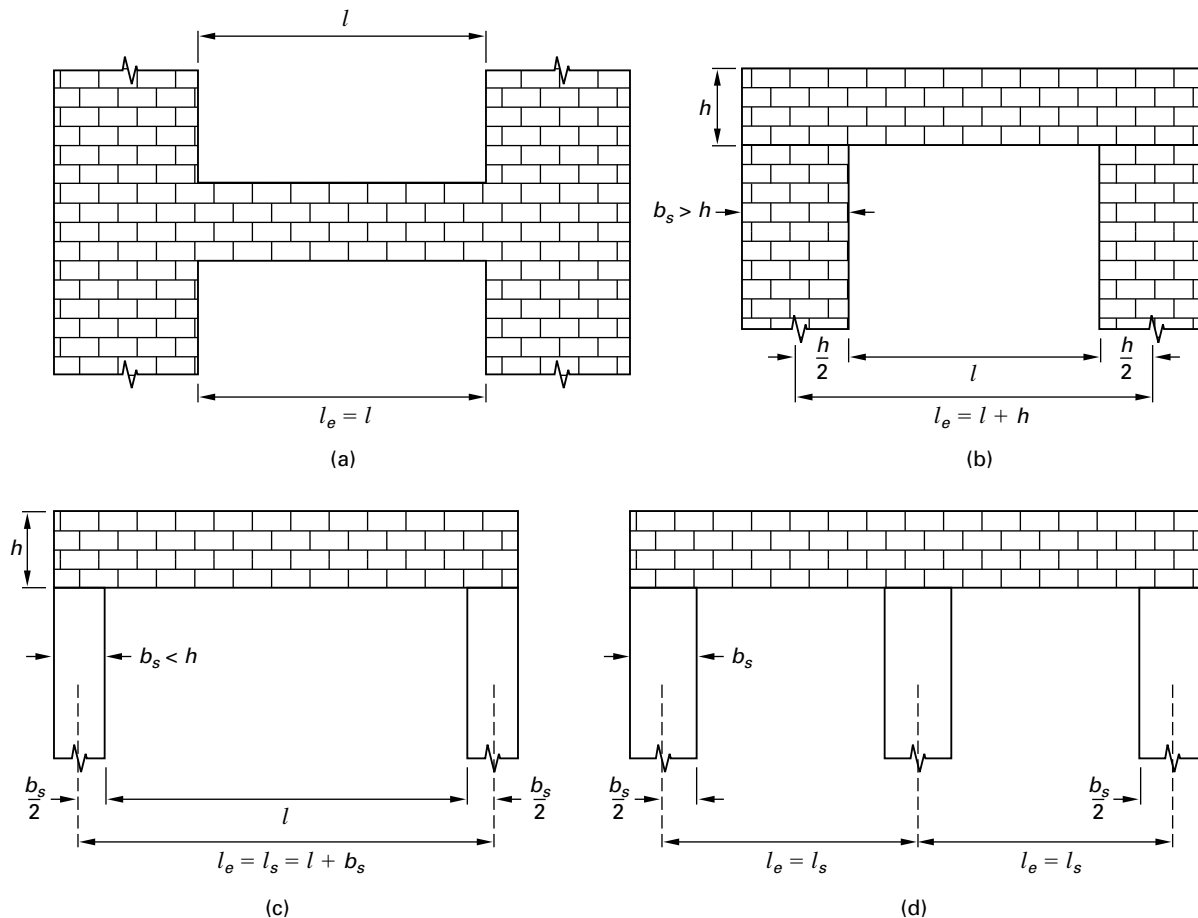
For a continuous beam, the effective span length is defined as the distance between the centers of the supports. As shown in Fig. 6.1(d),

$$l_e = l_s$$

The effective span length of beams built integrally with supports is customarily taken as being equal to the clear span. As shown in Fig. 6.1(a),

$$l_e = l$$

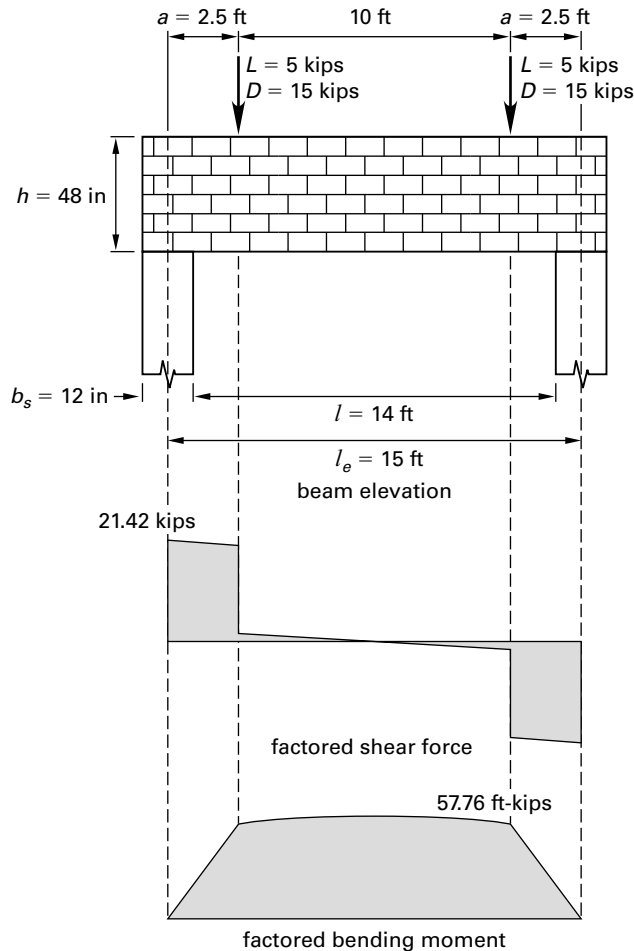
Figure 6.1 Effective Span Length



**Example 6.1**

The nominal 8 in, solid grouted concrete block masonry beam shown in the following illustration is simply supported over a clear span of 14 ft. The overall depth of the beam is 48 in, and its effective depth,  $d$ , is 45 in. The weight may be assumed to be  $69 \text{ lbf/ft}^2$ . Determine the maximum bending moment on the beam. Determine the shear force at a distance of  $d/2$  from the face of the support.

Illustration for Example 6.1

**Solution**

The effective span length is given by MSJC Sec. 1.13.1.2 as

$$\begin{aligned} l_e &= l_s \\ &= l + b_s \\ &= 14 \text{ ft} + 1 \text{ ft} \\ &= 15 \text{ ft} \end{aligned}$$

The beam self-weight is

$$\begin{aligned} w &= \left( 69 \frac{\text{lbf}}{\text{ft}^2} \right) (4 \text{ ft}) \\ &= 276 \text{ lbf/ft} \end{aligned}$$

The bending moment at midspan, produced by this self-weight, is

$$\begin{aligned} M_s &= \frac{wl_e^2}{8} \\ &= \frac{\left( 276 \frac{\text{lbf}}{\text{ft}} \right) (15 \text{ ft})^2}{8} \\ &= \frac{(8) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)}{8} \\ &= 7.76 \text{ ft-kips} \end{aligned}$$

The bending moment at midspan, produced by the concentrated dead loads, is

$$\begin{aligned} M_D &= Da \\ &= (15 \text{ kips})(2.5 \text{ ft}) \\ &= 37.5 \text{ ft-kips} \end{aligned}$$

The bending moment at midspan, produced by the concentrated live loads, is

$$\begin{aligned} M_L &= La \\ &= (5 \text{ kips})(2.5 \text{ ft}) \\ &= 12.5 \text{ ft-kips} \end{aligned}$$

The total moment at midspan is given by ASCE Sec. 2.4.1 as

$$\begin{aligned} M &= M_s + M_D + M_L \\ &= 7.76 \text{ ft-kips} + 37.5 \text{ ft-kips} \\ &\quad + 12.5 \text{ ft-kips} \\ &= 57.76 \text{ ft-kips} \end{aligned}$$

The shear force at a distance of  $d/2$  from the face of each support, produced by the beam self-weight, is

$$\begin{aligned} V_s &= \frac{w(l_e - d - b_s)}{2} \\ &= \frac{\left( 0.276 \frac{\text{kips}}{\text{ft}} \right) (15 \text{ ft} - 3.75 \text{ ft} - 1 \text{ ft})}{2} \\ &= 1.41 \text{ kips} \end{aligned}$$

The shear force at a distance of  $d/2$  from the face of each support, produced by the concentrated dead loads, is

$$\begin{aligned} V_D &= D \\ &= 15 \text{ kips} \end{aligned}$$

The shear force at a distance of  $d/2$  from each support, produced by the concentrated live loads, is

$$V_L = L = 5 \text{ kips}$$

The total shear force at a distance of  $d/2$  from the face of each support is given by ASCE Sec. 2.4.1 as

$$\begin{aligned} V &= V_s + V_D + V_L \\ &= 1.41 \text{ kips} + 15 \text{ kips} + 5 \text{ kips} \\ &= 21.41 \text{ kips} \end{aligned}$$

The relevant values are shown in the illustration.

## 2. DESIGN FOR FLEXURE

### Nomenclature

|        |                                                                                                                                                    |                     |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| $A_s$  | area of tension reinforcement                                                                                                                      | in <sup>2</sup>     |
| $b_w$  | width of beam                                                                                                                                      | in                  |
| $d$    | effective depth, distance from extreme compression fiber to centroid of tension reinforcement                                                      | in                  |
| $d_b$  | bar diameter                                                                                                                                       | in                  |
| $E_m$  | modulus of elasticity of masonry in compression, $900 f'_m$                                                                                        | lbf/in <sup>2</sup> |
| $E_s$  | modulus of elasticity of steel reinforcement, 29,000,000                                                                                           | lbf/in <sup>2</sup> |
| $f_b$  | calculated compressive stress in masonry due to flexure                                                                                            | lbf/in <sup>2</sup> |
| $f'_m$ | specified masonry compressive strength                                                                                                             | lbf/in <sup>2</sup> |
| $f_s$  | calculated tensile stress in reinforcement due to flexure                                                                                          | lbf/in <sup>2</sup> |
| $f_y$  | yield strength of reinforcement                                                                                                                    | lbf/in <sup>2</sup> |
| $F_b$  | allowable compressive stress in masonry due to flexure                                                                                             | lbf/in <sup>2</sup> |
| $F_s$  | allowable tensile stress in reinforcement due to flexure                                                                                           | lbf/in <sup>2</sup> |
| $h$    | overall dimension of member                                                                                                                        | in                  |
| $j$    | lever-arm factor, ratio of distance between centroid of flexural compressive forces and centroid of tensile forces to effective depth, $1 - (k/3)$ | —                   |
| $k$    | neutral axis depth factor, $\sqrt{2\rho n + (\rho n)^2} - \rho n$                                                                                  | —                   |
| $K$    | lesser of masonry cover, clear spacing of reinforcement, or 5 times the bar diameter                                                               | in                  |
| $l_c$  | distance between locations of lateral support                                                                                                      | in                  |
| $l_d$  | development length or lap length of straight reinforcement                                                                                         | in                  |
| $l_e$  | development length or lap length of a standard hook                                                                                                | in                  |
| $l_e$  | effective span                                                                                                                                     | in                  |
| $M$    | moment on the member                                                                                                                               | in-kips             |
| $n$    | modular ratio, $E_s/E_m$                                                                                                                           | —                   |
| $s_c$  | clear spacing of reinforcement                                                                                                                     | in                  |

### Symbols

|              |                                             |   |
|--------------|---------------------------------------------|---|
| $\gamma$     | reinforcement size factor                   | — |
| $\epsilon_m$ | strain in masonry                           | — |
| $\epsilon_s$ | strain in tension reinforcement             | — |
| $\rho$       | ratio of tension reinforcement, $A_s/b_w d$ | — |

### Dimensional Limitations of Beams

In accordance with MSJC Sec. 1.13.2, the maximum distance between lateral supports on the compression side of the beam must not exceed

$$l_c = 32b_w$$

In accordance with MSJC Sec. 2.3.3.3, the length of bearing of a beam must not be less than

$$b_r = 4 \text{ in}$$

### Beam Reinforcement Requirements

Limits are placed on the size of reinforcing bars, in order to control the bond stresses developed in the bars, to reduce congestion, and to aid grout consolidation. As specified in MSJC Sec. 1.15.2 and IBC Sec. 2107.5, the bar diameter,  $d_b$ , must not exceed the lesser of the following.

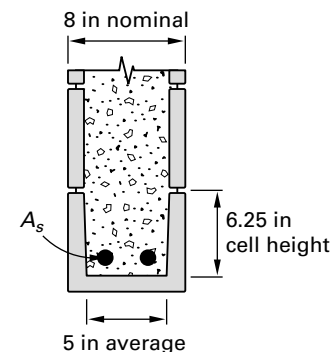
- one-eighth of the nominal member thickness
- one-quarter of the least clear dimension of the cell or course
- no. 11 bar

In accordance with MSJC Sec. 1.15.3.4, not more than 2 reinforcing bars may be bundled.

In accordance with MSJC Sec. 1.15.3.1, the clear distance between bars must not be less than the greater of  $d_b$  or 1 in. In addition, the thickness of grout between the reinforcement and the masonry unit must be a minimum of  $1/4$  in for fine grout, and a minimum of  $1/2$  in for coarse grout.

### Example 6.2

For the nominal 8 in beam shown in the illustration, determine the maximum permissible size of reinforcing bar.



*Solution*

From MSJC Sec. 1.15.2 and IBC Sec. 2107.5, the maximum size of reinforcing bar allowed is

$$\begin{aligned}d_b &= \text{no. 11 bar} \\d_b &= \frac{5 \text{ in}}{4} \\&= 1.25 \text{ in} \\d_b &= \frac{8 \text{ in}}{8} \\&= 1 \text{ in} \quad [\text{governs}]\end{aligned}$$

To conform to the governing bar diameter of 1 in, two no. 8 bars are satisfactory. This provides a clear spacing between bars of 2 in, and a thickness of grout between the reinforcement and the masonry unit of  $\frac{1}{2}$  in, which satisfies MSJC Secs. 1.15.3.1 and 1.15.5.5.

**Development Length and Splice Length of Reinforcement**

The required development length of compression and tension reinforcement is specified in MSJC Sec. 2.1.9.3 as

$$\begin{aligned}l_d &= \frac{0.13 d_b^2 f_y \gamma}{K \sqrt{f'_m}} \quad [\text{MSJC 2-12}] \\&\geq 12 \text{ in} \\ \gamma &= 1.0 \quad [\text{bar nos. 3, 4, and 5}] \\ \gamma &= 1.3 \quad [\text{bar nos. 6 and 7}] \\ \gamma &= 1.5 \quad [\text{bar nos. 8 through 11}]\end{aligned}$$

The equivalent development length of a standard hook in tension is specified in MSJC Sec. 2.1.9.5.1 as

$$l_e = 11.25 d_b$$

The minimum length of lap splices for reinforcing bars in tension or compression is given by IBC Sec. 2107.3 as

$$\begin{aligned}l_d &= 0.002 d_b f_s \\&\geq 12 \text{ in} \\&\geq 40 d_b\end{aligned}$$

When the design tensile stress in the reinforcement is greater than 80% of the allowable tension stress, lap length must be increased 50%.

Where epoxy coated bars are used, lap length must be increased 50%.

Welded or mechanical splices are required to develop a minimum of  $1.25 f_y$ .

**Example 6.3**

For the beam described in Ex. 6.2, in which the masonry has a compressive strength of 1500 lbf/in<sup>2</sup>, and the reinforcement consists of grade 60 bars, determine the

required development length for straight bars, and for bars provided with a standard hook.

*Solution*

The development length parameter,  $K$ , is the lesser of masonry cover, clear spacing of reinforcement, or 5 times the bar diameter. For masonry cover,

$$\begin{aligned}K &= \frac{b - s_c - 2d_b}{2} \\&= \frac{7.625 \text{ in} - 2.0 \text{ in} - (2)(1.0 \text{ in})}{2} \\&= 1.81 \text{ in}\end{aligned}$$

For clear spacing of reinforcement

$$\begin{aligned}K &= s_c \\&= 2 \text{ in} \quad [\text{from Ex. 6.2}]\end{aligned}$$

For the bar diameter,

$$\begin{aligned}K &= 5d_b \\&= (5)(1.0 \text{ in}) \\&= 5.0 \text{ in}\end{aligned}$$

$K = 1.81 \text{ in}$  governs.

The reinforcement size factor for a no. 8 bar is given by MSJC Sec. 2.1.9.3 as

$$\gamma = 1.5$$

The required development length is given by MSJC Eq. (2-12) as

$$\begin{aligned}l_d &= \frac{0.13 d_b^2 f_y \gamma}{K \sqrt{f'_m}} \\&= \frac{(0.13)(1.0 \text{ in})^2 \left(60,000 \frac{\text{lbf}}{\text{in}^2}\right)(1.5)}{(1.81 \text{ in}) \sqrt{1500 \frac{\text{lbf}}{\text{in}^2}}} \\&= 167 \text{ in}\end{aligned}$$

The equivalent development length provided by a standard hook is given by MSJC Sec. 2.1.9.5.1 as

$$\begin{aligned}l_e &= 11.25 d_b \\&= (11.25)(1.0 \text{ in}) \\&= 11.25 \text{ in}\end{aligned}$$

The required development length of the no. 8 bars, for bars provided with a standard hook, is

$$\begin{aligned}l_d &= 167 \text{ in} - 11.25 \text{ in} \\&= 155.75 \text{ in}\end{aligned}$$



### Design of Beams with Tension Reinforcement Only

The allowable stress design method, illustrated in Fig. 6.2 for a beam reinforced in tension, is used to calculate the stresses in a masonry beam under the action of the applied service loads, in order to ensure that these stresses do not exceed allowable values.

In the allowable stress design method, it is assumed that the strain distribution over the depth of the member is linear and that stresses in the concrete and the reinforcement are proportional to the strain. Tensile stress in the concrete is neglected.

The allowable compressive stress in the masonry caused by flexure is given by MSJC Sec. 2.3.3.2.2 as

$$F_b = \frac{f'_m}{3}$$

For grade 60 reinforcement, the allowable tensile stress is given by MSJC Sec. 2.3.2.1 as

$$F_s = 24,000 \text{ lbf/in}^2$$

The assumptions adopted are shown in Fig. 6.2. The depth of the neutral axis is obtained by equating tensile and compressive forces acting on the section and is given by

$$kd = \frac{2A_s f_s}{b_w f_b}$$

The tension reinforcement ratio is

$$\rho = \frac{A_s}{b_w d}$$

The neutral axis depth factor is then

$$k = \frac{2\rho f_s}{f_b}$$

From the strain diagram,

$$\begin{aligned} \frac{\epsilon_s}{\epsilon_m} &= \frac{d - kd}{kd} \\ &= \frac{1 - k}{k} \\ &= \frac{f_s E_m}{f_b E_s} \\ &= \frac{f_s}{n f_b} \end{aligned}$$

The modular ratio is given by

$$\begin{aligned} n &= \frac{E_s}{E_m} \\ &= \frac{29,000,000 \frac{\text{lbf}}{\text{in}^2}}{900 f'_m} \end{aligned}$$

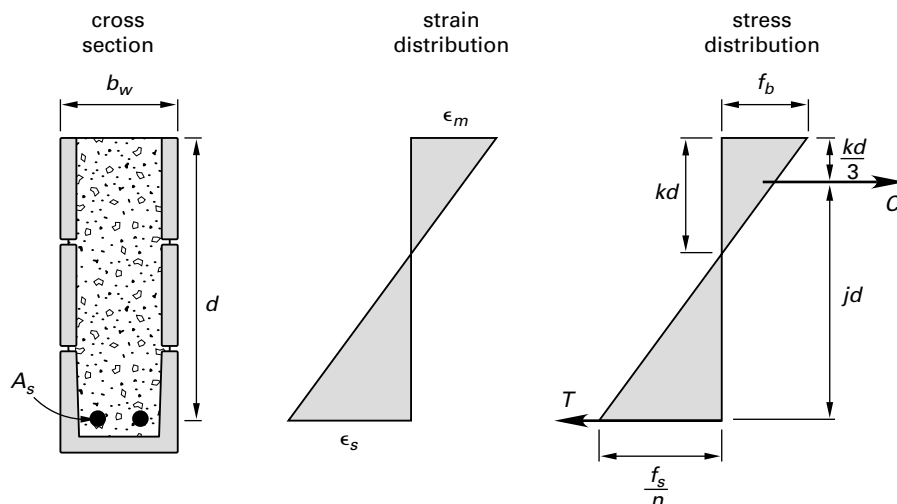
The ratio of steel stress to masonry stress is then

$$\frac{f_s}{f_b} = \frac{n(1 - k)}{k}$$

The neutral axis depth factor is given by

$$\begin{aligned} k &= \frac{2\rho n(1 - k)}{k} \\ &= \sqrt{2\rho n + (\rho n)^2} - \rho n \end{aligned}$$

**Figure 6.2** Allowable Stress Design of Concrete Masonry Beams



**Table 6.1** Values of the Neutral Axis Factor  $k$ 

| $\rho n$ | 0.000  | 0.001  | 0.002  | 0.003  | 0.004  | 0.005  | 0.006  | 0.007  | 0.008  | 0.009  |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0        | 0.0000 | 0.0437 | 0.0613 | 0.0745 | 0.0855 | 0.0951 | 0.1037 | 0.1115 | 0.1187 | 0.1255 |
| 0.01     | 0.1318 | 0.1377 | 0.1434 | 0.1488 | 0.1539 | 0.1589 | 0.1636 | 0.1682 | 0.1726 | 0.1769 |
| 0.02     | 0.1810 | 0.1850 | 0.1889 | 0.1927 | 0.1964 | 0.2000 | 0.2035 | 0.2069 | 0.2103 | 0.2136 |
| 0.03     | 0.2168 | 0.2199 | 0.2230 | 0.2260 | 0.2290 | 0.2319 | 0.2347 | 0.2375 | 0.2403 | 0.2430 |
| 0.04     | 0.2457 | 0.2483 | 0.2509 | 0.2534 | 0.2559 | 0.2584 | 0.2608 | 0.2632 | 0.2655 | 0.2679 |
| 0.05     | 0.2702 | 0.2724 | 0.2747 | 0.2769 | 0.2790 | 0.2812 | 0.2833 | 0.2854 | 0.2875 | 0.2895 |
| 0.06     | 0.2916 | 0.2936 | 0.2956 | 0.2975 | 0.2995 | 0.3014 | 0.3033 | 0.3051 | 0.3070 | 0.3088 |
| 0.07     | 0.3107 | 0.3125 | 0.3142 | 0.3160 | 0.3178 | 0.3195 | 0.3212 | 0.3229 | 0.3246 | 0.3263 |
| 0.08     | 0.3279 | 0.3296 | 0.3312 | 0.3328 | 0.3344 | 0.3360 | 0.3376 | 0.3391 | 0.3407 | 0.3422 |
| 0.09     | 0.3437 | 0.3452 | 0.3467 | 0.3482 | 0.3497 | 0.3511 | 0.3526 | 0.3540 | 0.3554 | 0.3569 |
| 0.10     | 0.3583 | 0.3597 | 0.3610 | 0.3624 | 0.3638 | 0.3651 | 0.3665 | 0.3678 | 0.3691 | 0.3705 |
| 0.11     | 0.3718 | 0.3731 | 0.3744 | 0.3756 | 0.3769 | 0.3782 | 0.3794 | 0.3807 | 0.3819 | 0.3832 |
| 0.12     | 0.3844 | 0.3856 | 0.3868 | 0.3880 | 0.3892 | 0.3904 | 0.3916 | 0.3927 | 0.3939 | 0.3951 |
| 0.13     | 0.3962 | 0.3974 | 0.3985 | 0.3996 | 0.4007 | 0.4019 | 0.4030 | 0.4041 | 0.4052 | 0.4063 |
| 0.14     | 0.4074 | 0.4084 | 0.4095 | 0.4106 | 0.4116 | 0.4127 | 0.4137 | 0.4148 | 0.4158 | 0.4169 |
| 0.15     | 0.4179 | 0.4189 | 0.4199 | 0.4209 | 0.4219 | 0.4229 | 0.4239 | 0.4249 | 0.4259 | 0.4269 |
| 0.16     | 0.4279 | 0.4288 | 0.4298 | 0.4308 | 0.4317 | 0.4327 | 0.4336 | 0.4346 | 0.4355 | 0.4364 |
| 0.17     | 0.4374 | 0.4383 | 0.4392 | 0.4401 | 0.4410 | 0.4419 | 0.4429 | 0.4437 | 0.4446 | 0.4455 |
| 0.18     | 0.4464 | 0.4473 | 0.4482 | 0.4491 | 0.4499 | 0.4508 | 0.4516 | 0.4525 | 0.4534 | 0.4542 |
| 0.19     | 0.4551 | 0.4559 | 0.4567 | 0.4576 | 0.4584 | 0.4592 | 0.4601 | 0.4609 | 0.4617 | 0.4625 |
| 0.20     | 0.4633 | 0.4641 | 0.4649 | 0.4657 | 0.4665 | 0.4673 | 0.4681 | 0.4689 | 0.4697 | 0.4705 |
| 0.21     | 0.4712 | 0.4720 | 0.4728 | 0.4736 | 0.4743 | 0.4751 | 0.4758 | 0.4766 | 0.4774 | 0.4781 |
| 0.22     | 0.4789 | 0.4796 | 0.4803 | 0.4811 | 0.4818 | 0.4825 | 0.4833 | 0.4840 | 0.4847 | 0.4855 |
| 0.23     | 0.4862 | 0.4869 | 0.4876 | 0.4883 | 0.4890 | 0.4897 | 0.4904 | 0.4911 | 0.4918 | 0.4925 |
| 0.24     | 0.4932 | 0.4939 | 0.4946 | 0.4953 | 0.4960 | 0.4966 | 0.4973 | 0.4980 | 0.4987 | 0.4993 |
| 0.25     | 0.5000 | 0.5007 | 0.5013 | 0.5020 | 0.5026 | 0.5033 | 0.5040 | 0.5046 | 0.5053 | 0.5059 |
| 0.26     | 0.5066 | 0.5072 | 0.5078 | 0.5085 | 0.5091 | 0.5097 | 0.5104 | 0.5110 | 0.5116 | 0.5123 |
| 0.27     | 0.5129 | 0.5135 | 0.5141 | 0.5147 | 0.5154 | 0.5160 | 0.5166 | 0.5172 | 0.5178 | 0.5184 |
| 0.28     | 0.5190 | 0.5196 | 0.5202 | 0.5208 | 0.5214 | 0.5220 | 0.5226 | 0.5232 | 0.5238 | 0.5243 |
| 0.29     | 0.5249 | 0.5255 | 0.5261 | 0.5267 | 0.5272 | 0.5278 | 0.5284 | 0.5290 | 0.5295 | 0.5301 |
| 0.30     | 0.5307 | 0.5312 | 0.5318 | 0.5323 | 0.5329 | 0.5335 | 0.5340 | 0.5346 | 0.5351 | 0.5357 |
| 0.31     | 0.5362 | 0.5368 | 0.5373 | 0.5379 | 0.5384 | 0.5389 | 0.5395 | 0.5400 | 0.5406 | 0.5411 |
| 0.32     | 0.5416 | 0.5422 | 0.5427 | 0.5432 | 0.5437 | 0.5443 | 0.5448 | 0.5453 | 0.5458 | 0.5464 |
| 0.33     | 0.5469 | 0.5474 | 0.5479 | 0.5484 | 0.5489 | 0.5494 | 0.5499 | 0.5505 | 0.5510 | 0.5515 |
| 0.34     | 0.5520 | 0.5525 | 0.5530 | 0.5535 | 0.5540 | 0.5545 | 0.5550 | 0.5554 | 0.5559 | 0.5564 |
| 0.35     | 0.5569 | 0.5574 | 0.5579 | 0.5584 | 0.5589 | 0.5593 | 0.5598 | 0.5603 | 0.5608 | 0.5613 |
| 0.36     | 0.5617 | 0.5622 | 0.5627 | 0.5632 | 0.5636 | 0.5641 | 0.5646 | 0.5650 | 0.5655 | 0.5660 |
| 0.37     | 0.5664 | 0.5669 | 0.5674 | 0.5678 | 0.5683 | 0.5687 | 0.5692 | 0.5696 | 0.5701 | 0.5705 |
| 0.38     | 0.5710 | 0.5714 | 0.5719 | 0.5723 | 0.5728 | 0.5732 | 0.5737 | 0.5741 | 0.5746 | 0.5750 |
| 0.39     | 0.5755 | 0.5759 | 0.5763 | 0.5768 | 0.5772 | 0.5776 | 0.5781 | 0.5785 | 0.5789 | 0.5794 |

Table 6.1<sup>4</sup> provides a design aid which tabulates  $\rho n$  against  $k$  and facilitates the analysis of a given section. For a given section, the known values of reinforcement area, masonry strength, and section dimensions enable the determination of  $\rho n$  and the corresponding value of  $k$  is obtained from Table 6.1.

The lever-arm of the internal resisting moment is obtained from Fig. 6.2 as

$$jd = d - \frac{kd}{3}$$

The lever-arm factor is

$$j = 1 - \frac{k}{3}$$

The resisting moment of the masonry is

$$M_m = Cjd$$

$$= \frac{f_b k j b_w d^2}{2}$$

<sup>4</sup>Williams, A., 2011 (See References and Codes)

For a given applied service moment  $M$ , the maximum masonry stress is given by

$$f_m = \frac{2M}{jkb_w d^2}$$

The resisting moment of the reinforcement is given by

$$\begin{aligned} M_s &= Tjd \\ &= f_s j \rho b_w d^2 \end{aligned}$$

For a given applied service moment  $M$ , the reinforcement stress is given by

$$f_s = \frac{M}{j \rho b_w d^2}$$

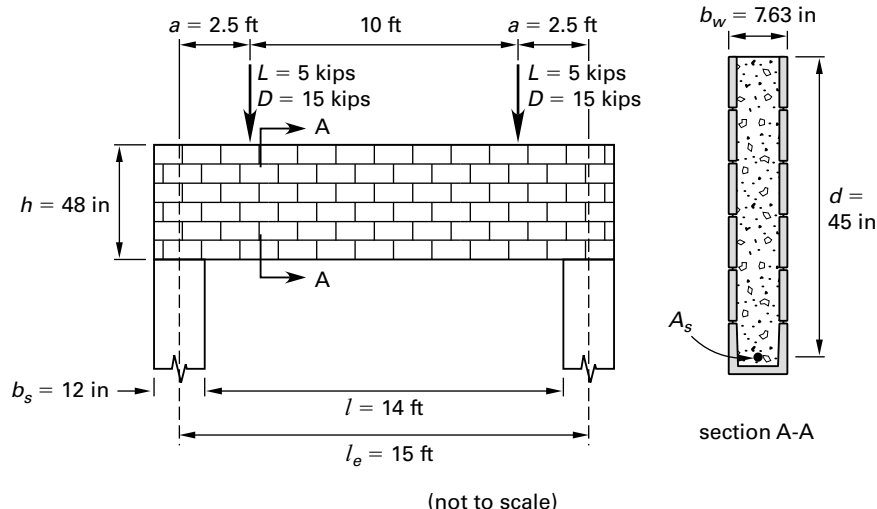
For a permissible masonry stress of  $F_b$ , and a permissible reinforcement stress of  $F_s$ , the service moment capacity of the section is the lesser of

$$\begin{aligned} M_m &= \frac{F_b j k b_w d^2}{2} \\ M_s &= F_s j \rho b_w d^2 \end{aligned}$$

#### Example 6.4

The 8 in, solid grouted concrete block masonry beam shown in the illustration is simply supported over a clear span of 14 ft. The overall depth of the beam is 48 in. Its effective depth,  $d$ , is 45 in. The unit weight may be assumed to be 69 lbf/ft<sup>3</sup>. The masonry has a compressive strength of 1500 lbf/in<sup>2</sup> and the reinforcement consists of grade 60 bars. Determine the number of no. 6 grade 60 reinforcing bars required to resist the applied loads.

*Illustration for Example 6.4*



*Solution*

The total moment at midspan is derived in Ex. 6.1 as

$$M = 57.76 \text{ ft-kips}$$

Try 2 no. 6 bars with an area of

$$A_s = 0.88 \text{ in}^2$$

The relevant parameters of the beam are

$$\begin{aligned} b_w &= 7.63 \text{ in} \\ d &= 45 \text{ in} \\ f'_m &= 1500 \text{ lbf/in}^2 \\ f_y &= 60,000 \text{ lbf/in}^2 \\ l_e &= 15 \text{ ft} \\ \frac{l_e}{b_w} &= \frac{(15 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{7.63 \text{ in}} \\ &= 23.6 \\ &< 32 \quad [\text{satisfies MSJC Sec. 1.13.2}] \end{aligned}$$

The allowable stresses are

$$\begin{aligned} F_b &= \frac{f'_m}{3} \\ &= \frac{1500 \frac{\text{lbf}}{\text{in}^2}}{3} \\ &= 500 \frac{\text{lbf}}{\text{in}^2} \\ F_s &= 24,000 \text{ lbf/in}^2 \end{aligned}$$

The modular ratio is

$$\begin{aligned} n &= \frac{E_s}{E_m} \\ &= \frac{29,000,000 \frac{\text{lbf}}{\text{in}^2}}{900 f'_m} \\ &= \frac{29,000,000 \frac{\text{lbf}}{\text{in}^2}}{(900) \left( 1500 \frac{\text{lbf}}{\text{in}^2} \right)} \\ &= 21.48 \end{aligned}$$

The tension reinforcement ratio is

$$\begin{aligned} \rho &= \frac{A_s}{b_w d} \\ &= \frac{0.88 \text{ in}^2}{(7.63 \text{ in})(45 \text{ in})} \\ &= 0.00256 \\ \rho n &= (0.00256)(21.48) \\ &= 0.0550 \end{aligned}$$

Using this value of  $\rho n$ , the neutral axis depth factor is obtained from Table 6.1 as

$$\begin{aligned} k &= \sqrt{2\rho n + (\rho n)^2} - \rho n \\ &= \sqrt{(2)(0.0550) + (0.0550)^2} - 0.0550 \\ &= 0.2811 \end{aligned}$$

The lever-arm factor is

$$\begin{aligned} j &= 1 - \frac{k}{3} \\ &= 1 - \frac{0.2811}{3} \\ &= 0.906 \end{aligned}$$

The moment capacity of the section is the lesser of

$$\begin{aligned} M_m &= \frac{F_b j k b_w d^2}{2} \\ &= \frac{\left( 500 \frac{\text{lbf}}{\text{in}^2} \right) (0.906) (0.2811) (7.63 \text{ in}) (45 \text{ in})^2}{(2) \left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)} \\ &= 81.98 \text{ ft-kips} \end{aligned}$$

$$\begin{aligned} M_s &= F_s j \rho b_w d^2 \\ &= \frac{\left( 24,000 \frac{\text{lbf}}{\text{in}^2} \right) (0.906) (0.00256) (7.63 \text{ in}) (45 \text{ in})^2}{\left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)} \\ &= 71.67 \text{ ft-kips} \quad [\text{governs}] \\ &> M \quad [\text{satisfactory}] \end{aligned}$$

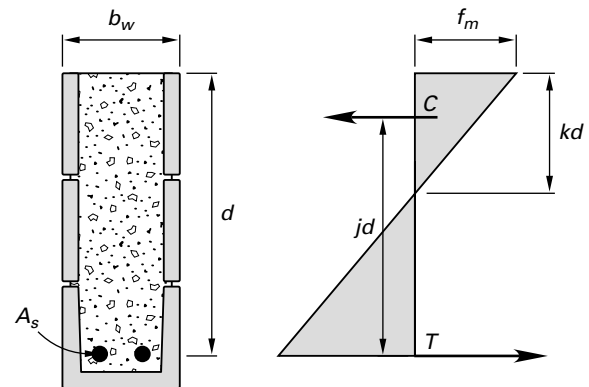
Two no. 6 bars are adequate.

### Analysis of Beams with Tension Reinforcement Only

The design strength of a masonry beam is readily determined from the given beam dimensions and the area of reinforcing steel (see Fig. 6.3). The neutral axis depth factor, obtained by equating tensile and compressive forces acting on the section, is

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

Figure 6.3 Analysis of Concrete Masonry Beams



The lever arm of the compressive and tensile forces is

$$jd = d - \frac{kd}{3}$$

For a permissible masonry stress of  $F_b$  and a permissible reinforcement stress of  $F_s$ , the service moment capacity of the section is the lesser of

$$\begin{aligned} M_m &= \frac{F_b j k b_w d^2}{2} \\ M_s &= F_s j \rho b_w d^2 \end{aligned}$$

For a given applied service moment  $M$ , the maximum masonry stress is given by

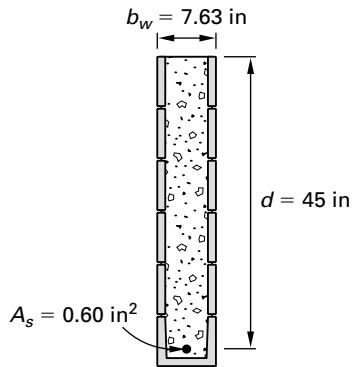
$$f_m = \frac{2M}{jkb_w d^2}$$

For a given applied service moment  $M$ , the reinforcement stress is given by

$$f_s = \frac{M}{j\rho b_w d^2}$$

### Example 6.5

The nominal 8 in, solid grouted concrete block masonry beam shown in the illustration has an effective depth,  $d$ , of 45 in. The masonry has a compressive strength of 1500 lbf/in<sup>2</sup>. The reinforcement consists of one no. 7 grade 60 bar. Determine the stresses in the beam if the applied moment is 48 ft-kips.



### Solution

The tension reinforcement ratio is

$$\begin{aligned}\rho &= \frac{A_s}{b_w d} \\ &= \frac{0.60 \text{ in}^2}{(7.63 \text{ in})(45 \text{ in})} \\ &= 0.00175 \\ \rho n &= (0.00175)(21.48) \\ &= 0.0376\end{aligned}$$

Using this value of  $\rho n$ , the neutral axis depth factor is obtained from Table 6.1 as

$$\begin{aligned}k &= \sqrt{2\rho n + (\rho n)^2} - \rho n \\ &= \sqrt{(2)(0.0376) + (0.0376)^2} - 0.0376 \\ &= 0.239\end{aligned}$$

The lever-arm factor is

$$\begin{aligned}j &= 1 - \frac{k}{3} \\ &= 1 - \frac{0.239}{3} \\ &= 0.920\end{aligned}$$

For an applied moment of 48 ft-kips, the maximum masonry stress is given by

$$\begin{aligned}f_m &= \frac{2M}{jkb_w d^2} \\ &= \frac{(2)(48,000 \text{ ft-lbf})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(0.920)(0.239)(7.63 \text{ in})(45 \text{ in})^2} \\ &= 339 \text{ lbf/in}^2 \\ &< 500 \text{ lbf/in}^2 \quad [\text{satisfactory}]\end{aligned}$$

For an applied moment of 48 ft-kips, the reinforcement stress is given by

$$\begin{aligned}f_s &= \frac{M}{j\rho b_w d^2} \\ &= \frac{(48,000 \text{ ft-lbf})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(0.920)(0.00175)(7.63 \text{ in})(45 \text{ in})^2} \\ &= 23,155 \text{ lbf/in}^2 \\ &< 24,000 \text{ lbf/in}^2 \quad [\text{satisfactory}]\end{aligned}$$

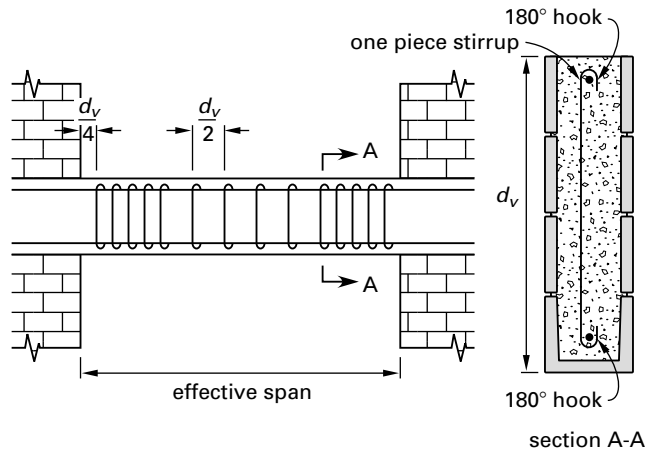
## 3. DESIGN FOR SHEAR

### Nomenclature

|        |                                                |                     |
|--------|------------------------------------------------|---------------------|
| $A_n$  | net area of cross section                      | in <sup>2</sup>     |
| $A_v$  | area of shear reinforcement                    | in <sup>2</sup>     |
| $b_w$  | width of beam                                  | in                  |
| $f'_m$ | specified masonry compressive strength         | lbf/in <sup>2</sup> |
| $f_v$  | calculated shear stress in masonry             | lbf/in <sup>2</sup> |
| $F_v$  | allowable shear stress in masonry              | lbf/in <sup>2</sup> |
| $l_e$  | effective span length of beam                  | ft                  |
| $s$    | spacing of shear reinforcement                 | in                  |
| $V$    | shear force at the section under consideration | kips                |

### Shear Reinforcement Requirements

When the shear stress on a section,  $f_v$ , exceeds the allowable shear stress of the masonry,  $F_v$ , MSJC Commentary Sec. 2.3.5 requires the provision of shear reinforcement to resist the total shear. As shown in Fig. 6.4, and specified by MSJC Sec. 3.3.4.2.3, shear reinforcement must comply with the following requirements.

**Figure 6.4** Shear Reinforcement Requirements


- To reduce congestion in the member, shear reinforcement must consist of a single bar with a standard 180° hook at each end.
- Shear reinforcement must be hooked around longitudinal reinforcement at each end to develop the shear reinforcement.
- To prevent brittle shear failure, the area of shear reinforcement must be not less than  $0.0007b_wd$ .
- The first shear reinforcing bar must be located not more than one-fourth the depth of the beam from the end of the beam, in order to intersect any diagonal crack formed at the support.
- To ensure that every potential shear crack is crossed by at least one stirrup, the spacing of shear reinforcing bars must not exceed one-half the beam depth, nor 48 in.

### Design of Beams for Shear

In accordance with MSJC Sec. 2.3.5.2.1, the shear stress in masonry is given by the expression

$$f_v = \frac{V}{b_w d} \quad [\text{MSJC 2-23}]$$

The allowable shear stress in a flexural member without shear reinforcement is given by MSJC Sec. 2.3.5.2.2 as

$$F_v = \sqrt{f'_m} \quad [\text{MSJC 2-24}]$$

$$\leq 50 \text{ lbf/in}^2$$

When this value of the shear stress is exceeded, shear reinforcement is provided to carry the full shear load without any contribution from the masonry. The area of shear reinforcement required is given by MSJC Sec. 2.3.5.3 as

$$\frac{A_v}{s} = \frac{V}{F_s d} \quad [\text{MSJC 2-30}]$$

The shear stress, with shear reinforcement designed to take the entire shear force, is limited by MSJC Sec. 2.3.5.2.3 to a maximum value of

$$F_v = 3\sqrt{f'_m} \quad [\text{MSJC 2-27}]$$

$$\leq 150 \text{ lbf/in}^2$$

When necessary, the dimensions of the masonry member must be increased to conform to this requirement.

As specified in MSJC Sec. 2.3.5.5, the maximum design shear for a beam may be calculated at a distance of  $d/2$  from the face of the support, provided that no concentrated load occurs between the face of the support and a distance of  $d/2$  from the face.

### Example 6.6

The nominal 8 in, solid grouted concrete block masonry beam shown in the illustration has an effective depth,  $d$ , of 45 in and an overall depth of 48 in. The masonry has a compressive strength of 1500 lbf/in<sup>2</sup>. The reinforcement consists of one no. 8 grade 60 bar. For the loading indicated, determine whether shear stirrups are required at the ends of the beam.

#### Solution

The critical section for shear for a beam, in accordance with MSJC Sec. 2.3.5.5, occurs at a distance of  $d/2$  from the face of the support. The shear at the critical section is derived in Ex. 6.1 as

$$V = 21.42 \text{ kips}$$

The shear stress at the critical section is given by MSJC Eq. (2-23) as

$$f_v = \frac{V}{b_w d}$$

$$= \frac{(21.42 \text{ kips}) \left( 1000 \frac{\text{lbf}}{\text{kips}} \right)}{(7.63 \text{ in})(45 \text{ in})}$$

$$= 62.39 \text{ lbf/in}^2$$

The allowable shear stress without shear reinforcement is given by MSJC Eq. (2-24) as

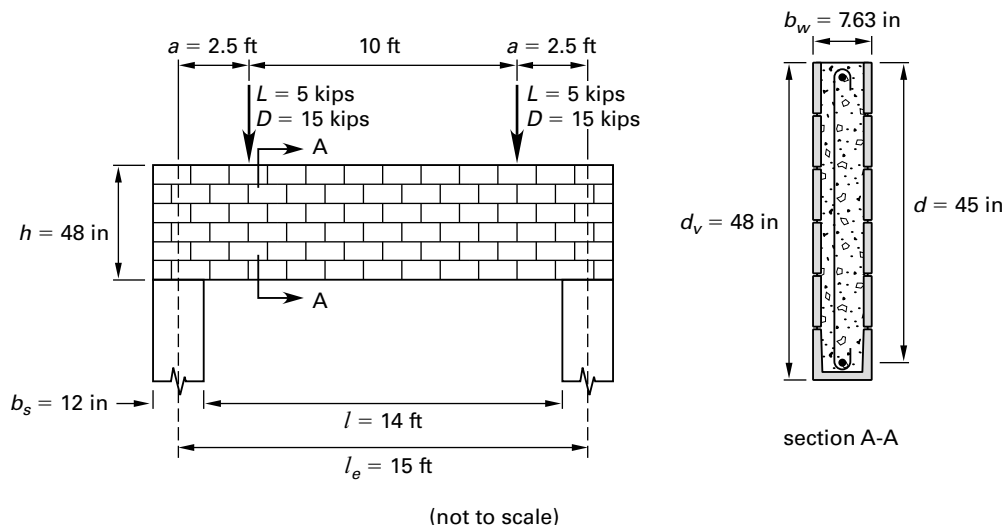
$$F_v = \sqrt{f'_m}$$

$$= \sqrt{1500 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 38.7 \text{ lbf/in}^2$$

$$< f_v \quad [\text{shear reinforcement is required}]$$

Illustration for Example 6.6



The shear stress with shear reinforcement provided to carry the total shear force is limited by MSJC Eq. (2-27) to

$$\begin{aligned}
 F_v &= 3\sqrt{f'_m} \\
 &= 3\sqrt{1500} \frac{\text{lb}}{\text{in}^2} \\
 &= 116 \text{ lb}/\text{in}^2 \\
 &\leq 150 \text{ lb}/\text{in}^2 \quad [\text{satisfies MSJC Sec. 2.3.5.2.3}] \\
 &> f_v \quad [\text{satisfactory}]
 \end{aligned}$$

The minimum area of shear reinforcement required is given by MSJC Eq. (2-30) as

$$\begin{aligned}
 \frac{A_v}{s} &= \frac{V}{F_s d} \\
 &= \frac{(21.55 \text{ kips}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{\left(24 \frac{\text{kips}}{\text{in}^2}\right) (45 \text{ in})} \\
 &= 0.239 \text{ in}^2/\text{ft}
 \end{aligned}$$

Providing no. 4 grade 60 stirrups at 8 in centers supplies a value of

$$\begin{aligned}
 \frac{A_v}{s} &= 0.300 \frac{\text{in}^2}{\text{ft}} \\
 &= 0.238 \text{ in}^2/\text{ft} \quad [\text{satisfactory}]
 \end{aligned}$$

#### 4. DESIGN OF MASONRY COLUMNS

##### Nomenclature

|       |                                       |                 |
|-------|---------------------------------------|-----------------|
| $a$   | distance between column reinforcement | in              |
| $A_n$ | net effective area of column          | in <sup>2</sup> |

|            |                                                 |                    |
|------------|-------------------------------------------------|--------------------|
| $A_{st}$   | area of laterally tied reinforcement            | in <sup>2</sup>    |
| $b$        | width of section                                | in                 |
| $d_{lat}$  | diameter of lateral ties                        | in                 |
| $d_{long}$ | diameter of longitudinal reinforcement          | in                 |
| $E_m$      | modulus of elasticity of masonry in compression | lb/in <sup>2</sup> |
| $E_s$      | modulus of elasticity of steel reinforcement    | lb/in <sup>2</sup> |
| $f'_m$     | specified masonry compressive strength          | lb/in <sup>2</sup> |
| $f_y$      | yield strength of reinforcement                 | lb/in <sup>2</sup> |
| $h$        | effective height of column                      | in                 |
| $P$        | unfactored axial load                           | kips               |
| $P_a$      | allowable axial strength                        | kips               |
| $r$        | radius of gyration                              | in                 |
| $s$        | center-to-center spacing of items               | in                 |
| $t$        | nominal thickness of member                     | in                 |

##### Symbols

|        |                     |   |
|--------|---------------------|---|
| $\rho$ | reinforcement ratio | — |
|--------|---------------------|---|

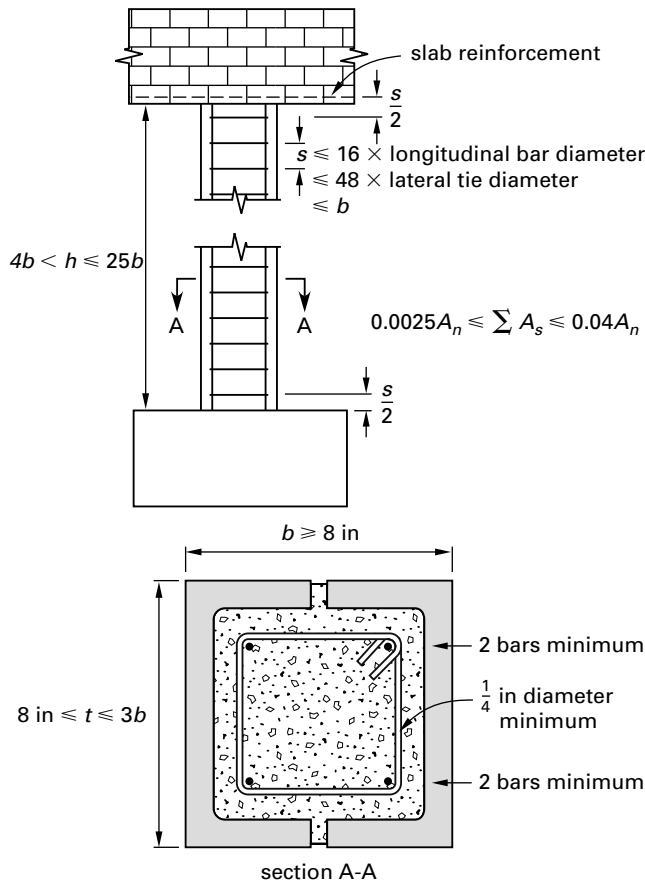
#### Dimensional Limitations of Columns

Limitations are imposed on column dimensions in MSJC Secs. 1.6, 1.14, and 2.1.6.1 to prevent lateral instability. As shown in Fig. 6.5, the minimum nominal column width is limited to 8 in. The ratio of the distance between lateral supports and least nominal width must not exceed 25. The minimum nominal column depth is limited to 8 in, and the maximum depth must not exceed three times the nominal width.

#### Column Reinforcement Requirements

Limitations are imposed on column longitudinal reinforcement in MSJC Sec. 1.14.1.2. As shown in Fig. 6.5, reinforcement must comply with the following requirements.

Figure 6.5 Column Details



- Longitudinal reinforcement is limited to a maximum of 4% of the net column area of cross section to reduce congestion in the column.
- To prevent brittle failure, the area of longitudinal reinforcement must not be less than a minimum of 0.25% of the net column area of cross section.
- At least four longitudinal reinforcing bars must be provided, one in each corner of the column.

Lateral ties, for the confinement of longitudinal reinforcement in a column, are specified by MSJC Sec. 1.14.1.3. As shown in Fig. 6.5, lateral ties must comply with the following requirements.

- Lateral ties for the confinement of longitudinal reinforcement must not be less than  $\frac{1}{4}$  in diameter.
- Lateral ties must be placed at a spacing not exceeding the lesser of 16 longitudinal bar diameters, 48 lateral tie diameters, or the least cross-sectional dimension of the column.
- Lateral ties must be arranged such that every corner and alternative longitudinal bar will have support provided by the corner of a lateral tie, and no bar will be farther than 6 in clear on each side from a supported bar.

- Lateral ties must be located not more than one-half lateral tie spacing above the top of the footing or slab in any story, and must be placed not more than one-half lateral tie spacing below the horizontal reinforcement in the beam or slab reinforcement above.
- Where beams or brackets frame into a column from four directions, lateral ties may be terminated not more than 3 in below the lowest reinforcement in the shallowest beam or bracket.

### Axial Compression in Columns

The allowable axial compressive strength of an axially loaded reinforced masonry column, is given by MSJC Sec. 2.3.3.2.1. For columns having a ratio of effective height to radius of gyration not greater than 99, the allowable axial strength is

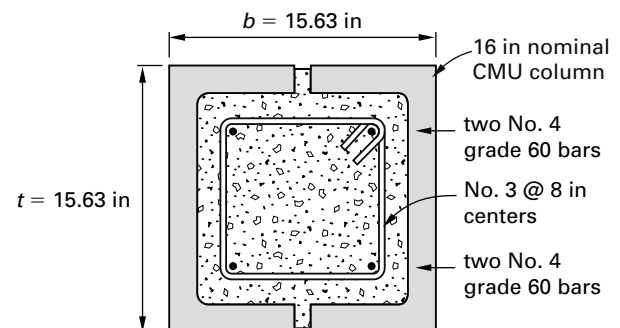
$$P_a = (0.25 f'_m A_n + 0.65 A_{st} F_s) \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \quad [\text{MSJC 2-20}]$$

For columns having a ratio of effective height to radius of gyration greater than 99, the allowable axial load is

$$P_a = (0.25 f'_m A_n + 0.65 A_{st} F_s) \left( \frac{70r}{h} \right)^2 \quad [\text{MSJC 2-21}]$$

### Example 6.7

The nominal 16 in square, solid grouted concrete block masonry column shown in the illustration has a specified strength of 3000 lbf/in<sup>2</sup>. It is reinforced with four no. 4 grade 60 bars. The column has a height of 15 ft, and may be considered pinned at each end. Determine the design axial strength of the column and the required size and spacing of lateral ties. Neglect the effects of accidental eccentricity.



### Solution

The relevant properties of the column are the following. The effective column width is

$$b = 15.63 \text{ in}$$



The effective column height is

$$h = 15 \text{ ft}$$

The reinforcement area is

$$A_s = 0.80 \text{ in}^2$$

The effective column area is

$$\begin{aligned} A_n &= b^2 \\ &= (15.63 \text{ in})^2 \\ &= 244 \text{ in}^2 \\ \rho &= \frac{A_{st}}{A_n} \\ &= \frac{0.80 \text{ in}^2}{244 \text{ in}^2} \\ &= 0.0033 \\ &< 0.04 \\ &> 0.0025 \quad [\text{satisfies MSJC Sec. 1.14.1.2}] \end{aligned}$$

The radius of gyration of the column is given by MSJC Commentary Sec. 1.9.3 as

$$\begin{aligned} r &= \sqrt{\frac{I_n}{A_n}} \\ &= 0.289b \\ &= (0.289)(15.63 \text{ in}) \\ &= 4.52 \text{ in} \end{aligned}$$

The slenderness ratio of the column is

$$\begin{aligned} \frac{h}{r} &= \frac{(15 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{4.52 \text{ in}} \\ &= 39.82 \\ &< 99 \quad [\text{MSJC Eq. (2-20) is applicable}] \end{aligned}$$

The allowable axial strength is

$$\begin{aligned} P_a &= (0.25f'_m A_n + 0.65A_{st}F_s) \left(1 - \left(\frac{h}{140r}\right)^2\right) \\ &\quad [\text{MSJC 2-20}] \\ &= \left( (0.25) \left(3 \frac{\text{kips}}{\text{in}^2}\right) (244 \text{ in}^2) \right. \\ &\quad \left. + (0.65)(0.80 \text{ in}^2) \left(24 \frac{\text{kips}}{\text{in}^2}\right) \right) \\ &\quad \times \left(1.0 - \left(\frac{39.82}{140}\right)^2\right) \\ &= 180 \text{ kips} \end{aligned}$$

As specified by MSJC Sec. 1.14.1.3, lateral ties for the confinement of longitudinal reinforcement must not be

less than  $\frac{1}{4}$  in diameter. Using no. 3 bars for the ties, the spacing must not exceed the lesser of

$$\begin{aligned} s &= 48d_{\text{lat}} \\ &= (48)(0.375 \text{ in}) \\ &= 18 \text{ in} \\ s &= 16 \text{ in} \quad [\text{least cross-sectional column dimension}] \\ s &= 16d_{\text{long}} \\ &= (16)(0.5 \text{ in}) \\ &= 8 \text{ in} \quad [\text{governs}] \end{aligned}$$

## 5. DESIGN OF MASONRY SHEAR WALLS

### Nomenclature

|          |                                                                   |                   |
|----------|-------------------------------------------------------------------|-------------------|
| $A_g$    | gross cross-sectional area of the wall using specified dimensions | $\text{in}^2$     |
| $A_s$    | area of reinforcement                                             | $\text{in}^2$     |
| $A_{sh}$ | area of horizontal reinforcement                                  | $\text{in}^2$     |
| $A_{sv}$ | area of vertical reinforcement                                    | $\text{in}^2$     |
| $A_v$    | area of shear reinforcement                                       | $\text{in}^2$     |
| $b$      | width of section                                                  | $\text{in}$       |
| $d$      | effective depth of tension reinforcement                          | $\text{in}$       |
| $d_v$    | depth of wall in direction of shear                               | $\text{in}$       |
| $f'_m$   | specified masonry compressive strength                            | $\text{lbf/in}^2$ |
| $f_v$    | calculated shear stress in masonry                                | $\text{lbf/in}^2$ |
| $F_s$    | allowable stress in reinforcement                                 | $\text{lbf/in}^2$ |
| $F_v$    | allowable shear stress in masonry                                 | $\text{lbf/in}^2$ |
| $h$      | effective height of shear wall                                    | $\text{in}$       |
| $M$      | unfactored bending moment associated with $V$                     | $\text{in-kips}$  |
| $s$      | spacing of reinforcement                                          | $\text{in}$       |
| $V$      | unfactored shear force                                            | $\text{kips}$     |

### Shear Wall Types

The following are several types of shear walls classified in MSJC Sec. 1.6. Determining which type of wall to adopt depends on the seismic design category of the structure.

- *Ordinary plain (unreinforced) masonry shear walls* are shear walls designed to resist lateral forces without reinforcement or where stresses in the reinforcement, if present, are neglected. This type of wall may be used only in seismic design categories A and B.
- *Detailed plain (unreinforced) masonry shear walls* are shear walls with specific minimum reinforcement and connection requirements that are designed to resist lateral forces with the stresses in the reinforcement neglected. This type of wall may be used only in seismic design categories A and B. The reinforcement requirements are specified in MSJC Sec. 1.17.3.2.3.1 as horizontal and vertical reinforcement of at least no. 4 bars at a maximum spacing of 120 in. Additional

reinforcement is required at wall openings and corners.

- *Ordinary reinforced masonry shear walls* are shear walls with the minimum reinforcement specified in MSJC Sec. 1.17.3.2.3.1 that are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall may be used only in seismic design categories A, B, or C. The maximum permitted height in seismic design category C is 160 ft.
- *Intermediate reinforced masonry shear walls* are shear walls with the minimum reinforcement specified in MSJC Sec. 1.17.3.2.3.1 with the exception that the spacing of vertical reinforcement is limited to a maximum of 48 in. The walls are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall may be used only in seismic design category A, B, or C. There is no limitation on height in seismic design category C.
- *Special reinforced masonry shear walls* are shear walls with the minimum reinforcement specified in MSJC Sec. 1.17.3.2.6 that are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall must be used in seismic design categories D, E, and F. When used in bearing wall or building frame systems, the maximum permitted height in seismic design category D and E is 160 ft and in seismic design category F is 100 ft.

### Special Reinforced Shear Wall Reinforcement Requirements

Reinforcement is provided in special reinforced masonry shear walls in order to provide ductile behavior in the walls under seismic loads. To ensure this in seismic design categories D, E, and F, MSJC Sec. 1.17.3.2.6 requires walls to be reinforced with uniformly distributed vertical and horizontal reinforcement. Shear reinforcement shall be anchored around vertical reinforcement with a standard 180° hook. The minimum required combined area of shear reinforcement and vertical reinforcement is

$$A_{sh} + A_{sv} = 0.002A_g$$

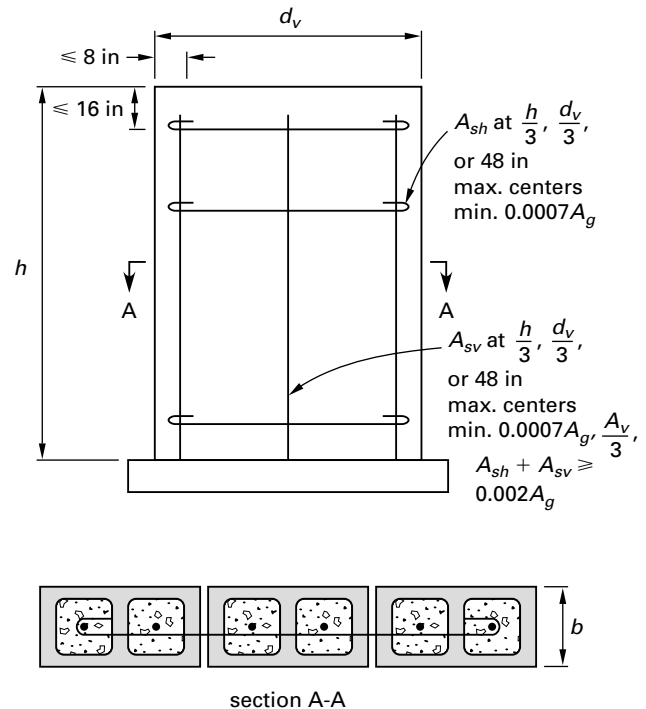
The following requirements for horizontal shear reinforcement are shown in Fig. 6.6.

- The maximum spacing must not exceed the lesser of one-third the length of the wall, one-third the height of the wall, 48 in, or 24 in in stack bond masonry walls.
- For masonry laid in running bond, the minimum cross-sectional area must not be less than 0.0007 times the gross cross-sectional area of the wall, using specified dimensions.

For masonry laid in other than running bond, the minimum cross-sectional area must not be less than

0.0015 times the gross cross-sectional area of the wall.

**Figure 6.6** Reinforcement Details for Special Reinforced Masonry Shear Walls Laid in Running Bond



The requirements for vertical reinforcement follow.

- The maximum spacing must not exceed the lesser of one-third the length of the wall, one-third the height of the wall, 48 in, or 24 in in stack bond masonry walls.
- The minimum cross-sectional area must not be less than the greater of 0.0007 times the gross cross-sectional area of the wall, using specified dimensions, or one-third the required shear reinforcement.

As specified by MSJC Sec. 1.17.3.2.6.1.2, special reinforced shear walls that are designed by the allowable stress method must be designed for 1.5 times the seismic force calculated by IBC Ch. 16.

### Shear Capacity of a Shear Wall without Shear Reinforcement

The allowable shear stress depends on the ratio  $M/Vd$ , where  $M$  is the moment acting at the location where the applied shear force  $V$  is calculated. In a masonry wall without shear reinforcement and with  $M/Vd < 1.0$ , the allowable shear stress is given by MSJC Sec. 2.3.5.2.2(b) as

$$F_v = \left( \frac{1}{3} \right) \left( 4 - \frac{M}{Vd} \right) \sqrt{f'_m} \quad [\text{MSJC 2-25}]$$

$$\leq \left( 80 - \frac{45M}{Vd} \right) \quad [\text{lb/in}^2]$$

In a masonry wall without shear reinforcement and with  $M/Vd \geq 1.0$ , the allowable shear stress is given by MSJC Sec. 2.3.5.2.2(b) as

$$F_v = \sqrt{f'_m} \quad [\text{MSJC 2-26}]$$

$$\leq 35 \text{ lbf/in}^2$$

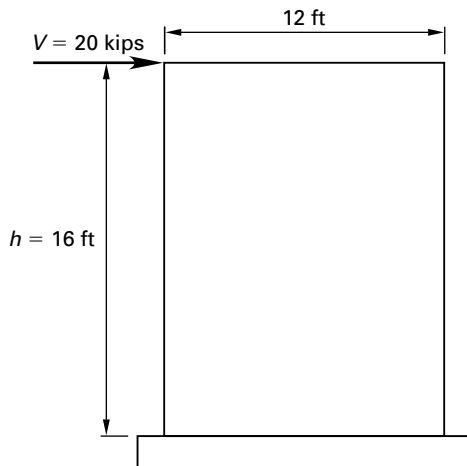
The shear stress in the masonry is determined from MSJC Sec. 2.3.5.2.1 and MSJC Commentary Sec. 2.3.5.3 as

$$f_v = \frac{V}{bd} \quad [\text{MSJC 2-23}]$$

For shear walls without shear reinforcement and with shear parallel to the plane of the wall, MSJC Commentary Sec. 2.3.5.3 specifies the substitution of the overall depth of the wall  $d_v$  in place of the effective depth  $d$ . Similarly, for shear walls with horizontal shear reinforcement and with vertical reinforcement uniformly distributed along the depth of the wall,  $d_v$  may be substituted for  $d$ .

### Example 6.8

The nominal 8 in, solid grouted concrete block masonry shear wall shown in the illustration has a specified strength of 3000 lbf/in<sup>2</sup>.



The wall has a height of 16 ft, and is assigned to seismic design category B. A wind load of 20 kips acts at the top of the wall and this is the governing shear load. Determine the shear reinforcement required.

### Solution

The shear stress in the masonry wall is given by MSJC Eq. (2-23) and MSJC Commentary Sec. 2.3.5.3 as

$$f_v = \frac{V}{bd_v}$$

$$= \frac{20,000 \text{ lbf}}{(7.63 \text{ in})(144 \text{ in})}$$

$$= 18.20 \text{ lbf/in}^2$$

The allowable stress is obtained by applying MSJC Sec. 2.3.5.2.2. The ratio

$$\frac{M}{Vd} = \frac{Vh}{Vd_v}$$

$$= \frac{h}{d_v}$$

$$= \frac{(16 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{144 \text{ in}}$$

$$= 1.33 \quad [\text{MSJC Eq. (2-26) is applicable}]$$

Applying MSJC Eq. (2-26), the allowable stress is the smaller value given by

$$F_v = \sqrt{f'_m}$$

$$= \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 54.8 \text{ lbf/in}^2$$

Or,

$$F_v = 35 \frac{\text{lbf}}{\text{in}^2} \quad [\text{governs}]$$

$$> f_v \quad [\text{satisfactory}]$$

The masonry takes all the shear force, and nominal reinforcement is required as detailed in MSJC Sec. 1.17.3.2.3.1 for a structure assigned to seismic design category A or B.

### Shear Capacity of a Shear Wall with Shear Reinforcement

When the allowable shear stress of the masonry is exceeded, shear reinforcement must be provided to carry the entire shear force without any contribution from the masonry. In a masonry wall with shear reinforcement designed to carry the entire shear force, and with  $M/Vd < 1.0$ , the shear stress is limited by MSJC Sec. 2.3.5.2.3(b) to

$$F_v = \left(\frac{1}{2}\right)\left(4 - \frac{M}{Vd}\right)\sqrt{f'_m} \quad [\text{MSJC 2-28}]$$

$$\leq 120 - \frac{45M}{Vd} \quad [\text{lbf/in}^2]$$

In a masonry wall, with shear reinforcement designed to carry the entire shear force and with  $M/Vd \geq 1.0$ , the shear stress is limited by MSJC Sec. 2.3.5.2.3(b) to

$$F_v = 1.5\sqrt{f'_m} \quad [\text{MSJC 2-29}]$$

$$\leq 75 \text{ lbf/in}^2$$

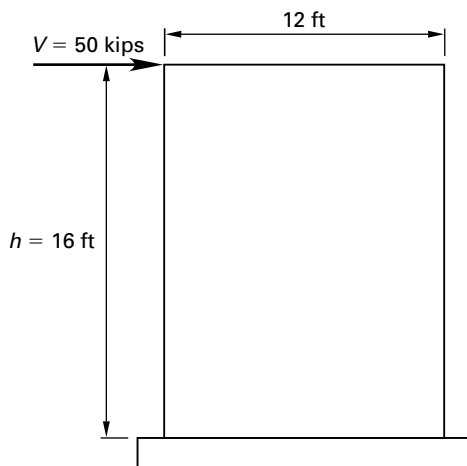
The area of shear reinforcement required when the allowable shear stress in the masonry is exceeded, is given by MSJC Sec. 2.3.5.3 as

$$\frac{A_v}{s} = \frac{V}{F_s d} \quad [\text{MSJC 2-30}]$$

The spacing of shear reinforcement must not exceed the lesser of  $d/2$  or 48 in. Reinforcement must be provided perpendicular to the shear reinforcement and must be at least equal to  $A_v/3$ . This perpendicular reinforcement must be uniformly distributed and must not exceed a spacing of 8 ft.

### Example 6.9

The nominal 8 in solid grouted, concrete block masonry shear wall shown in the illustration has a strength of 3000 lbf/in<sup>2</sup>. An in-plane wind load of 50 kips acts at the top of the wall, and this is the governing shear load. The wall is located in a structure assigned to seismic design category D and is laid in running bond. Determine the shear reinforcement required in the wall.



### Solution

Since the structure is assigned to seismic design category D, the reinforcement details of a special reinforced shear wall must be provided. From Ex. 6.8, the allowable shear stress of the masonry section without shear reinforcement is

$$F_v = 35 \frac{\text{lbf}}{\text{in}^2}$$

The shear stress in the masonry wall is given by MSJC Eq. (2-23) and MSJC Commentary Sec. 2.3.5.3 as

$$\begin{aligned} f_v &= \frac{V}{bd_v} \\ &= \frac{50,000 \text{ lbf}}{(7.63 \text{ in})(144 \text{ in})} \\ &= 45.51 \text{ lbf/in}^2 \\ &> F_v \end{aligned}$$

Reinforcement is required to carry the entire shear force. In a masonry wall with shear reinforcement designed to carry the entire shear force, and with  $M/Vd > 1.0$ , the allowable stress is limited by MSJC Eq. (2-29) to the lesser of

$$\begin{aligned} F_v &= 1.5\sqrt{f'_m} \\ &= 1.5\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 82.16 \text{ lbf/in}^2 \end{aligned}$$

Or,

$$\begin{aligned} F_v &= 75 \frac{\text{lbf}}{\text{in}^2} \quad [\text{governs}] \\ &> f_v \quad [\text{satisfies MSJC Sec. 2.3.5.2.3(b)}] \end{aligned}$$

The minimum area of shear reinforcement required is given by MSJC Eq. (2-30) as

$$\begin{aligned} \frac{A_v}{s} &= \frac{V}{F_s d} \\ &= \frac{(50 \text{ kips}) \left(12 \frac{\text{in}}{\text{ft}}\right)}{\left(24 \frac{\text{kips}}{\text{in}^2}\right) (144 \text{ in})} \\ &= 0.174 \text{ in}^2/\text{ft} \end{aligned}$$

Providing no. 5 horizontal bars at 16 in on center gives a reinforcement area of

$$\begin{aligned} A_{sh} &= 0.233 \frac{\text{in}^2}{\text{ft}} \\ &> 0.174 \frac{\text{in}^2}{\text{ft}} \quad [\text{satisfies MSJC Sec. 2.3.5.3}] \\ &> 0.0007 A_g \quad [\text{satisfies MSJC Sec. 1.17.3.2.6(c)}] \end{aligned}$$

To comply with MSJC Sec. 1.17.3.2.6(c), the vertical reinforcement must not be less than

$$\begin{aligned} A_v &= \frac{A_v}{3} \\ &= \frac{0.174 \frac{\text{in}^2}{\text{ft}}}{3} \\ &= 0.058 \text{ in}^2/\text{ft} \end{aligned}$$

Providing no. 5 vertical bars at 48 in on center gives a reinforcement area of

$$A_{sv} = 0.078 \frac{\text{in}^2}{\text{ft}}$$

$$\left[ \begin{array}{l} \text{this will be augmented by flexural} \\ \text{reinforcement that is not considered here} \end{array} \right]$$

$$> 0.058 \frac{\text{in}^2}{\text{ft}} \quad [\text{satisfactory}]$$

$$> 0.0007A_g \quad [\text{satisfies MSJC Sec. 1.17.3.2.6(c)}]$$

The sum of the horizontal and vertical reinforcement areas provided is

$$A_{sh} + A_{sv} = 0.233 \frac{\text{in}^2}{\text{ft}} + 0.078 \frac{\text{in}^2}{\text{ft}}$$

$$= 0.311 \text{ in}^2/\text{ft} \quad [\text{satisfactory}]$$

The required sum, in accordance with MSJC Sec. 1.17.3.2.6(c), is

$$A_{sh} + A_{sv} = 0.002A_g$$

$$= (0.002)(7.63 \text{ in})(12 \text{ in})$$

$$= 0.18 \text{ in}^2/\text{ft}$$

$$< 0.311 \text{ in}^2/\text{ft} \quad [\text{satisfactory}]$$

## 6. WALL DESIGN FOR OUT-OF-PLANE LOADS

### Nomenclature

|            |                                                                         |                   |
|------------|-------------------------------------------------------------------------|-------------------|
| $a$        | depth of equivalent rectangular stress block                            | in                |
| $A_{\max}$ | maximum area of reinforcement that will satisfy MSJC Sec. 3.3.3.5.1     | $\text{in}^2$     |
| $A_{se}$   | effective area of reinforcing steel                                     | $\text{in}^2$     |
| $b$        | effective thickness of wall                                             | in                |
| $c$        | depth of neutral axis                                                   | in                |
| $C_m$      | force in masonry stress block                                           | kips              |
| $d$        | effective depth                                                         | in                |
| $d_v$      | depth of wall in direction of shear                                     | in                |
| $e$        | eccentricity of applied axial load                                      | in                |
| $e_u$      | eccentricity of applied factored load                                   | in                |
| $E_s$      | modulus of elasticity of reinforcement                                  | $\text{lbf/in}^2$ |
| $E_m$      | modulus of elasticity of masonry                                        | $\text{lbf/in}^2$ |
| $f'_m$     | specified masonry compressive strength                                  | $\text{lbf/in}^2$ |
| $f_r$      | modulus of rupture of masonry                                           | $\text{lbf/in}^2$ |
| $f_s$      | stress in reinforcement                                                 | $\text{lbf/in}^2$ |
| $f_y$      | yield strength of reinforcement                                         | $\text{lbf/in}^2$ |
| $h$        | wall height                                                             | in                |
| $I_{cr}$   | moment of inertia of cracked transformed section about the neutral axis | $\text{in}^4$     |
| $I_g$      | moment of inertia of gross wall section                                 | $\text{in}^4$     |
| $L_w$      | length of wall                                                          | in                |
| $M_{cr}$   | cracking moment                                                         | ft-kips           |

|                  |                                                         |                 |
|------------------|---------------------------------------------------------|-----------------|
| $M_n$            | nominal bending moment strength                         | ft-kips         |
| $M_{\text{ser}}$ | service moment                                          | ft-kips         |
| $M_u$            | factored bending moment                                 | ft-kips         |
| $n$              | modular ratio                                           | —               |
| $P$              | unfactored axial load                                   | kips            |
| $P_f$            | service load level from tributary floor and roof loads  | kips            |
| $P_u$            | sum of $P_{uw}$ and $P_{uf}$                            | kips            |
| $P_{uf}$         | factored load from tributary floor or roof loads        | kips            |
| $P_{uw}$         | factored weight of wall tributary to section considered | kips            |
| $P_w$            | service level load tributary to the section considered  | kips            |
| $S_n$            | section modulus of net wall section                     | $\text{in}^3$   |
| $V_u$            | factored shear force                                    | lbf             |
| $w_u$            | factored lateral load                                   | $\text{lbf/ft}$ |

### Symbols

|                 |                                                                                                       |    |
|-----------------|-------------------------------------------------------------------------------------------------------|----|
| $\delta_u$      | deflection at midheight of wall caused by factored loads and including $P$ -delta effects             | in |
| $\epsilon_{mu}$ | maximum usable compressive strain of masonry, 0.0035 for clay masonry and 0.0025 for concrete masonry | —  |
| $\epsilon_s$    | strain in reinforcement                                                                               | —  |
| $\epsilon_y$    | strain at yield in tension reinforcement                                                              | —  |
| $\rho_{\max}$   | maximum reinforcement ratio that will satisfy MSJC Sec. 3.3.3.5.1                                     | —  |
| $\phi$          | strength reduction factor                                                                             | —  |

### Flexural Strength

For the design of slender masonry walls, NCEES requires the use of the strength design method.

The design flexural strength for a wall with out-of-plane loading is given by MSJC Eq. (3-27) as

$$M_u \leq \phi M_n$$

An allowance is made for the axial load on the wall, and the effective reinforcement area is calculated as

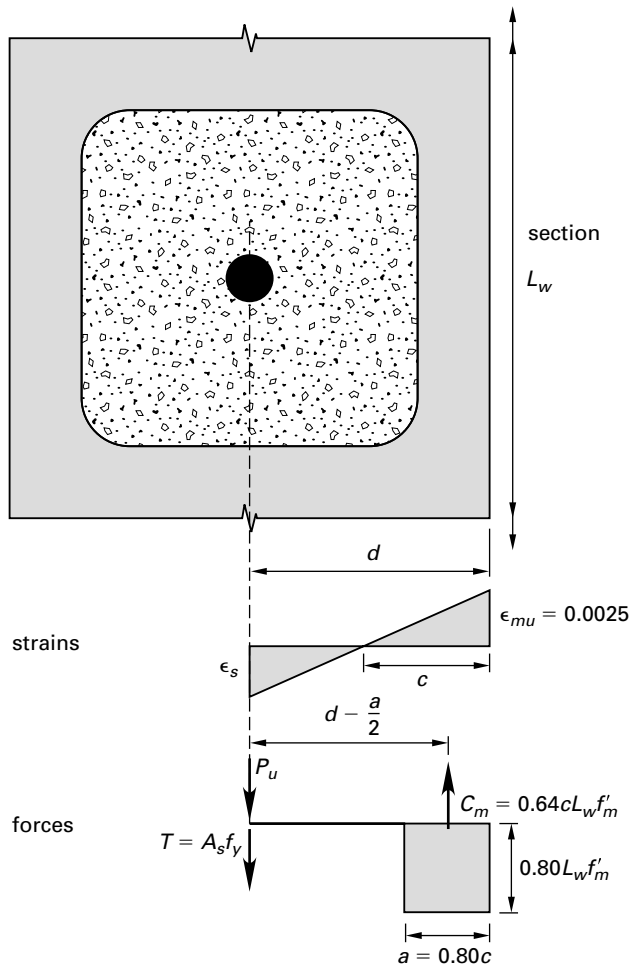
$$A_{se} = \frac{P_u + A_s f_y}{f_y}$$

With the reinforcing steel placed in the center of the wall as shown in Fig. 6.7, and using the design assumptions of MSJC Sec. 3.3.2, the nominal moment is given by MSJC Eq. (3-28) as

$$M_n = A_{se} f_y \left( d - \frac{a}{2} \right)$$

$$a = \frac{P_u + A_s f_y}{0.80 f'_m L_w} \quad [\text{MSJC 3-29}]$$

**Figure 6.7** Nominal Moment for Out-of-Plane Loading of Concrete Masonry



### Example 6.10

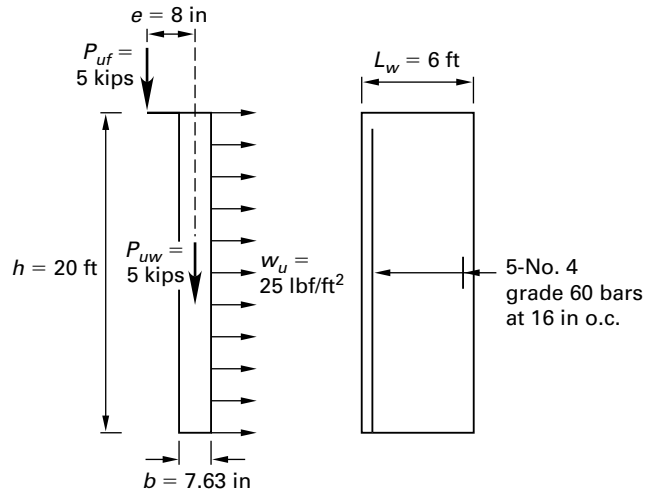
The nominal 8 in, solid grouted concrete block masonry wall shown in the illustration has a specified strength of 3000 lbf/in<sup>2</sup>. It is reinforced longitudinally with five no. 4 grade 60 bars placed centrally in the wall. The wall has a height of 20 ft, and is simply supported at the top and bottom. The factored applied loads are shown in the illustration. The lateral load is due to wind. Determine the design flexural strength of the wall.

#### Solution

The total factored axial load at the midheight of the wall is

$$\begin{aligned} P_u &= P_{uf} + P_{uw} \\ &= 5 \text{ kips} + 5 \text{ kips} \\ &= 10 \text{ kips} \end{aligned}$$

#### Illustration for Example 6.10



The reinforcement area in the wall is

$$A_s = (5)(0.20 \text{ in}^2) = 1.00 \text{ in}^2$$

For strength level loads, the equivalent reinforcement area is

$$\begin{aligned} A_{se} &= \frac{P_u + A_s f_y}{f_y} = \frac{10 \text{ kips} + (1.00 \text{ in}^2) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)}{60 \frac{\text{kips}}{\text{in}^2}} \\ &= 1.17 \text{ in}^2 \end{aligned}$$

The depth of the rectangular stress block is given by MSJC Eq. (3-29) as

$$\begin{aligned} a &= \frac{A_{se} f_y}{0.80 f'_m L_w} = \frac{(1.17 \text{ in}^2) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)}{(0.80) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) (6 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)} \\ &= 0.41 \text{ in} \end{aligned}$$

The nominal moment strength is given by MSJC Eq. (3-28) as

$$\begin{aligned} M_n &= A_{se} f_y \left( d - \frac{a}{2} \right) \\ &= \frac{(1.17 \text{ in}^2) \left( 60 \frac{\text{kips}}{\text{in}^2} \right) \left( 3.82 \text{ in} - \frac{0.41 \text{ in}}{2} \right)}{12 \frac{\text{in}}{\text{ft}}} \\ &= 21.15 \text{ ft-kips} \end{aligned}$$

The design moment strength is

$$\phi M_n = (0.9)(21.15 \text{ ft-kips}) = 19.04 \text{ ft-kips}$$

### Maximum Reinforcement Ratio for Walls

The reinforcement ratio in a wall subject to out-of-plane loading must not exceed the value necessary to satisfy the requirements of MSJC Sec. 3.3.3.5.1. When  $M_u/V_u d_v \geq 1$ , the maximum reinforcement ratio is determined using the following design assumptions.

- Strain in the extreme tension reinforcement is 1.5 times the strain associated with the reinforcement yield stress,  $f_y$ .
- Unfactored gravity axial loads are included in the analysis using the combination

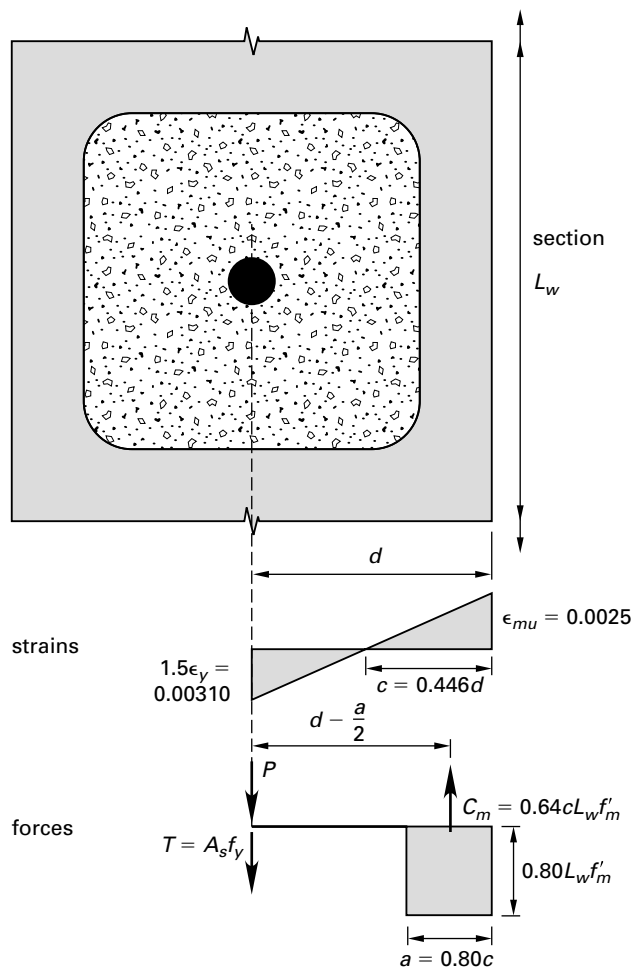
$$P = D + 0.75L + 0.525Q_E$$

From the strain distribution shown in Fig. 6.8, the neutral axis depth and the depth of the equivalent rectangular stress block are

$$c = \frac{0.0025d}{0.00560} = 0.446d$$

$$a = 0.80c = 0.357d$$

**Figure 6.8** Maximum Reinforcement Requirements for Concrete Masonry Walls



The force in the equivalent rectangular stress block is

$$\begin{aligned} C_m &= 0.80aL_w f'_m \\ &= 0.286L_w d f'_m \end{aligned}$$

The force in the reinforcing bars is given by MSJC Sec. 3.3.3.5.1 as

$$T = A_{\max} f_y$$

Equating compressive and tensile forces acting on the section gives

$$\begin{aligned} P &= C_m - T \\ &= 0.286L_w d f'_m - A_{\max} f_y \end{aligned}$$

The maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$A_{\max} = \frac{0.286L_w d f'_m - P}{f_y}$$

### Example 6.11

The nominal 8 in, solid grouted concrete block masonry wall described in Ex. 6.10 has a specified strength of 3000 lbf/in<sup>2</sup>. It is reinforced with five no. 4 grade 60 bars. Determine whether the reinforcement area provided satisfies MSJC Sec. 3.3.3.5.1.

#### Solution

From Ex. 6.10, the factored tributary roof load is

$$P_{uf} = 5 \text{ kips}$$

The service level tributary roof load is

$$\begin{aligned} P_f &= \frac{5 \text{ kips}}{1.2} \\ &= 4.17 \text{ kips} \end{aligned}$$

From Ex. 6.10, the factored wall dead load at the mid-height of the wall is

$$P_{uw} = 5 \text{ kips}$$

The service level roof dead load is

$$\begin{aligned} P_w &= \frac{5 \text{ kips}}{1.2} \\ &= 4.17 \text{ kips} \end{aligned}$$



The axial load combination specified in MSJC Sec. 3.3.3.5.1 for determining the maximum reinforcement limit is

$$\begin{aligned} P &= D + 0.75L + 0.525Q_E \\ &\approx P_f + P_w \\ &= 4.17 \text{ kips} + 4.17 \text{ kips} \\ &= 8.34 \text{ kips} \end{aligned}$$

The relevant dimensions are obtained from Ex. 6.10 as

$$\begin{aligned} b &= 7.63 \text{ in} \\ d &= 3.82 \text{ in} \end{aligned}$$

The maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$\begin{aligned} A_{\max} &= \frac{0.286L_w d f'_m - P}{f_y} \\ &= \frac{(0.286)(3.82)(6 \text{ ft})}{\left(12 \frac{\text{in}}{\text{ft}}\right) \left(3 \frac{\text{kips}}{\text{in}^2}\right) - 8.34 \text{ kips}} \\ &= \frac{60 \frac{\text{kips}}{\text{in}^2}}{60 \frac{\text{kips}}{\text{in}^2}} \\ &= 3.79 \text{ in}^2 \\ &> 1.0 \text{ in}^2 \text{ provided [satisfactory]} \end{aligned}$$

## Design Loads

Slender masonry walls are designed for factored applied loads, taking into consideration the  $P$ -delta effects caused by the vertical loads and the lateral deflection of the wall. The design method of MSJC Sec. 3.3.5.4 assumes the wall is simply supported, and uniformly laterally loaded, with the critical section at the mid-height of the wall. In addition, MSJC Eq. (3-24) limits the factored axial load stress at the midheight of the wall to a maximum value of

$$\frac{P_u}{A_g} = 0.20f'_m$$

$P_u$  is the factored load caused by applied loads and wall self-weight.

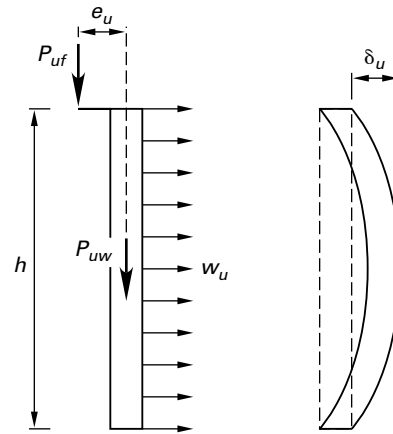
When the slenderness ratio exceeds 30,

$$\frac{P_u}{A_g} \leq 0.05f'_m$$

As shown in Fig. 6.9, the factored moment at the mid-height of the wall is given by MSJC Eq. (3-25) as

$$M_u = \frac{w_u h^2}{8} + \frac{P_{uf} e_u}{2} + P_u \delta_u$$

Figure 6.9 Wall with Out-of-Plane Loading



The deflection at the midheight of the wall,  $\delta_u$ , caused by factored loads and including  $P$ -delta effects, is derived from MSJC Eqs. (3-32) and (3-31) as

$$\begin{aligned} \delta_u &= \frac{5M_{cr}h^2}{48E_m I_g} + \frac{5h^2(M_u - M_{cr})}{48E_m I_{cr}} \quad [M_{cr} < M_u < M_n] \\ \delta_u &= \frac{5M_u h^2}{48E_m I_g} \quad [M_u \leq M_{cr}] \end{aligned}$$

The cracking moment,  $M_{cr}$ , is

$$M_{cr} = S_n f_r$$

The moment of inertia of a cracked transformed section about the neutral axis is

$$I_{cr} = \frac{L_w c^3}{3} + n A_{se} (d - c)^2$$

The modular ratio is

$$n = \frac{E_s}{E_m}$$

The distance of the reinforcing bar from the neutral axis is  $d - c$ . The effective area of reinforcing steel is

$$A_{se} = \frac{P_u + A_s f_y}{f_y}$$

The moment of inertia of the gross wall section is

$$I_g = \frac{L_w b^3}{12}$$

The section modulus of the net wall section is

$$S_n = \frac{L_w b^2}{6}$$



The modulus of rupture of masonry,  $f_r$ , is given in MSJC Table 3.1.8.2. The modulus of elasticity of reinforcement is

$$E_s = 29,000 \text{ kips/in}^2 \quad [\text{MSJC Sec. 1.8.2.1}]$$

The modulus of elasticity for concrete masonry is

$$E_m = 900f'_m \quad [\text{MSJC Sec. 1.8.2.2.1}]$$

An iterative process is required<sup>5</sup> until the values for  $M_u$  and  $\delta_u$  converge.

### Example 6.12

The nominal 8 in, solid grouted concrete block masonry wall described in Ex. 6.10 has a specified strength of 3000 lbf/in<sup>2</sup>. It is reinforced with five no. 4 grade 60 bars. The factored loads acting on the wall are indicated in the illustration for Ex. 6.10. Determine the factored design moment at the midheight of the wall.

#### Solution

Assume a deflection at midheight caused by factored loads of

$$\delta_{u1} = 0.10 \text{ in}$$

The modulus of rupture for out-of-plane forces on a fully grouted masonry wall is given by MSJC Table 3.1.8.2 as

$$f_r = 158 \frac{\text{lbf}}{\text{in}^2} \quad [\text{type N Portland cement/lime mortar}]$$

The section modulus of the net wall section is

$$\begin{aligned} S_n &= \frac{b^2 L_w}{6} \\ &= \frac{(7.63 \text{ in})^2 (72 \text{ in})}{6} \\ &= 699 \text{ in}^3 \end{aligned}$$

Ignoring the effects of axial load, the nominal cracking moment strength is

$$\begin{aligned} M_{cr} &= S_n f_r \\ &= \frac{(699 \text{ in}^3) (158 \frac{\text{lbf}}{\text{in}^2})}{\left(12 \frac{\text{in}}{\text{ft}}\right) \left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 9.20 \text{ ft-kips} \end{aligned}$$

The nominal wall moment is

$$\begin{aligned} M_n &= 21.15 \text{ ft-kips} \quad [\text{from Ex. 6-10}] \\ \frac{M_n}{M_{cr}} &= \frac{21.15 \text{ ft-kips}}{9.20 \text{ ft-kips}} \\ &= 2.3 \\ &> 1.3 \quad [\text{complies with MSJC Sec. 3.3.4.2.2.2}] \end{aligned}$$

The applied strength level moment at the midheight of the wall is given by MSJC Eq. (3-25) as

$$\begin{aligned} M_{u1} &= \frac{w_u h^2}{8} + \frac{P_{uf} e_u}{2} + P_u \delta_{u1} \\ &= \frac{\left(0.025 \frac{\text{kips}}{\text{ft}^2}\right) (6 \text{ ft}) (20 \text{ ft})^2 \left(12 \frac{\text{in}}{\text{ft}}\right)}{8} \\ &\quad + \frac{(5 \text{ kips}) (8 \text{ in})}{2} + (10 \text{ kips}) (0.10 \text{ in}) \\ &= \frac{111 \text{ in-kips}}{12 \frac{\text{in}}{\text{ft}}} \\ &= 9.25 \text{ ft-kips} \end{aligned}$$

The deflection corresponding to the factored moment is determined in accordance with MSJC Eq. (3-32). The moment of inertia of the gross wall section is

$$\begin{aligned} I_g &= \frac{L_w b^3}{12} \\ &= \frac{(72 \text{ in}) (7.63 \text{ in})^3}{12} \\ &= 2665 \text{ in}^4 \end{aligned}$$

From Ex. 6.10, the depth of the equivalent rectangular stress block is

$$a = 0.41 \text{ in}$$

The depth of the neutral axis is

$$\begin{aligned} c &= \frac{a}{0.80} \\ &= \frac{0.41 \text{ in}}{0.80} \\ &= 0.51 \text{ in} \end{aligned}$$

The modulus of elasticity of reinforcement is given by MSJC Sec. 1.8.2.1.1 as

$$E_s = 29,000 \frac{\text{kips}}{\text{in}^2}$$

<sup>5</sup>Ekwueme, C.G., 2003 (See References and Codes)

The modulus of elasticity of concrete masonry is given by MSJC Sec. 1.8.2.2 as

$$\begin{aligned} E_m &= 900f'_m \\ &= (900) \left( 3 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 2700 \text{ kips/in}^2 \end{aligned}$$

The modular ratio is

$$\begin{aligned} n &= \frac{E_s}{E_m} \\ &= \frac{29,000 \frac{\text{kips}}{\text{in}^2}}{2700 \frac{\text{kips}}{\text{in}^2}} \\ &= 10.74 \end{aligned}$$

The moment of inertia of the cracked transformed section about the neutral axis is

$$\begin{aligned} I_{cr} &= \frac{L_w c^3}{3} + n A_{se} (d - c)^2 \\ &= \frac{(72 \text{ in})(0.51 \text{ in})^3}{3} \\ &\quad + (10.74)(1.17 \text{ in}^2)(3.815 \text{ in} - 0.51 \text{ in})^2 \\ &= 140 \text{ in}^4 \end{aligned}$$

Because  $M_{u1}$  is greater than  $M_{cr}$ , the midheight deflection corresponding to the factored moment is derived from MSJC Eq. (3-32) as

$$\begin{aligned} \delta_u &= \frac{5M_{cr}h^2}{48E_m I_g} + \frac{5h^2(M_{u1} - M_{cr})}{48E_m I_{cr}} \\ &= \frac{(5)(110.40 \text{ in-kips})(240 \text{ in})^2}{(48) \left( 2700 \frac{\text{kips}}{\text{in}^2} \right) (2665 \text{ in}^4)} \\ &\quad + \frac{(5)(0.60 \text{ in-kips})(240 \text{ in})^2}{(48) \left( 2700 \frac{\text{kips}}{\text{in}^2} \right) (140 \text{ in}^4)} \\ &= 0.102 \text{ in} \\ &\approx \delta_{u1} \quad [\text{satisfactory}] \end{aligned}$$

The original assumptions are correct, and the factored applied moment is

$$\begin{aligned} M_{u1} &= 9.25 \text{ ft-kips} \\ &< \phi M_n \quad [\text{satisfactory}] \end{aligned}$$

The flexural capacity is adequate.

## Service Load Deflections

The maximum permissible deflection at the midheight of the wall, caused by service level vertical and lateral loads, and including  $P$ -delta effects, is given by MSJC Eq. (3-30) as

$$\delta_s = 0.007h$$

When the applied service moment,  $M_{ser}$ , exceeds the cracking moment,  $M_{cr}$ , the service deflection is given by MSJC Eq. (3-32) as

$$\delta_s = \frac{5M_{cr}h^2}{48E_m I_g} + \frac{5h^2(M_{ser} - M_{cr})}{48E_m I_{cr}}$$

The service moment at the midheight of the wall, including  $P$ -delta effects, is

$$M_{ser} = \frac{wh^2}{8} + \frac{P_f e}{2} + P\delta_s$$

The service level lateral load is  $w$ . The unfactored axial load is

$$P = P_f + P_w$$

The cracked moment of inertia of the wall section, assuming the stress in the masonry is essentially elastic, is

$$I_{cr} = \frac{L_w c^3}{3} + n A'_{se} (d - c)^2$$

The depth to the neutral axis is

$$\begin{aligned} c &= kd \\ k &= \sqrt{2n\rho_e + (n\rho_e)^2} - n\rho_e \\ n &= \frac{E_s}{E_m} \\ \rho_e &= \frac{A'_{se}}{L_w d} \end{aligned}$$

The equivalent reinforcement area at working load is

$$A'_{se} = \frac{P + A_s f_y}{f_y}$$

When the applied service moment is less than the cracking moment, the service deflection is given by MSJC Eq. (3-31) as

$$\delta_s = \frac{5M_{ser}h^2}{48E_m I_g}$$

An iterative process is required until the values for  $\delta_s$  and the values for  $M_{ser}$  converge.

**Example 6.13**

Determine whether the midheight deflection of the slender wall of Ex. 6.10 under service level loads is within the permissible limits.

*Solution*

The maximum permissible deflection at midheight of the wall caused by service level vertical and lateral loads, including  $P$ -delta effects, is given by MSJC Eq. (3-30) as

$$\begin{aligned}\delta_s &= 0.007h \\ &= (0.007)(20 \text{ ft}) \left(12 \frac{\text{in}}{\text{ft}}\right) \\ &= 1.68 \text{ in}\end{aligned}$$

From Ex. 6.12, the midheight deflection produced by the factored loads is

$$\begin{aligned}\delta_u &= 0.10 \text{ in} \\ &< \delta_s\end{aligned}$$

The deflection under service loads is within the permissible limit.

**7. DESIGN OF ANCHOR BOLTS<sup>6</sup>****Nomenclature**

|            |                                                                        |                 |
|------------|------------------------------------------------------------------------|-----------------|
| $A_b$      | nominal cross-sectional area of the bolt                               | in <sup>2</sup> |
| $A_o$      | overlap of projected areas                                             | in <sup>2</sup> |
| $A_{pt}$   | projected area of tensile breakout surface                             | in <sup>2</sup> |
| $b_a$      | tensile force on an anchor bolt                                        | kips            |
| $b_v$      | shear force on an anchor bolt                                          | kips            |
| $B_{ab}$   | axial capacity in tension of an anchor bolt when governed by breakout  | kips            |
| $B_{as}$   | design capacity in steel tensile yield                                 | kips            |
| $B_{vb}$   | design capacity in shear of an anchor bolt when governed by breakout   | kips            |
| $B_{vc}$   | capacity in shear of an anchor bolt when governed by crushing          | kips            |
| $B_{vpry}$ | design capacity in shear pryout                                        | kips            |
| $B_{vs}$   | design capacity in shear of an anchor bolt when governed by bolt steel | kips            |
| $d_b$      | bolt diameter                                                          | in              |
| $f_y$      | bolt yield stress                                                      | —               |
| $l_b$      | effective embedment depth of anchor bolt                               | in              |
| $l_{be}$   | anchor bolt edge distance                                              | in              |
| $r$        | radius of projected area                                               | in              |
| $s$        | bolt spacing                                                           | in              |

**Symbols**

|          |                                                                                          |     |
|----------|------------------------------------------------------------------------------------------|-----|
| $\theta$ | half the angle subtended by the chord at the intersection of overlapping projected areas | deg |
|----------|------------------------------------------------------------------------------------------|-----|

**Headed Anchor Bolts in Tension**

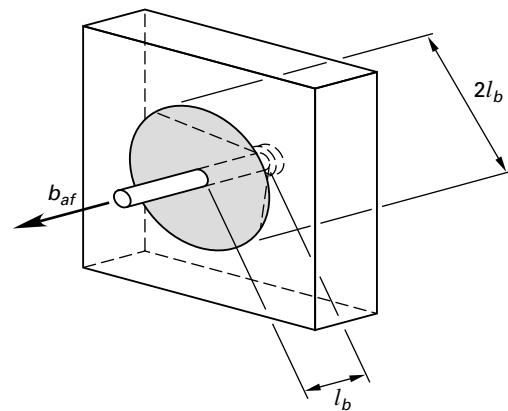
In accordance with MSJC Commentary Sec. 1.16.2, anchors that are solidly grouted in masonry fail in tension by the pullout of a conically shaped section of masonry (see Fig. 6.10). The failure surface slopes at 45°. The bolt's projected area, given by MSJC Eq. (1-2), is a circle of radius equal to the embedment length of the bolt. The projected area is

$$A_{pt} = \pi l_b^2$$

The effective embedment depth of the anchor bolt, measured from the surface of the masonry to the bearing surface of the bolt head, is  $l_b$ .

The anchor bolt edge distance, measured from the edge of the masonry to the center of the bolt is  $l_{be}$ .

**Figure 6.10** Masonry Failure in Tension



For tensile strength governed by masonry breakout, the design capacity in tension is given by MSJC Eq. (2-1) as

$$B_{ab} = 1.25 A_{pt} \sqrt{f'_m}$$

The design capacity in steel tensile yield is

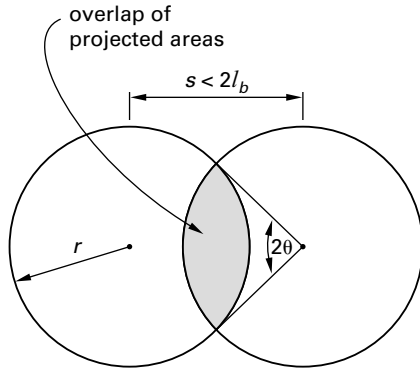
$$B_{as} = 0.6 A_b f_y$$

The minimum effective embedment length is specified by MSJC Sec. 1.16.6 as the greater of four bolt diameters or 2 in.

When bolts are spaced at less than twice the embedment length apart, the projected areas of the bolts overlap, and MSJC Sec. 1.16.2 requires the combined projected area to be reduced by the overlapping area. As shown in Fig. 6.11, the overlapping area is

$$A_o = \left( \frac{\pi \theta}{90^\circ} - \sin 2\theta \right) r^2$$

<sup>6</sup>Ekwueme, C. G., 2010 (See References and Codes)

**Figure 6.11** *Overlap of Projected Areas*

The angle subtended at the center of the projected area by the chord of the intersecting circles is  $2\theta$ , where

$$\theta = \cos^{-1}\left(\frac{s}{2r}\right)$$

$$r = l_b$$

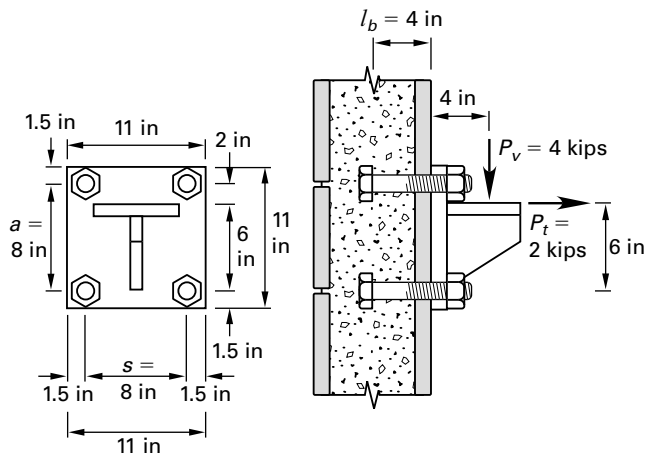
Similarly, when a bolt is located less than the embedment length from the edge of a member, that portion of the projected area falling in an open cell shall be deducted from the calculated area.

For tensile strength governed by the bolt steel, the design capacity in tension is given by MSJC Eq. (2-2) as

$$B_{as} = 0.6A_b f_y$$

### Example 6.14

The anchorage for a bracket to a nominal 8 in, solid grouted concrete block masonry wall is shown in the illustration. The masonry compressive strength is 3000 lbf/in<sup>2</sup>. Anchor bolts are  $\frac{7}{8}$  in diameter, ASTM A307 type C, with a minimum specified yield strength of 36 kips/in<sup>2</sup>, and an effective cross-sectional area at the root of the threads of 0.46 in<sup>2</sup>. For the loads indicated in the illustration, determine whether the bolts are adequate for the tensile forces on the bracket.



### Solution

By taking moments about the bottom bolts, the tensile force in each top bolt is obtained as

$$b_a = \frac{4P_v + 6P_t}{2a}$$

$$= \frac{(4 \text{ in})(4 \text{ kips}) + (6 \text{ in})(2 \text{ kips})}{(2)(8 \text{ in})}$$

$$= 1.75 \text{ kips}$$

For tensile strength governed by the  $\frac{7}{8}$  in bolts, the design capacity of one bolt in tension is given by MSJC Eq. (2-2) as

$$B_{as} = 0.6A_b f_y$$

$$= (0.6)(0.46 \text{ in}^2) \left( 36 \frac{\text{kips}}{\text{in}^2} \right)$$

$$= 9.94 \text{ kips}$$

$$> b_a \quad [\text{satisfactory}]$$

The effective embedment length of an anchor bolt, measured from the surface of the masonry to the bearing surface of the bolt head, is

$$l_b = 4 \text{ in}$$

$$> 4d_b \quad [\text{satisfies MSJC Sec. 1.16.6}]$$

$$> 2 \text{ in} \quad [\text{satisfies MSJC Sec. 1.16.6}]$$

The spacing of the bolts is

$$s = 8 \text{ in}$$

$$= 2l_b$$

The projected areas of the bolts do not overlap. MSJC Sec. 1.16.2 gives the projected area of one bolt as

$$A_{pt} = \pi l_b^2$$

$$= (3.14)(4 \text{ in})^2$$

$$= 50.24 \text{ in}^2$$

For tensile strength governed by masonry breakout, the design capacity in tension is given by MSJC Eq. (2-1) as

$$B_{ab} = 1.25A_{pt} \sqrt{f'_m}$$

$$= \frac{(1.25)(50.24 \text{ in}^2) \sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 3.44 \text{ kips} \quad [\text{governs}]$$

$$> b_a \quad [\text{satisfactory}]$$

### Headed Anchor Bolts in Shear

The projected area in shear for bolts is given by MSJC Eq. (1-3) as

$$A_{pv} = \frac{\pi l_{be}^2}{2}$$

The masonry design capacity in shear crushing is given by MSJC Eq. (2-7) as

$$B_{vc} = 350 \sqrt{f'_m A_b}$$

The design capacity in shear breakout is given by MSJC Eq. (2-6) as

$$B_{vb} = 1.25 A_{pv} \sqrt{f'_m}$$

For shear strength governed by the bolt steel yielding, the design capacity in shear is given by MSJC Eq. (2-9) as

$$B_{vs} = 0.36 A_b f_y$$

The design capacity in shear pryout is given by MSJC Eq. (2-8) as

$$\begin{aligned} B_{vpry} &= 2.0 B_{ab} \\ &= 2.5 A_{pt} \sqrt{f'_m} \end{aligned}$$

#### Example 6.15

For the bracket anchored to a nominal 8 in, solid grouted concrete block masonry wall described in Ex. 6.14, the masonry compressive strength is 3000 lbf/in<sup>2</sup>. The threaded portions of the anchor bolts are located inside the shear plane. The bolts are 7/8 in diameter, ASTM A307 type C, with minimum specified yield strength of 36 kips/in<sup>2</sup>. For the loads indicated in the illustration for Ex. 6.14, determine whether the bolts are adequate for the shear forces on the bracket.

#### Solution

The shear force on each bolt is

$$\begin{aligned} b_v &= \frac{P_v}{4} \\ &= \frac{4 \text{ kips}}{4} \\ &= 1.0 \text{ kip} \end{aligned}$$

The bolts are far from a free edge of the wall and masonry breakout in shear does not govern. The shear strength is

governed by shear crushing. The design capacity in shear crushing is given by MSJC Eq. (2-7) as

$$\begin{aligned} B_{vc} &= 0.350 \sqrt{f'_m A_b} \\ &= 0.350 \sqrt{\left(3000 \frac{\text{lbf}}{\text{in}^2}\right) (0.46 \text{ in}^2)} \\ &= 2.13 \text{ kips} \\ &> b_v \quad [\text{satisfactory}] \end{aligned}$$

### Headed Anchor Bolts in Combined Tension and Shear

For combined tension and shear, MSJC Eq. (2-10) shall be satisfied

$$\frac{b_a}{B_a} + \frac{b_v}{B_v} \leq 1$$

The governing design axial capacity of an anchor bolt is  $B_a$ . The governing design shear capacity of an anchor bolt is  $B_v$ . In addition, the design capacity in shear and tension shall each exceed the applied loads.

#### Example 6.16

For the bracket anchored to a nominal 8 in, solid grouted concrete block masonry wall described in Ex. 6.14, the masonry compressive strength is 3000 lbf/in<sup>2</sup>. Anchor bolts are 7/8 in diameter, ASTM A307 type C, with a minimum specified yield strength of 36 kips/in<sup>2</sup>. For the loads indicated in the illustration for Ex. 6.14, determine whether the bolts are adequate for the combined tension and shear forces.

#### Solution

For combined tension and shear, MSJC Eq. (2-10) must be satisfied

$$\frac{b_a}{B_a} + \frac{b_v}{B_v} \leq 1$$

The left-hand side of the expression is

$$\begin{aligned} \frac{1.75 \text{ kips}}{3.44 \text{ kips}} + \frac{1.0 \text{ kip}}{2.13 \text{ kips}} &= 0.51 + 0.47 \\ &= 0.98 \\ &< 1.00 \quad [\text{satisfactory}] \end{aligned}$$

## 8. DESIGN OF PRESTRESSED MASONRY

### Nomenclature

|          |                                                               |                      |
|----------|---------------------------------------------------------------|----------------------|
| $A_n$    | net cross-sectional area of masonry                           | in <sup>2</sup>      |
| $A_{ps}$ | area of prestressing steel                                    | in <sup>2</sup>      |
| $A_s$    | area of reinforcement                                         | in <sup>2</sup>      |
| $E_m$    | modulus of elasticity of masonry                              | kips/in <sup>2</sup> |
| $f_a$    | calculated compressive stress in masonry caused by axial load | lbf/in <sup>2</sup>  |

|           |                                                                                  |                      |
|-----------|----------------------------------------------------------------------------------|----------------------|
| $f_b$     | calculated compressive stress in masonry caused by flexure                       | lbf/in <sup>2</sup>  |
| $f'_m$    | specified compressive strength of masonry                                        | lbf/in <sup>2</sup>  |
| $f'_{mi}$ | specified compressive strength of masonry at time of prestress transfer          | lbf/in <sup>2</sup>  |
| $f_{ps}$  | stress in prestressed tendon at nominal strength                                 | kips/in <sup>2</sup> |
| $f_{psi}$ | initial stress in prestressed tendon                                             | kips/in <sup>2</sup> |
| $f_{pu}$  | specified tensile strength of prestressing tendons                               | kips/in <sup>2</sup> |
| $f_{py}$  | specified yield strength of prestressing tendons                                 | kips/in <sup>2</sup> |
| $f_s$     | stress in prestressing tendons                                                   | kips/in <sup>2</sup> |
| $f_{se}$  | effective stress in prestressed tendon after allowance for all prestress losses  | kips/in <sup>2</sup> |
| $f_y$     | specified yield stress of non-prestressed reinforcement                          | kips/in <sup>2</sup> |
| $F_a$     | allowable compressive stress in masonry caused by axial load                     | lbf/in <sup>2</sup>  |
| $F_b$     | allowable compressive stress in masonry caused by flexure                        | lbf/in <sup>2</sup>  |
| $F_t$     | allowable tensile stress in masonry caused by flexure                            | lbf/in <sup>2</sup>  |
| $h$       | effective height of wall                                                         | ft                   |
| $I_n$     | moment of inertia of net cross-sectional area                                    | in <sup>4</sup>      |
| $l_p$     | clear span of the prestressed member in the direction of the prestressing tendon | ft                   |
| $M$       | maximum moment at the section under consideration                                | ft-kips              |
| $M_n$     | nominal flexural strength                                                        | ft-kips              |
| $M_u$     | factored moment                                                                  | ft-kips              |
| $P$       | sum of $P_w$ and $P_f$                                                           | kips                 |
| $P_f$     | unfactored load from tributary floor or roof loads                               | kips                 |
| $P_{ps}$  | prestressing tendon force at time and location relevant for design               | kips                 |
| $P_u$     | factored axial load                                                              | kips                 |
| $P_w$     | unfactored weight of wall tributary to section considered                        | kips                 |
| $r$       | radius of gyration                                                               | in                   |
| $S_n$     | section modulus of net cross-sectional area                                      | in <sup>3</sup>      |
| $w_u$     | factored lateral load                                                            | lbf/ft               |

### Symbols

|        |                           |   |
|--------|---------------------------|---|
| $\phi$ | strength reduction factor | — |
|--------|---------------------------|---|

### General Considerations

A masonry member is prestressed by tensioning a prestressing tendon that is anchored within the member. This produces a compression in the member, increasing its flexural tensile capacity, and augmenting its resistance to lateral loading.<sup>7</sup> Compared with conventional reinforced masonry construction, prestressed masonry

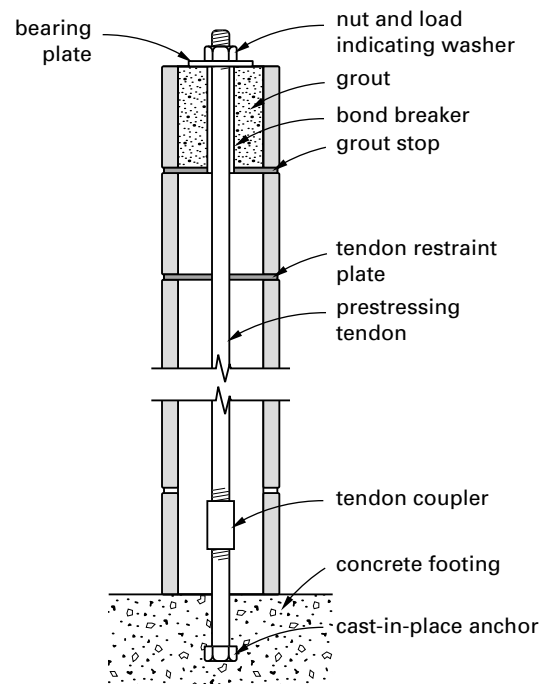
reduces construction time and costs by eliminating most of the grout and conventional reinforcement. This also reduces the weight of the masonry, resulting in smaller seismic forces and cost savings in foundations. The technique is particularly well suited to walls, and design consists of an initial allowable stress approach followed by a strength check. The addition of prestressing to an existing masonry structure is a relatively simple technique, and can be used to strengthen deficient structures.

Prestressed masonry members must be designed for the three design stages: transfer, service, and ultimate. At the transfer design stage, a prestressing force is applied to the member. Immediate prestress losses result from the elastic deformation of the masonry and anchor set. At the serviceability design stage, all time-dependent prestress losses have occurred as a result of creep, shrinkage of the masonry, and relaxation of the tendon stress. At the strength design stage, a rectangular stress block is assumed in the masonry, with a maximum strain in concrete masonry of 0.0025.

### Construction Technique

Figure 6.12 shows a section through a typical prestressed masonry wall. Prestressing tendons are anchored at the foot and top of the wall. Tendons are normally unbonded, as shown, and require corrosion protection in walls that are subject to a moist and corrosive environment. Corrosion protection may be provided by coating the tendon with a corrosion-inhibiting material, and enclosing the tendon in a continuous plastic sheath.

Figure 6.12 Prestressed Masonry Construction



<sup>7</sup>Durning, T.A., 2000 (See References and Codes)



Alternatively, tendons may be galvanized threaded high-tensile steel rods.

Tendons are attached by a threaded coupler at the foot of the wall to an anchor bolt embedded in the concrete foundation. An inspection port is provided at the location of the coupler to enable the tendon to be connected to the coupler after the block wall has been constructed to full height. Blocks are usually laid in face-shell mortar bedding. In accordance with MSJC Sec. 4.8.2, couplers shall develop 95% of the specified tensile strength of the prestressing tendons. When a moment connection is not required at the base of the wall, anchor bolts may be located in the first block course of masonry.

The tendons are normally laterally restrained in order to conform to the lateral deformation of the wall. This has the advantage that the prestressing force does not contribute to the elastic instability of the wall, as lateral displacement of the wall is counterbalanced by an equal and opposite restraint from the prestressing tendons. In addition, the ultimate strength of the wall is increased by ensuring that the wall and the tendon deform together. MSJC Commentary Sec. 4.4.2 stipulates that three restraints along the length of a tendon provide adequate restraint. Restraint is provided by a steel plate with a central hole. The sides of the plate are embedded in the joint between courses. Alternatively, a cell can be filled with grout and a bond breaker applied to the tendon to allow it to move freely within the cell.

Similarly, the topmost cell is filled with grout and a steel bearing plate bedded on top. The size of bearing plate is determined from the requirement of MSJC Sec. 4.8.4.2 that bearing stresses shall not exceed 50% of the masonry compressive strength at the time of transfer. The tendon may be stressed by means of an hydraulic jack, or by tightening a nut with a standard wrench. In the former method, the load is measured by a load cell or a calibrated gauge on the jack. The force produced by tightening a nut may be measured with a direct tension indicator washer. This is a washer with dimples formed on its top face. A hardened steel washer is placed on top of the indicator washer, and as the nut is tightened, the dimples are compressed. The required force in the tendon has been produced when the gap between the two washers reaches a specified amount as measured by a feeler gauge. MSJC Specification Sec. 3.6B requires that the elongation of the tendons be measured and compared with the elongation anticipated for the applied prestressing force. If the discrepancy between the two methods exceeds 7%, the cause must be determined and corrected.

### Serviceability Design Stage

For laterally restrained tendons, the permissible stresses under service loads after all prestressing losses have occurred are specified in MSJC Secs. 4.3, 4.4, and 2.2.3. The allowable compressive stress in a member that is subjected to axial load and flexure, having a

height-to-radius of gyration ratio not greater than 99, is given by MSJC Eq. (2-15) as

$$F_a = 0.25f'_m \left( 1 - \left( \frac{h}{140r} \right)^2 \right)$$

For members having a height-to-radius of gyration ratio greater than 99, the allowable compressive stress is given by MSJC Eq. (2-16) as

$$F_a = (0.25f'_m) \left( \frac{70r}{h} \right)^2$$

The allowable compressive stress in a member caused by flexure is given by MSJC Eq. (2-17) as

$$F_b = \frac{f'_m}{3}$$

The compressive stress produced in the wall by floor and roof loads, wall self-weight, and the effective prestressing force after all losses is

$$f_a = \frac{P + P_{ps}}{A_n}$$

The flexural stress produced in the wall by applied lateral loads and eccentric axial load is

$$f_b = \frac{M}{S_n}$$

The resulting combined stress caused by compression and flexure shall satisfy the interaction expression of MSJC Eq. (2-13), which is

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

The allowable tensile stress,  $F_t$ , caused by flexure is given by MSJC Table 2.2.3.2. In accordance with MSJC Sec. 2.2.3.2,

$$f_b - f_a \leq F_t$$

For laterally restrained tendons, the member cannot buckle under its own prestressing force. The Euler critical load is given by MSJC Eq. (2-18) as

$$P_e = \frac{\pi^2 E_m I_n \left( 1 - \frac{0.577e}{r} \right)^3}{h^2}$$

To ensure elastic stability, the axial load on a wall is limited by MSJC Eq. (2-14) to a value of

$$P \leq \frac{P_e}{4}$$

For laterally restrained tendons, the effective prestressing force,  $P_{ps}$ , is not considered in the calculation of the axial force,  $P$ .

At the time the prestress is applied, the permissible stress in prestressing tendons at anchorages and couplers is given by MSJC Sec. 4.3.3 as

$$\begin{aligned} f_s &\leq 0.78f_{py} \\ &\leq 0.70f_{pu} \end{aligned}$$

The total loss of prestress for concrete masonry after long-term service load conditions is given by MSJC Commentary Sec. 4.3.4 as 30–35%.

### Example 6.17

A nominal 8 in. concrete block masonry wall, laid in face-shell type N Portland cement/lime mortar bedding, has a specified strength of 2000 lbf/in<sup>2</sup>. The wall is 16 ft high, is pinned at the top and bottom, and weighs 45 lbf/ft<sup>2</sup>. The wind load on the wall, which may act in either direction, is 20 lbf/ft<sup>2</sup>. The wall supports an axial load of 500 lbf/ft. Assuming the total loss of prestress after long-term service is 35%, determine the spacing required for 7/16 in diameter steel rod tendons. The tendons are laterally restrained.

#### Solution

The design is based on a 1 ft length of wall. The relevant details<sup>8</sup> are

$$\begin{aligned} A_{ps} &= 0.142 \text{ in}^2 \quad [\text{for one rod}] \\ f_{py} &= 100 \text{ kips/in}^2 \\ f_{pu} &= 122 \text{ kips/in}^2 \end{aligned}$$

$$\begin{aligned} A_n &= 30 \text{ in}^2 && [\text{face-shell mortar bedding}] \\ I_n &= 309 \text{ in}^4 && [\text{face-shell mortar bedding}] \\ S_n &= 81 \text{ in}^3 && [\text{face-shell mortar bedding}] \\ r &= 3.21 \text{ in} && [\text{face-shell mortar bedding}] \\ E_m &= 900f'_m \\ &= 1800 \text{ kips/in}^2 \end{aligned}$$

The weight of the wall above midheight is

$$\begin{aligned} P_w &= \left(45 \frac{\text{lbf}}{\text{ft}}\right)(8 \text{ ft}) \\ &= 360 \text{ lbf} \end{aligned}$$

The axial load at the top of the wall is

$$P_f = 500 \text{ lbf}$$

The total axial load at midheight of the wall is

$$\begin{aligned} P &= P_w + P_f \\ &= 360 \text{ lbf} + 500 \text{ lbf} \\ &= 860 \text{ lbf} \end{aligned}$$

The compressive stress produced in the wall by floor and roof loads, wall self-weight, and the effective prestressing force after all losses is

$$\begin{aligned} f_a &= \frac{P + P_{ps}}{A_n} \\ &= \frac{860 \text{ lbf} + P_{ps}}{30 \text{ in}^2} \\ &= 28.67 \frac{\text{lbf}}{\text{in}^2} + \frac{P_{ps}}{30 \text{ in}^2} \end{aligned}$$

The bending moment acting on the wall is

$$\begin{aligned} M &= \frac{wh^2}{8} \\ &= \frac{\left(20 \frac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})^2}{8} \\ &= 640 \text{ ft-lbf} \end{aligned}$$

The flexural stress produced in the wall by applied lateral load is

$$\begin{aligned} f_b &= \frac{M}{S_n} \\ &= \frac{(640 \text{ ft-lbf})\left(12 \frac{\text{in}}{\text{ft}}\right)}{81 \text{ in}^3} \\ &= 94.81 \text{ lbf/in}^2 \end{aligned}$$

The slenderness ratio of the wall is

$$\begin{aligned} \frac{h}{r} &= \frac{(16 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{3.21 \text{ in}} \\ &= 59.81 \end{aligned}$$

The allowable compressive stress in a member having a height-to-radius of gyration ratio not greater than 99, and subjected to axial load and flexure, is given by MSJC Eq. (2-15) as

$$\begin{aligned} F_a &= 0.25f'_m \left(1 - \left(\frac{h}{140r}\right)^2\right) \\ &= (0.25)\left(2000 \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \left(\frac{59.81}{140}\right)^2\right) \\ &= 408.74 \text{ lbf/in}^2 \end{aligned}$$

<sup>8</sup>Masonry Society, 2010 (See References and Codes)



The allowable compressive stress in a member caused by flexure is given by MSJC Eq. (2-17) as

$$F_b = \frac{f'_m}{3} = \frac{2000 \frac{\text{lbf}}{\text{in}^2}}{3} = 666.67 \text{ lbf/in}^2$$

The combined stress caused by compression and flexure shall satisfy MSJC Eq. (2-13), which is

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

$$\frac{28.67 \frac{\text{lbf}}{\text{in}^2} + \frac{P_{ps}}{30 \text{ in}^2}}{408.74 \frac{\text{lbf}}{\text{in}^2}} + \frac{94.81 \frac{\text{lbf}}{\text{in}^2}}{666.67 \frac{\text{lbf}}{\text{in}^2}} \leq 1.0$$

$$0.070 + \frac{P_{ps}}{12,262} + 0.14 \leq 1.0$$

$$P_{ps} \leq \frac{9687 \frac{\text{lbf}}{\text{kip}}}{1000 \frac{\text{lbf}}{\text{kip}}} \leq 9.69 \text{ kips}$$

The allowable tensile stress caused by flexure for type N Portland cement/lime mortar is given by MSJC Table 2.2.3.2 as

$$F_t = 19 \frac{\text{lbf}}{\text{in}^2}$$

The difference between calculated compressive stress caused by axial load and flexure is

$$f_b - f_a \leq 19 \frac{\text{lbf}}{\text{in}^2}$$

$$94.82 \frac{\text{lbf}}{\text{in}^2} - \left( 28.67 \frac{\text{lbf}}{\text{in}^2} + \frac{P_{ps}}{30 \text{ in}^2} \right) \leq 19 \frac{\text{lbf}}{\text{in}^2}$$

$$P_{ps} \geq \frac{1415 \frac{\text{lbf}}{\text{kip}}}{1000 \frac{\text{lbf}}{\text{kip}}} \geq 1.41 \text{ kips}$$

At the time of application of the prestress, the maximum permissible stress in prestressing tendons at anchorages and couplers is given by MSJC Sec. 4.3.3 as the lesser of

$$f_{psi} = 0.70f_{pu}$$

$$= (0.70) \left( 122 \frac{\text{kips}}{\text{in}^2} \right)$$

$$= 85.4 \text{ kips/in}^2$$

$$f_{psi} = 0.78f_{py}$$

$$= (0.78) \left( 100 \frac{\text{kips}}{\text{in}^2} \right)$$

$$= 78 \text{ kips/in}^2 \quad [\text{governs}]$$

Allowing for a total long-term loss of prestress of 35%, the final tendon force is

$$P_{ps} = 0.65f_{psi}A_{ps}$$

$$= (0.65) \left( 78 \frac{\text{kips}}{\text{in}^2} \right) (0.142 \text{ in}^2)$$

$$= 7.2 \text{ kips}$$

Providing a  $\frac{7}{16}$  in diameter tendon at 2 ft on center gives an effective final prestressing force per foot of

$$P_{ps} = 3.60 \text{ kips}$$

$$> 1.41 \text{ kips} \quad [\text{satisfactory}]$$

$$< 9.66 \text{ kips} \quad [\text{satisfactory}]$$

For laterally restrained tendons, the Euler critical load is given by MSJC Eq. (2-18) as

$$P_e = \frac{\pi^2 E_m I_n \left( 1 - \frac{0.577e}{r} \right)^3}{h^2}$$

$$= \frac{\pi^2 \left( 1800 \frac{\text{kips}}{\text{in}^2} \right) (309 \text{ in}^4) (1 - 0)^3}{(192 \text{ in})^2}$$

$$= 148.91 \text{ kips}$$

To ensure elastic stability, the axial load on a wall is limited by MSJC Eq. (2-14) to a maximum value of

$$P = \frac{P_e}{4}$$

$$= \frac{148.91 \text{ kips}}{4}$$

$$= 37.23 \text{ kips}$$

$$> 860 \text{ lbf} \quad [\text{satisfactory}]$$

The wall is stable.

### Transfer Design Stage

The allowable stresses in the masonry at transfer are given by MSJC Sec. 4.4.1.2 as

$$F_{ai} = 1.2F_a$$

$$F_{bi} = 1.2F_b$$

MSJC Eq. (2-13) for combined compression and flexure becomes

$$\frac{f_{ai}}{F_{ai}} + \frac{f_{bi}}{F_{bi}} \leq 1.0$$

The allowable tensile stress  $F_t$  caused by flexure is given by MSJC Table 2.2.3.2. Then, in accordance with MSJC Sec. 2.2.3.2,

$$f_{bi} - f_{ai} \leq F_t$$

Immediately after transfer, the permissible stress in prestressing tendons is given by MSJC Sec. 4.3.2 as

$$\begin{aligned} f_{psi} &\leq 0.82f_{py} \\ &\leq 0.74f_{pu} \end{aligned}$$

However, for post-tensioned tendons with couplers, the permissible stress is governed by the maximum allowed at the time the prestress is applied, given by MSJC Sec. 4.3.3 as

$$\begin{aligned} f_{psi} &\leq 0.78f_{py} \\ &\leq 0.70f_{pu} \end{aligned}$$

MSJC Commentary Sec. 4.3.4 gives the initial loss of prestress for concrete masonry at transfer as 5–10%.

### Example 6.18

The nominal 8 in. concrete block masonry wall described in Ex. 6.17 is post-tensioned with  $\frac{7}{16}$  in diameter, steel rod tendons at 2 ft centers. Assuming the initial loss of prestress at transfer is 5%, determine whether the stresses at transfer are satisfactory. The specified masonry strength at transfer is 1500 lbf/in<sup>2</sup>.

#### Solution

At the time the prestress is applied, the allowable stress is governed by MSJC Sec. 4.3.3. The maximum permissible stress was obtained in Ex. 6.17 as

$$f_{psi} = 78.0 \frac{\text{kips}}{\text{in}^2}$$

Allowing for an initial loss of prestress of 5%, the tendon force immediately after transfer is

$$\begin{aligned} P_{psi} &= 0.95f_{psi}A_{ps} \\ &= (0.95) \left( 78.0 \frac{\text{kips}}{\text{in}^2} \right) (0.142 \text{ in}^2) \\ &= 10.52 \text{ kips} \end{aligned}$$

For tendons spaced at 2 ft on center, the initial force per foot of wall is

$$P_{psi} = 5.26 \text{ kips}$$

The total axial load at the midheight of the wall is

$$P = 860 \text{ lbf}$$

The compressive stress produced in the wall by floor and roof loads, wall self-weight, and the effective prestressing force after initial losses is

$$\begin{aligned} f_{ai} &= \frac{P + P_{psi}}{A_n} \\ &= \frac{860 \text{ lbf} + 5260 \text{ lbf}}{30 \text{ in}^2} \\ &= 204 \text{ lbf/in}^2 \end{aligned}$$

The flexural stress produced in the wall by applied lateral load at transfer is

$$f_{bi} = 94.82 \text{ lbf/in}^2$$

The allowable compressive stress in the wall at transfer, caused by combined compression and flexure, is given by MSJC Sec. 4.4.1.2 and MSJC Eq. (2-15) as

$$\begin{aligned} F_{ai} &= \frac{1.2F_a f'_{mi}}{f'_m} \\ &= \frac{(1.2) \left( 408.74 \frac{\text{lbf}}{\text{in}^2} \right) \left( 1500 \frac{\text{lbf}}{\text{in}^2} \right)}{2000 \frac{\text{lbf}}{\text{in}^2}} \\ &= 367.87 \text{ lbf/in}^2 \end{aligned}$$

The allowable compressive stress in the wall at transfer, caused by flexure, is given by MSJC Sec. 4.4.1.2 and MSJC Eq. (2-17) as

$$\begin{aligned} F_{bi} &= \frac{1.2f'_{mi}}{3} \\ &= \frac{(1.2) \left( 1500 \frac{\text{lbf}}{\text{in}^2} \right)}{3} \\ &= 600 \text{ lbf/in}^2 \end{aligned}$$

The interaction equation for combined compression and flexure is

$$\frac{f_{ai}}{F_{ai}} + \frac{f_{bi}}{F_{bi}} \leq 1.0$$

The left-hand side of the expression is evaluated as

$$\begin{aligned} \frac{204 \frac{\text{lbf}}{\text{in}^2}}{367.87 \frac{\text{lbf}}{\text{in}^2}} + \frac{94.82 \frac{\text{lbf}}{\text{in}^2}}{600 \frac{\text{lbf}}{\text{in}^2}} &= 0.713 \\ &< 1.0 \quad [\text{satisfactory}] \end{aligned}$$

The allowable tensile stress caused by flexure,  $F_t$ , is given by MSJC Table 2.2.3.2, and in accordance with MSJC Sec. 2.2.3.2,

$$\begin{aligned} f_{bi} - f_{ai} &\leq F_t \\ 94.82 \frac{\text{lbf}}{\text{in}^2} - 204 \frac{\text{lbf}}{\text{in}^2} &\leq 19 \frac{\text{lbf}}{\text{in}^2} \\ -109.18 \text{ lbf/in}^2 &< 19 \text{ lbf/in}^2 \\ &\text{[satisfactory]} \end{aligned}$$

The stresses at transfer are satisfactory.

### Strength Design Stage

The design flexural strength for a prestressed member is given by MSJC Sec. 4.4.3.3 as

$$\phi M_n = 0.8 M_n$$

The nominal strength of a masonry member is determined as detailed in MSJC Sec. 4.4.3.

For walls with laterally restrained, unbonded tendons, the stress in the tendons at nominal load is given by MSJC Eq. (4-3) as

$$\begin{aligned} f_{ps} &= f_{se} + (1,000,000) \left( \frac{d}{l_p} \right) \sqrt{1 - (1.4) \left( \frac{f_{pu} A_{ps}}{b d f'_m} \right)} \\ &\leq f_{ps} \\ &\geq f_{se} \end{aligned}$$

An equivalent rectangular stress block is assumed in the masonry, with a stress of  $0.80 f'_m$ . The depth of the stress block is given by MSJC Eq. (4-1) as

$$a = \frac{f_{ps} A_{ps} + f_y A_s + P_u}{0.80 f'_m b}$$

The nominal moment is given by MSJC Eq. (4-2) as

$$M_n = (f_{ps} A_{ps} + f_y A_s + P_u) \left( d - \frac{a}{2} \right)$$

To ensure a ductile failure, MSJC Sec. 4.4.3.6 specifies a maximum depth for the stress block of

$$a = 0.425d$$

Also,

$$a \leq \text{face shell thickness}$$

### Example 6.19

The nominal 8 in, concrete block masonry wall described in Ex. 6.17 is post-tensioned with  $7/16$  in diameter, steel rod tendons at 2 ft centers. Determine whether the design moment strength is adequate. The factored axial dead load at midheight is  $P_u = 900$  lbf/ft, and the factored lateral force is  $w_u = 32$  lbf/ft<sup>2</sup>.

### Solution

Consider one foot length of wall.

The factored moment is

$$\begin{aligned} M_u &= \frac{w_u h^2}{8} \\ &= \frac{\left( 32 \frac{\text{lbf}}{\text{ft}} \right) (16 \text{ ft})^2}{8} \\ &= 1024 \text{ ft-lbf} \end{aligned}$$

From Ex. 6.17, the effective stress in the tendons after all losses is

$$\begin{aligned} f_{se} &= 0.65 f_{psi} \\ &= (0.65) \left( 78 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 50.70 \text{ kips/in}^2 \end{aligned}$$

The area of the tendons per foot of wall is

$$\begin{aligned} A_{ps} &= \frac{0.142 \text{ in}^2}{2 \text{ ft}} \\ &= 0.071 \text{ in}^2 \end{aligned}$$

For walls with laterally restrained, unbonded tendons, the stress in the tendons at nominal load per foot of wall is given by MSJC Eq. (4-3) as

$$\begin{aligned} f_{ps} &= f_{se} + (1,000,000) \left( \frac{d}{l_p} \right) \sqrt{1 - (1.4) \left( \frac{f_{pu} A_{ps}}{b d f'_m} \right)} \\ &= 50.70 \frac{\text{kips}}{\text{in}^2} + (1000) \left( \frac{3.82 \text{ in}}{(16 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)} \right) \\ &\quad \times \sqrt{1 - (1.4) \left( \frac{\left( 122 \frac{\text{kips}}{\text{in}^2} \right) (0.071 \text{ in}^2)}{(12 \text{ in}) (3.82 \text{ in}) \left( 2 \frac{\text{kips}}{\text{in}^2} \right)} \right)} \\ &= 69.23 \frac{\text{kips}}{\text{in}^2} \quad \text{[governs]} \\ &\leq f_{py} \\ &= 100 \text{ kips/in}^2 \end{aligned}$$

The force in the tendons at nominal load per foot of wall is

$$\begin{aligned} f_{ps} A_{ps} &= \left( 69,240 \frac{\text{lbf}}{\text{in}^2} \right) (0.071 \text{ in}^2) \\ &= 4916 \text{ lbf} \end{aligned}$$

The depth of the stress block is given by MSJC Eq. (4-1) as

$$\begin{aligned} a &= \frac{f_{ps}A_{ps} + f_yA_s + P_u}{0.80f'_m b} \\ &= \frac{4916 \text{ lbf} + 0 + 900 \text{ lbf}}{(0.80)\left(2000 \frac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})} \\ &= 0.303 \text{ in} \\ &< \text{face shell thickness} \quad [\text{satisfactory}] \end{aligned}$$

$$\begin{aligned} a &= \left(\frac{0.303 \text{ in}}{3.82 \text{ in}}\right)d \\ &= 0.08d \\ &< 0.425d \quad [\text{satisfies MSJC Sec. 4.4.3.6}] \end{aligned}$$

The nominal moment is given by MSJC Eq. (4-2) as

$$\begin{aligned} M_n &= (f_{ps}A_{ps} + f_yA_s + P_u)\left(d - \frac{a}{2}\right) \\ &= \left(\frac{4916 \text{ lbf} + 0 + 900 \text{ lbf}}{1000 \frac{\text{lbf}}{\text{kips}}}\right) \\ &\quad \times \left(3.82 \text{ in} - \frac{0.303 \text{ in}}{2}\right) \\ &= 21.34 \text{ in-kips} \end{aligned}$$

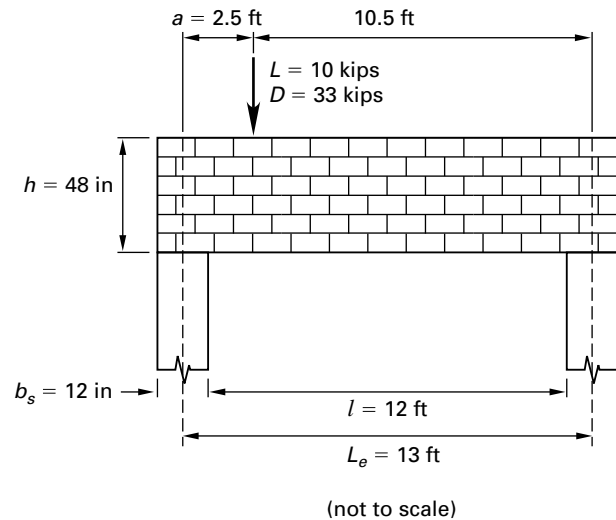
The design flexural strength for a prestressed member is given by MSJC Sec. 4.4.3.3 as

$$\begin{aligned} \phi M_n &= 0.8M_n \\ &= (0.8)(21.34 \text{ in-kips})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right) \\ &= 1423 \text{ ft-lbf} \\ &> M_u \quad [\text{satisfactory}] \end{aligned}$$

The design moment strength is adequate.

## PRACTICE PROBLEMS

Problems 1–3 refer to the nominal 8 in, solid grouted concrete block masonry beam shown. The beam is simply supported over a clear span of 12 ft. The overall depth of the beam is 48 in, and its effective depth,  $d$ , is 45 in. The unit weight may be assumed to be 80 lbf/ft<sup>3</sup>. The masonry has a compressive strength of 1500 lbf/in<sup>2</sup>, and the reinforcement consists of two no. 7 grade 60 bars.



1. What is most nearly the maximum nonfactored bending moment on the beam caused by the applied loads shown?

- (A) 20 ft-kips
- (B) 66 ft-kips
- (C) 91 ft-kips
- (D) 110 ft-kips

2. What is most nearly the moment capacity of the section?

- (A) 92.0 ft-kips
- (B) 96.0 ft-kips
- (C) 100 ft-kips
- (D) 104 ft-kips

3. What is most nearly the spacing required for no. 5 stirrups at the critical section?

- (A) 2 in
- (B) 4 in
- (C) 6 in
- (D) 8 in

Problems 4 and 5 refer to a nominal 8 in square, solid grouted, concrete block masonry column with a specified strength of  $f'_m = 1500 \text{ lbf/in}^2$  and reinforced with four no. 4 grade 60 bars. The column supports an axial load of  $P = 25 \text{ kips}$ , has a height of  $h = 12 \text{ ft}$ , and may be considered pinned at each end. Neglect accidental eccentricity.

4. Which of the given statements is/are true?

- I. The allowable axial load is approximately 24 kips.
  - II. The allowable axial load is approximately 27 kips.
  - III. The column is adequate.
  - IV. The column is inadequate.
- (A) I and III only
  - (B) I and IV only
  - (C) II and III only
  - (D) II and IV only

5. What is most nearly the minimum required size and spacing of lateral ties?

- (A) lateral ties = 0.25 in diameter, spacing not more than 8 in
- (B) lateral ties = 0.25 in diameter, spacing not more than 10 in
- (C) lateral ties < 0.25 in diameter, spacing not more than 8 in
- (D) lateral ties < 0.25 in diameter, spacing not more than 10 in

## SOLUTIONS

1. The effective span length is given by MSJC Sec. 1.13 as

$$\begin{aligned} L_e &= L_s \\ &= l + b_s \\ &= 12 \text{ ft} + 1 \text{ ft} \\ &= 13 \text{ ft} \end{aligned}$$

The maximum moment occurs at point  $P$ , the point of application of the concentrated load, a distance  $a = 2.5 \text{ ft}$  from the center of the left-hand support.

The beam self-weight is

$$\begin{aligned} w &= \left(80 \frac{\text{lbf}}{\text{ft}^2}\right)(4 \text{ ft}) \\ &= 320 \text{ lbf/ft} \end{aligned}$$

The bending moment produced by this self-weight at point  $P$  is

$$\begin{aligned} M_s &= \frac{wa(L_e - a)}{2} \\ &= \frac{\left(320 \frac{\text{lbf}}{\text{ft}}\right)(2.5 \text{ ft})(13 \text{ ft} - 2.5 \text{ ft})}{(2)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 4.2 \text{ ft-kips} \end{aligned}$$

The bending moment produced by the concentrated dead load at point  $P$  is

$$\begin{aligned} M_D &= \frac{Da(L_e - a)}{L_e} \\ &= \frac{(33 \text{ kips})(2.5 \text{ ft})(13 \text{ ft} - 2.5 \text{ ft})}{13 \text{ ft}} \\ &= 66.63 \text{ ft-kips} \end{aligned}$$

The bending moment produced by the concentrated live load at point  $P$  is

$$\begin{aligned} M_L &= \frac{La(L_e - a)}{L_e} \\ &= \frac{(10 \text{ kips})(2.5 \text{ ft})(13 \text{ ft} - 2.5 \text{ ft})}{13 \text{ ft}} \\ &= 20.19 \text{ ft-kips} \end{aligned}$$

The total moment at point  $P$  is

$$\begin{aligned} M &= M_s + M_D + M_L \\ &= 4.2 \text{ ft-kips} + 66.63 \text{ ft-kips} \\ &\quad + 20.19 \text{ ft-kips} \\ &= 91.02 \text{ ft-kips} \quad (91 \text{ ft-kips}) \end{aligned}$$

**The answer is (C).**

**2.** The total moment at point  $P$  is derived in Prob. 1 as

$$M = 91.02 \text{ ft-kips}$$

The relevant parameters of the beam are

$$\begin{aligned} b_w &= 7.63 \text{ in} \\ d &= 45 \text{ in} \\ f'_m &= 1500 \frac{\text{lbf}}{\text{in}^2} \\ f_y &= 60,000 \frac{\text{lbf}}{\text{in}^2} \\ L_e &= 13 \text{ ft} \end{aligned}$$

The allowable stresses are

$$\begin{aligned} F_b &= \frac{f'_m}{3} \\ &= \frac{1500 \frac{\text{lbf}}{\text{in}^2}}{3} \\ &= 500 \frac{\text{lbf}}{\text{in}^2} \\ F_s &= 24,000 \text{ lbf/in}^2 \end{aligned}$$

The modular ratio is

$$\begin{aligned} n &= \frac{E_s}{E_m} \\ &= \frac{29,000,000 \frac{\text{lbf}}{\text{in}^2}}{900 f'_m} \\ &= \frac{29,000,000 \frac{\text{lbf}}{\text{in}^2}}{(900) \left( 1500 \frac{\text{lbf}}{\text{in}^2} \right)} \\ &= 21.48 \end{aligned}$$

Assuming the reinforcement consists of two no. 7 bars, the tension reinforcement ratio is

$$\begin{aligned} \rho &= \frac{A_s}{b_w d} \\ &= \frac{1.20 \text{ in}^2}{(7.63 \text{ in})(45 \text{ in})} \\ &= 0.00349 \\ \rho n &= (0.00349)(21.48) \\ &= 0.0750 \end{aligned}$$

Using this value of  $\rho n$ , the neutral axis depth factor is obtained from Table 6.1 as

$$k = 0.3197$$

The lever-arm factor is

$$\begin{aligned} j &= 1 - \frac{k}{3} \\ &= 1 - \frac{0.3197}{3} \\ &= 0.893 \end{aligned}$$

The moment capacity of the section is the lesser of

$$\begin{aligned} M_s &= F_s j \rho b_w d^2 \\ &= \frac{\left( 24,000 \frac{\text{lbf}}{\text{in}^2} \right) (0.893) (0.00349) (7.63 \text{ in}) (45 \text{ in})^2}{\left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)} \\ &= 96.31 \text{ ft-kips} \end{aligned}$$

$$\begin{aligned} M_m &= \frac{F_b j k b_w d^2}{2} \\ &= \frac{\left( 500 \frac{\text{lbf}}{\text{in}^2} \right) (0.893) (0.3197) (7.63 \text{ in}) (45 \text{ in})^2}{(2) \left( 12 \frac{\text{in}}{\text{ft}} \right) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)} \\ &= 91.90 \text{ ft-kips} \quad (92 \text{ ft-kips}) \quad [\text{governs}] \\ &> M \quad [\text{satisfactory}] \end{aligned}$$

Two no. 7 bars are adequate.

**The answer is (A).**

3. The critical section for shear occurs at a distance of  $d/2$  from the face of the support. This is a distance from the left-hand effective support given by

$$\begin{aligned} x &= \frac{b_s + d}{2} \\ &= \frac{1 \text{ ft} + 3.75 \text{ ft}}{2} \\ &= 2.38 \text{ ft} \end{aligned}$$

The shear force produced by the beam self-weight at a distance  $x$  from the left-hand effective support is

$$\begin{aligned} V_s &= w \left( \frac{L_e}{2} - x \right) \\ &= \left( 0.320 \frac{\text{kips}}{\text{ft}} \right) \left( \frac{13 \text{ ft}}{2} - 2.38 \text{ ft} \right) \\ &= 1.32 \text{ kips} \end{aligned}$$

The shear force produced by the concentrated dead load at a distance  $x$  from the left-hand effective support is

$$\begin{aligned} V_D &= \frac{D(L_e - a)}{L_e} \\ &= \frac{(33 \text{ kips})(13 \text{ ft} - 2.5 \text{ ft})}{13 \text{ ft}} \\ &= 26.65 \text{ kips} \end{aligned}$$

The shear force produced by the concentrated live load at a distance  $x$  from the left-hand effective support is

$$\begin{aligned} V_L &= \frac{L(L_e - a)}{L_e} \\ &= \frac{(10 \text{ kips})(13 \text{ ft} - 2.5 \text{ ft})}{13 \text{ ft}} \\ &= 8.08 \text{ kips} \end{aligned}$$

The total shear force at a distance  $x$  from the left-hand effective support is

$$\begin{aligned} V &= V_s + V_D + V_L \\ &= 1.32 \text{ kips} + 26.65 \text{ kips} + 8.08 \text{ kips} \\ &= 36.05 \text{ kips} \end{aligned}$$

The shear stress at the critical section is given by MSJC Eq. (2-23) as

$$\begin{aligned} f_v &= \frac{V}{b_w d} = \frac{(36.05 \text{ kips}) \left( 1000 \frac{\text{lbf}}{\text{kip}} \right)}{(7.63 \text{ in})(45 \text{ in})} \\ &= 105 \text{ lbf/in}^2 \end{aligned}$$

The allowable shear stress without shear reinforcement is given by MSJC Eq. (2-24) as

$$\begin{aligned} F_v &= \sqrt{f'_m} \\ &= \sqrt{1500 \frac{\text{lbf}}{\text{in}^2}} \\ &= 38.7 \frac{\text{lbf}}{\text{in}^2} \\ &< f_v \quad [\text{shear reinforcement is required}] \end{aligned}$$

The shear stress with shear reinforcement provided to carry the total shear force is limited by MSJC Eq. (2-27) to

$$\begin{aligned} F_v &= 3\sqrt{f'_m} \\ &= 3\sqrt{1500 \frac{\text{lbf}}{\text{in}^2}} \\ &= 116 \frac{\text{lbf}}{\text{in}^2} \\ &\leq 150 \frac{\text{lbf}}{\text{in}^2} \quad [\text{satisfies MSJC Sec. 2.3.5.2.3}] \\ &> f_v \quad [\text{satisfactory}] \end{aligned}$$

The minimum area of shear reinforcement required is given by MSJC Eq. (2-30) as

$$\begin{aligned} \frac{A_v}{s} &= \frac{V}{F_s d} \\ &= \frac{(36.05 \text{ kips}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{\left( 24 \frac{\text{kips}}{\text{in}^2} \right) (45 \text{ in})} \\ &= 0.401 \text{ in}^2/\text{ft} \end{aligned}$$

Providing no. 5 grade 60 stirrups at 8 in centers supplies a value of

$$\begin{aligned} \frac{A_v}{s} &= 0.470 \frac{\text{in}^2}{\text{ft}} \\ &> 0.401 \text{ in}^2/\text{ft} \quad [\text{satisfactory}] \end{aligned}$$

**The answer is (D).**

4. The effective column width is

$$b = 7.63 \text{ in}$$

The effective column height is

$$b = 12 \text{ ft}$$

The reinforcement area is

$$A_s = 0.80 \text{ in}^2$$

The effective column area is

$$\begin{aligned} A_s &= b^2 \\ &= (7.63 \text{ in})^2 \\ &= 58 \text{ in}^2 \end{aligned}$$

The reinforcement ratio is

$$\begin{aligned} \rho &= \frac{A_s}{A_n} \\ &= \frac{0.80 \text{ in}^2}{58 \text{ in}^2} \\ &= 0.014 \\ &< 0.04 \\ &> 0.0025 \quad [\text{satisfies MSJC Sec. 1.14.1.2}] \end{aligned}$$

The allowable reinforcement stress is

$$\begin{aligned} F_s &= 0.4f_y \\ &= (0.4) \left( 60 \frac{\text{kips}}{\text{in}^2} \right) \\ &= 24 \text{ kips/in}^2 \end{aligned}$$

The radius of gyration of the column is

$$\begin{aligned} r &= \sqrt{\frac{I_n}{A_n}} = \sqrt{\frac{b^2}{12}} \\ &= (0.289)(7.63 \text{ in}) \\ &= 2.21 \text{ in} \end{aligned}$$

The slenderness ratio of the column is

$$\begin{aligned} \frac{h}{r} &= \frac{(12 \text{ ft}) \left( 12 \frac{\text{in}}{\text{ft}} \right)}{2.21 \text{ in}} \\ &= 65.16 \\ &< 99 \quad [\text{MSJC Eq. (2-20) is applicable}] \end{aligned}$$

The allowable column load is given by

$$\begin{aligned} P_a &= (0.25f'_m A_n + 0.65A_{st}F_s) \left( 1.0 - \left( \frac{h}{140r} \right)^2 \right) \\ &= \left( (0.25) \left( 1.5 \frac{\text{kips}}{\text{in}^2} \right) (58 \text{ in}^2) \right. \\ &\quad \left. + (0.65)(0.80 \text{ in}^2) \left( 24 \frac{\text{kips}}{\text{in}^2} \right) \right) \\ &\quad \times \left( 1.0 - \left( \frac{65.16}{140} \right)^2 \right) \\ &= 26.81 \text{ kips} \quad (27 \text{ kips}) \\ &> P \quad [\text{satisfactory}] \end{aligned}$$

The column is adequate.

**The answer is (C).**

**5.** As specified by MSJC Sec. 1.14.1.3, lateral ties for the confinement of longitudinal reinforcement cannot be less than  $\frac{1}{4}$  in diameter. The spacing must not exceed the lesser of

$$\begin{aligned} s &= 48 \text{ lateral tie diameters} \\ &= (48)(0.375 \text{ in}) \\ &= 18 \text{ in} \end{aligned}$$

$$\begin{aligned} s &= \text{the least cross-sectional dimension of the column} \\ &= 8 \text{ in} \end{aligned}$$

$$\begin{aligned} s &= 16 \text{ longitudinal bar diameters} \\ &= (16)(0.5 \text{ in}) \\ &= 8 \text{ in} \quad [\text{governs}] \end{aligned}$$

**The answer is (A).**





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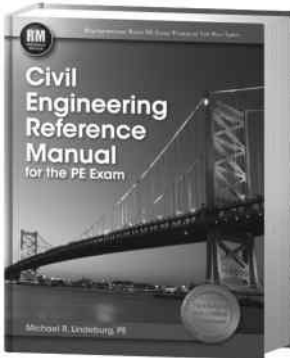
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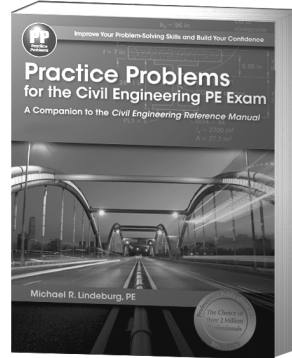
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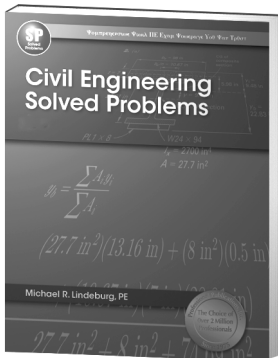
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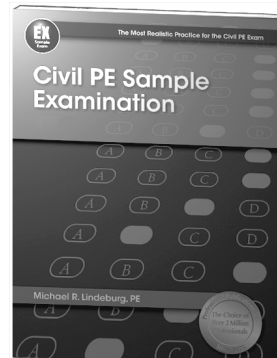
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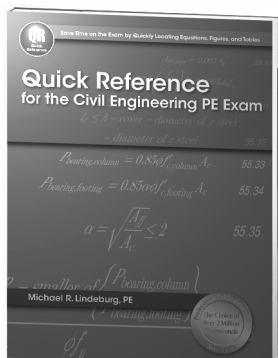
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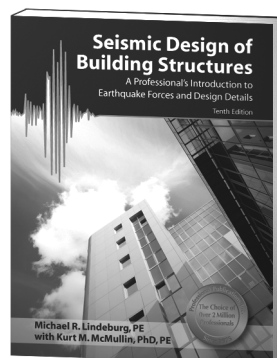
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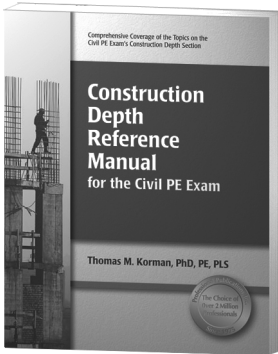
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## Construction

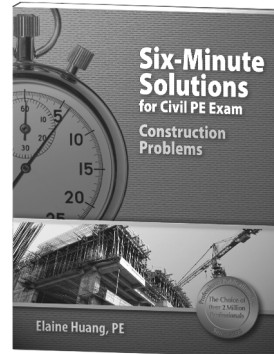
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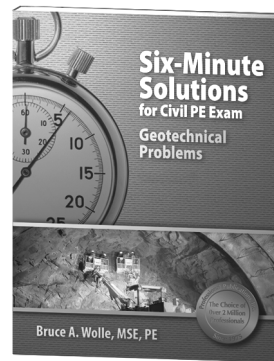
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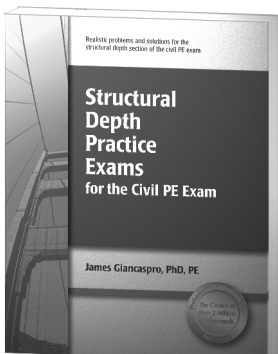
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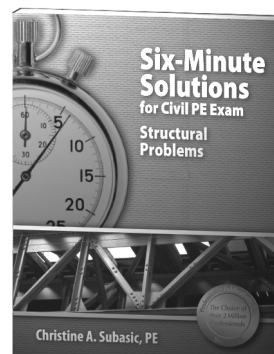
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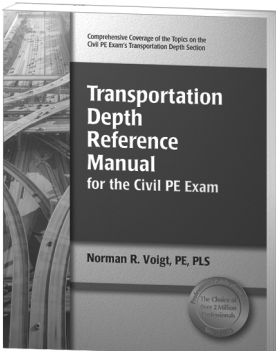
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## Transportation

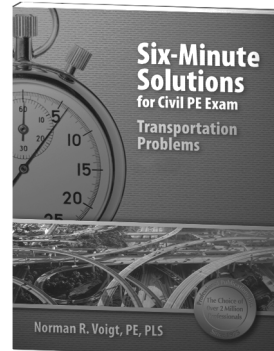
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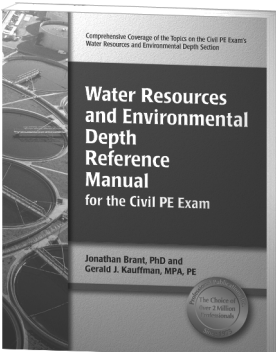
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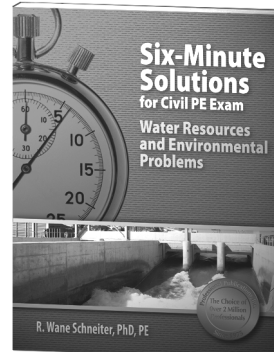
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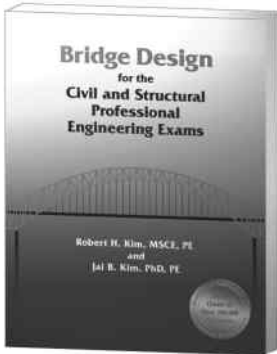
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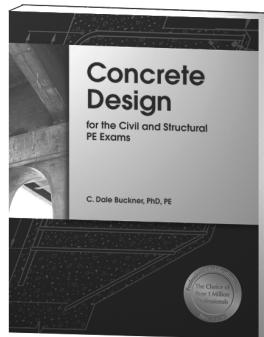
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Robert H. Kim, MSCE, PE; Jai B. Kim, PhD, PE

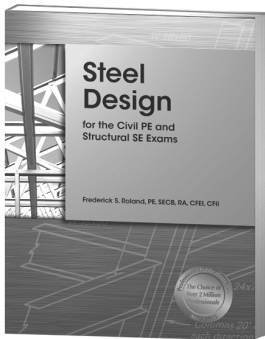
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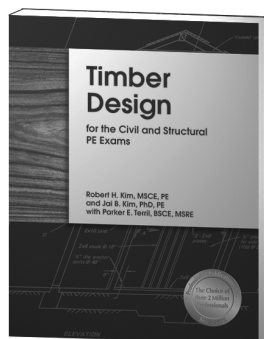
- Comprehensive concrete design review
- A complete overview of the relevant codes and standards
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### Steel Design for the Civil PE and Structural SE Exams

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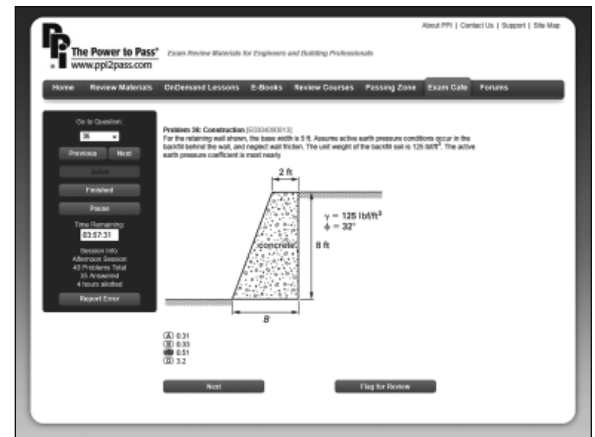


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# Structural Depth Reference Manual for the Civil PE Exam

Third Edition

## Comprehensive Coverage of the Topics on the Civil PE Exam's Structural Depth Section

The *Structural Depth Reference Manual* prepares you for the structural depth section of the Civil PE exam. This book provides a concise but comprehensive review of the structural depth section exam topics. It highlights the most useful equations in the exam-adopted codes and standards, and provides guidelines for selecting and applying these equations. More than 90 example problems demonstrate the application of concepts and equations. The end-of-chapter problems provide ample opportunity to practice solving exam-like problems, and step-by-step solutions allow you to check your solution approach. For structural steel design problems, both ASD and LRFD solution methods are shown.

Quick access to supportive information is critical to exam success. This book's thorough index directs you to the codes and concepts you will need during the exam. Throughout the book, cross references connect concepts and point you to additional relevant tables, figures, equations, and codes.

### Referenced Codes and Standards

*Building Code Requirements for Structural Concrete* (ACI 318)

*Building Code Requirements for Masonry Structures* (ACI 530)

*International Building Code without Supplements* (IBC)

*Minimum Design Loads for Buildings and Other Structures* (ASCE 7)

*National Design Specification for Wood Construction, with Commentary and Supplement* (NDS)

*Steel Construction Manual*

### Topics Covered

- Design of Reinforced Masonry
- Foundations
- Reinforced Concrete Design
- Design of Wood Structures
- Prestressed Concrete Design
- Structural Steel Design

### About the Author

Alan Williams, PhD, SE, FICE, C Eng, is a registered professional engineer and structural engineer in California, a Fellow and Life Member of the Institution of Civil Engineers, a Chartered Engineer in the United Kingdom, and a member of the Structural Engineers Association of Southern California. He obtained his bachelor of science degree and doctorate from University of Leeds in England. Dr. Williams has extensive experience teaching and practicing structural engineering, including designing and constructing bridges, schools, and industrial and commercial structures. He has worked as a senior engineer with the California Department of Transportation and as a Principal for Structural Safety with the California Division of the State Architect. Dr. Williams has published several textbooks and papers on structural engineering design, structural analysis, seismic design, and reinforced and prestressed concrete design.

### Also Available for Civil PE Exam Candidates

*Six-Minute Solutions for Civil PE Exam Structural Problems*

*Civil PE Sample Examination*

*Structural Engineering Solved Problems*

*Steel Design for the Civil PE and Structural SE Exams*

*Concrete Design for the Civil and Structural PE Exams*

*Timber Design for the Civil and Structural PE Exams*

*Bridge Design for the Civil and Structural PE Exams*

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